

# Exam Summer Semester 2023

## Student Group

| First Name | Surname | Matrikel Nr. |
|------------|---------|--------------|
|            |         |              |
|            |         |              |
|            |         |              |

## Table of Contents

|                                                                                                                              |    |
|------------------------------------------------------------------------------------------------------------------------------|----|
| <b>Exam Summer Semester 2023</b>                                                                                             | 3  |
| Additional permitted Aids                                                                                                    | 3  |
| Hits                                                                                                                         | 3  |
| Tasks                                                                                                                        | 3  |
| Exercise E1 Resistivity and temperature dependent Resistance (written test, approx. 7 % of a 60-minute written test, SS2023) | 3  |
| Exercise E2 Resistivity and temperature dependent Resistance (written test, approx. 7 % of a 60-minute written test, SS2023) | 4  |
| Exercise E3 Analyzing a Scope Plot (written test, approx. 12 % of a 60-minute written test, SS2023)                          | 6  |
| Exercise E4 Complex voltage dividers (written test, approx. 16 % of a 60-minute written test, SS2023)                        | 6  |
| Exercise E5 Pure Resistor Network Simplification (written test, approx. 12 % of a 60-minute written test, SS2023)            | 7  |
| Exercise E6 Pure Resistor Network Simplification I (written test, approx. 14 % of a 60-minute written test, SS2023)          | 9  |
| Exercise E7 Equivalent Linear Source (written test, approx. 10 % of a 60-minute written test, SS2023)                        | 12 |
| Exercise E8 (Dis)Charging Capacities (written test, approx. 14 % of a 60-minute written test, SS2023)                        | 15 |
| Exercise E9 Impedances at Frequencies (written test, approx. 14 % of a 60-minute written test, SS2023)                       | 16 |
| Exercise E10 Efficiency (written test, approx. 14 % of a 60-minute written test, SS2023)                                     | 17 |
| Exercise E11 Analyzing a Scope Plot (written test, approx. 12 % of a 60-minute written test, SS2023)                         | 18 |
| Exercise E12 Complex voltage dividers (written test, approx. 16 % of a 60-minute written test, SS2023)                       | 19 |
| Exercise E13 Pure Resistor Network Simplification (written test, approx. 12 % of a 60-minute written test, SS2023)           | 20 |

|                                                                                                                            |    |
|----------------------------------------------------------------------------------------------------------------------------|----|
| Exercise E14 Pure Resistor Network Simplification I (written test, approx. 14 % of a 60-minute written test, SS2023) ..... | 22 |
| Exercise E15 Equivalent Linear Source (written test, approx. 10 % of a 60-minute written test, SS2023) .....               | 25 |
| Exercise E16 (Dis)Charging Capacities (written test, approx. 14 % of a 60-minute written test, SS2023) .....               | 28 |
| Exercise E17 Impedances at Frequencies (written test, approx. 14 % of a 60-minute written test, SS2023) .....              | 29 |
| Exercise E18 Efficiency (written test, approx. 14 % of a 60-minute written test, SS2023) .....                             | 30 |

# Exam Summer Semester 2023

## Additional permitted Aids

- non-programmable calculator,
- formulary (2 DIN A4 pages)

## Hits

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

## Tasks

### Exercise E1 Resistivity and temperature dependent Resistance (written test, approx. 7 % of a 60-minute written test, SS2023)

The resistivity of a dielectric material is temperature dependent. It is described by the Arrhenius law in an exponential form. In this case, a best fit would be a B25 = 15000 \$ for between the 20 °C and 100 °C.

Solution  
The resistivity of the dielectric material is  $\rho_{PP}(20 \text{ } ^\circ\text{C}) = 10^{17} \text{ } \Omega\text{m}$ .

For the given material the temperature coefficients in the range of 20 °C and 55 °C are given as  $\alpha = -0.048 \text{ } 1/\text{K}$  and  $\beta = +0.00057 \text{ } 1/\text{K}^2$ .

$$\begin{aligned} R(55 \text{ } ^\circ\text{C}) &= R(20 \text{ } ^\circ\text{C}) \cdot (1 + \alpha \cdot \Delta T + \beta \cdot T^2 + \dots) \\ &= 80 \text{ } \Omega \cdot (1 - 0.048 \text{ } 1/\text{K} \cdot (35 \text{ } \text{K}) + 0.00057 \text{ } 1/\text{K}^2 \cdot \Delta T^2) \end{aligned}$$

Calculate the resistance for the dielectric material for  $20 \text{ }^\circ\text{C}$ .

Solution

$$\begin{aligned} R(20 \text{ }^\circ\text{C}) &= \rho \cdot \frac{d}{A} \\ &= 10^{17} \text{ } \Omega \cdot \frac{0.8 \cdot 10^{-6} \text{ m}}{1 \text{ m}^2} \end{aligned}$$

### Exercise E2 Resistivity and temperature dependent Resistance (written test, approx. 7 % of a 60-minute written test, SS2023)

The resistivity of a dielectric material is given by the Arrhenius equation:  $\rho = \rho_0 \cdot \exp\left(\frac{E_a}{k_B T}\right)$ . The activation energy  $E_a$  is  $0.5 \text{ eV}$ . Calculate the resistance  $R$  of a dielectric material with a thickness  $d = 1 \text{ } \mu\text{m}$  and a cross-sectional area  $A = 1 \text{ cm}^2$  at  $T = 300 \text{ K}$  and  $T = 400 \text{ K}$ .

Solution: The plates of \$d=0\$ and \$100\text{ }\mu\text{m}\$ (rm °C\$)

The resistivity of the dielectric material is \$10^{-6}\text{ }\Omega\text{m}\$ (rm °C\$) and \$10^{17}\text{ }\Omega\text{m}\$ (rm °C\$).

For the given material the temperature coefficients in the range of \$20\text{ }\sim\text{ }\text{ }^\circ\text{C}\$ and \$55\text{ }\sim\text{ }\text{ }^\circ\text{C}\$ are given as

\$\alpha = 48(20\text{ }\sim\text{ }\text{ }^\circ\text{C})\$ and \$\beta = 10(55\text{ }\sim\text{ }\text{ }^\circ\text{C})\$.

\$\rho(T) = \rho\_0 \cdot (1 - \alpha \cdot (T - T\_0) + \beta \cdot (T - T\_0)^2)\$  
 \$\rho\_0 = 10^{-6}\text{ }\Omega\text{m}\$, \$T\_0 = 20\text{ }\text{ }^\circ\text{C}\$, \$\alpha = 48\text{ }\text{ }^\circ\text{C}^{-1}\$, \$\beta = 10\text{ }\text{ }^\circ\text{C}^{-2}\$  
 \$\rho(35\text{ }\text{ }^\circ\text{C}) = 10^{-6} \cdot (1 - 48 \cdot (35 - 20) + 10 \cdot (35 - 20)^2)\$  
 \$= 10^{-6} \cdot (1 - 720 + 2250) = 10^{-6} \cdot 1531 = 1.531 \cdot 10^{-3}\text{ }\Omega\text{m}\$

Calculate the resistance for the dielectric material for \$20\text{ }\sim\text{ }\text{ }^\circ\text{C}\$.

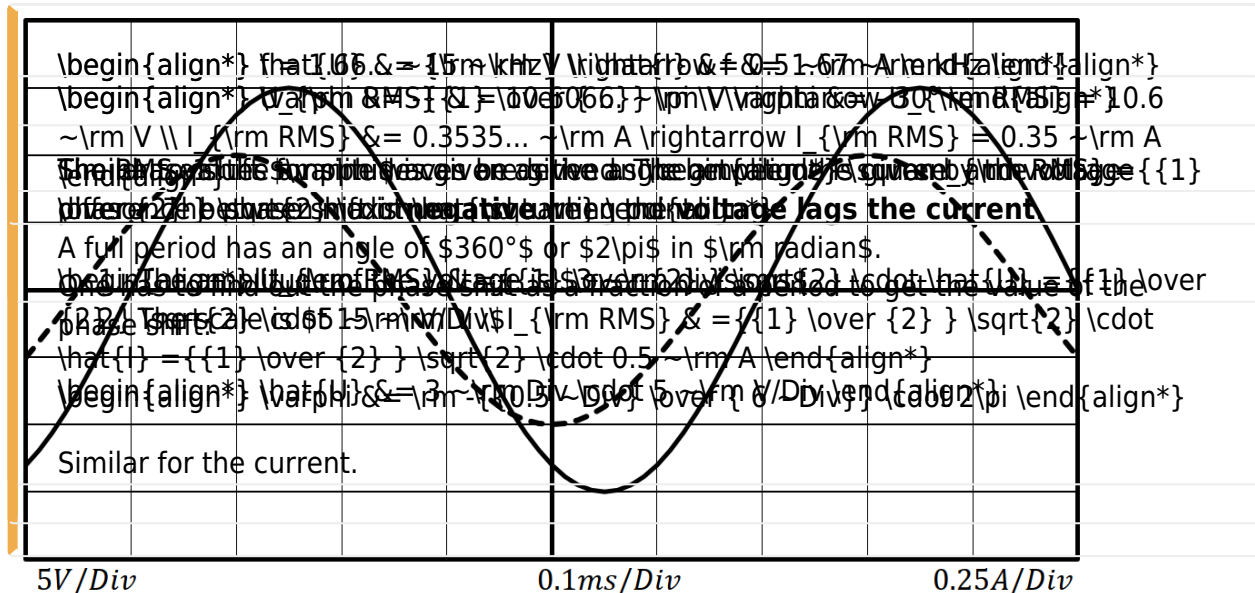
Solution

\$R(20\text{ }\sim\text{ }\text{ }^\circ\text{C}) = \rho \cdot \frac{d}{A} = 10^{-6}\text{ }\Omega\text{m} \cdot \frac{0.8 \cdot 10^{-6}\text{ m}}{1\text{ }\text{ m}^2} = 8 \cdot 10^{-13}\text{ }\Omega\$

### Exercise E3 Analyzing a Scope Plot (written test, approx. 12 % of a 60-minute written test, SS2023)

3. What is the phase difference between the voltage and current (in degrees and degree)?  
Result: The measured current curve shall be visible as a dashed line.

**The continuous line shows the voltage.**  
Solution



Use the correct symbols and units in your answers!

1. Calculate the frequency  $f$  of the periodic signals.

Solution

Frequency  $f$  is given by the period  $T$ . The period can be measured in the image of the scope.

1. The sine waves repeat after  $6 \text{ divisions}$  (e.g. from falling turning point to falling turning point of one curve)
2. The scale is  $0.1 \text{ ms/Div}$

$$f = \frac{1}{T} \quad T = 6 \text{ Div} \cdot 0.1 \text{ ms/Div} \\ \Rightarrow f = \frac{1}{6 \text{ Div} \cdot 0.1 \text{ ms/Div}}$$

### Exercise E4 Complex voltage dividers (written test, approx. 16 % of a 60-minute written test, SS2023)

Calculate the two impedances  $Z_L$  and  $Z_C$  and the resulting phase shift  $\varphi$  and the total impedance  $Z$ . Choose an appropriate scaling factor and write it down.

- $R = 1.1 \text{ k}\Omega$

Solution  $L = 3.5 \text{ mH}$

Result  $\underline{U} = 0.5 \text{ V} \cdot e^{j(\omega t - \varphi)}$  with  $\varphi = 3.14 \text{ rad}$

$$\underline{U} = 0.5 \text{ V} - j \cdot 1.5 \text{ V}$$

At the cutoff frequency the absolute values of impedances  $|Z_L|$  is equal to  $|Z_C|$ . This leads to  $\omega L = \frac{1}{\omega C}$  with  $\omega = 2\pi \cdot f$  and  $L = 3.5 \text{ mH}$  and  $C = 10 \text{ nF}$  we get  $f = \frac{1}{2\pi \cdot L \cdot C} = \frac{1}{2\pi \cdot 3.5 \cdot 10^{-3} \cdot 10^{-8}} = 4.5 \text{ kHz}$



.. Calculate the impedance  $Z_L$ .

Solution

$$Z_L = j \cdot \omega \cdot L = j \cdot 2\pi \cdot 150 \text{ kHz} \cdot 3.5 \text{ mH}$$

### Exercise E5 Pure Resistor Network Simplification (written test, approx. 12 % of a 60-minute written test, SS2023)

Calculate the voltage  $U_K$ , when switch  $S$  is closed.

Result

The values in the circuit are

Solution

- $R_1 = 60 \text{ }\Omega$

•  $R_2 = 40 \text{ }\Omega$  •  $R_3 = 150 \text{ }\Omega$  •  $R_4 = 100 \text{ }\Omega$  •  $R_5 = 100 \text{ }\Omega$

$$R_1 = 60 \text{ }\Omega$$

The voltage divider for node  $K$  has the same proportionality as the voltage

divider for node  $A$ . Therefore, the potential of  $K$  is the same as for  $A$ .

There will be no current flow through  $R_3$ . The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node  $K$ , when switch  $S$  is open.  
It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:



Once the switch  $S$  is opened, the upper part is a parallel circuit. Therefore,  $R_{\text{eq}}$  is given as:

$$R_{\text{eq}} = (R_1 + R_2) \parallel (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (60 \, \Omega + 40 \, \Omega) + 100 \, \Omega$$

### Exercise E6 Pure Resistor Network Simplification I (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below shall be given as  $U_{\text{AB}} = 60 \, \text{V}$ . What is the value for  $I_{\text{AB}}$  in the circuit?

Solution

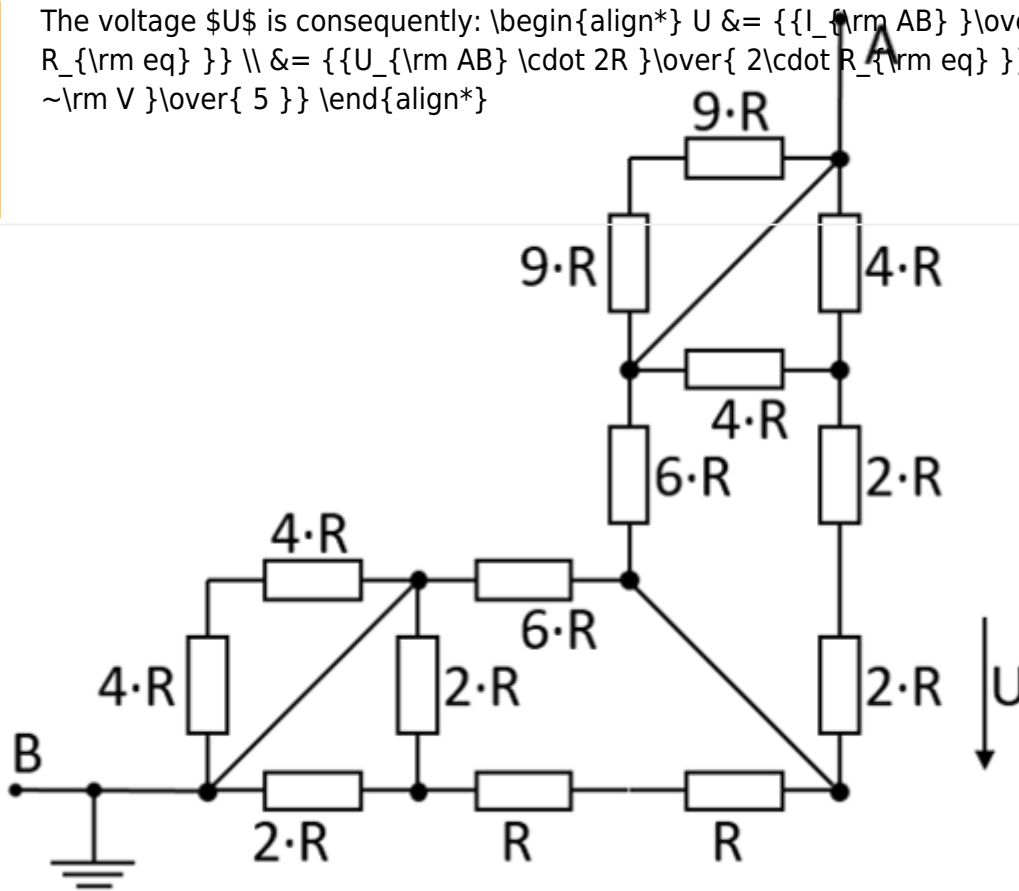
$$I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}} = \frac{60 \, \text{V}}{60 \, \Omega} = 1 \, \text{A}$$

The current through the circuit is given as  $I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}}$ .

This current has to flow in summary through parallel branches. The voltage  $U$  in question in the upper right branch given by  $(4R \parallel 4R) + 2R + 2R$ . Its resistance is just the same as the upper left branch  $6R$ .

Therefore, half of the current flows to the left half to the right side.

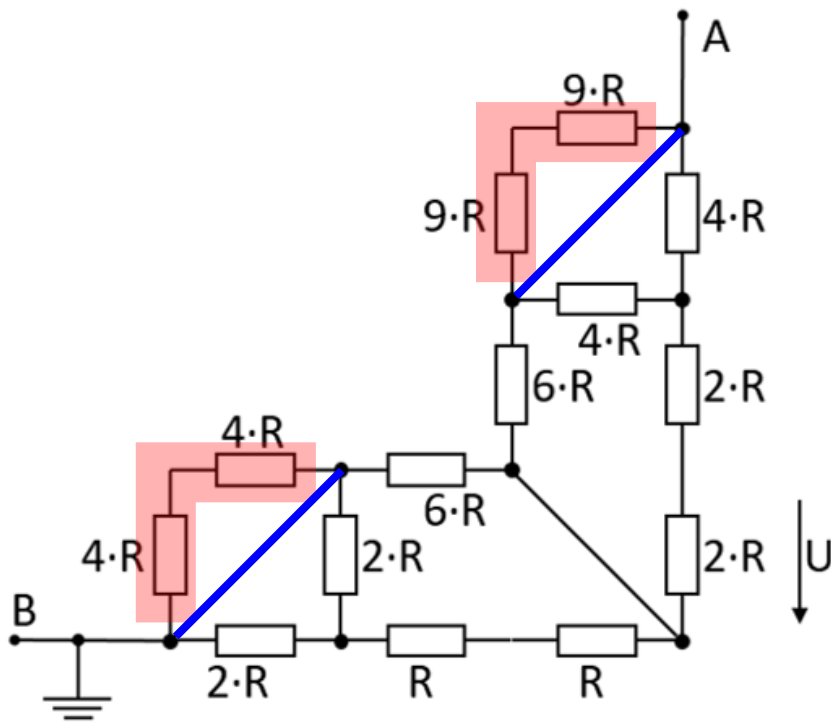
The voltage  $U$  is consequently: 
$$U = \frac{I_{\text{arm AB}}}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{U_{\text{AB}} \cdot 2R}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{60 \text{ V}}{5}$$



1. What is the equivalent resistance  $R_{\text{eq}}$ ?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

Therefore,  $R_{\text{eq}}$  is given as: 
$$R_{\text{eq}} = (2R \parallel 2R + R +$$

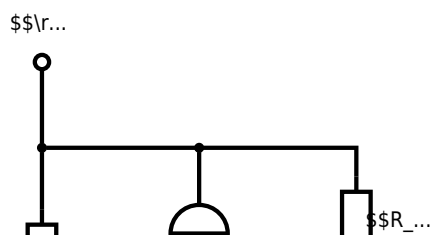
$$R) \parallel 6R \text{ \&\& } + 6R \parallel (2R + 2R + 4R \parallel 4R) \text{ \&\& } = (R + R + R) \parallel 6R \text{ \&\& } + 6R \parallel (2R + 2R + 2R) \text{ \&\& } \\ \&= 3R \parallel 6R \text{ \&\& } + 6R \parallel 6R \text{ \&\& } = \left\{ \frac{3R \cdot 6R}{3R + 6R} \right\} \text{ \&\& } + 3R \text{ \&\& } \end{align*}$$

### Exercise E7 Equivalent Linear Source (written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification.

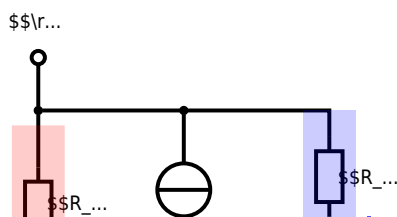
Calculate the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{s}}$  of an equivalent linear voltage source.

- $R_1 = 5 \text{ } \Omega$
  - $U_1 = 10 \text{ V}$
  - $R_2 = 5 \text{ } \Omega$
  - $I_3 = 0.5 \text{ A}$
  - $R_4 = 10 \text{ } \Omega$
  - $U_5 = 4 \text{ V}$
- $$U_{\text{AB}} = 1.11 \text{ V} \quad R_{\text{i}} = 5.55 \text{ } \Omega$$



### Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by  $I_x = \frac{U_x}{R_x}$

$$\begin{aligned} I_0 &= \frac{U_0}{R_1} = \frac{10 \text{ V}}{5 \text{ } \Omega} = 2 \text{ A} \\ I_5 &= \frac{U_5}{R_4} = \frac{4 \text{ V}}{10 \text{ } \Omega} = 0.4 \text{ A} \end{aligned}$$

$I_3$  and  $I_0$  can be combined to  $I_{03} = I_0 - I_3$  facing upwards:

$$I_{03} = 1.5 \text{ A}$$

Then, the linear current source  $I_{03}$  with  $R_1$  gets transformed into a linear voltage source with  $U_{03} = R_1 \cdot I_{03}$  facing down.

$$U_{03} = 7.5 \text{ V}$$

Then, the resistors  $R_1$  and  $R_2$  can be combined to  $R_{12} = R_1 + R_2$ .

After this, the next step is to make a linear current source out of  $U_{03}$  and  $R_{12}$ . The current will be  $I_{0123} = \frac{U_{03}}{R_{12}}$ , facing up again.

$$I_{0123} = 0.6 \text{ A}$$

The second-last step is the sum up of the current sources  $I_{0123}$  and  $I_5$  as  $I_{01235} = I_{0123} - I_5$  and the resistors as  $R_{124} = R_{12} || R_4$ .

$$I_{01235} = 0.2 \text{ A} \quad R_{124} = 5.55 \text{ } \Omega$$

The final step is the back-transformation to a linear voltage source, with  $U_{\text{AB}} = R_{124} \cdot I_{01235}$ .

The simplest and fastest (= for exams) is to work with interim results in the calculation.

Here, there is also a full final formula given:

$$\begin{aligned} U_{\text{AB}} &= U_{\text{AB}} = I_{01235} \cdot R_{124} \quad \&= (I_{0123} - I_5) \cdot (R_{12} \parallel R_4) \quad \&= \left( \frac{U_{03}}{R_{12}} - I_5 \right) \cdot (R_{12} \parallel R_4) \\ &= \left( \frac{R_1 \cdot I_{03}}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \quad \&= \left( \frac{R_1 \cdot \left( \frac{U_0}{R_1} - I_3 \right)}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \end{aligned}$$

### Exercise E8 (Dis)Charging Capacities (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit shown in the drawing consists of a DC voltage source  $U = 10 \text{ V}$ , a capacitor  $C = 200 \text{ nF}$ , and three resistors  $R_1 = 8 \text{ k}\Omega$ ,  $R_2 = 17 \text{ k}\Omega$ , and  $R_3 = 1 \text{ k}\Omega$ . The switch  $S_1$  is initially open and the capacitor is fully discharged. At  $t = 0$ , the switch  $S_1$  is closed. What is the new time constant?

- $\tau = 10 \text{ ns}$

Solution:  $\tau = 8.0 \text{ ns}$

Solution:  $\tau = 8.0 \text{ ns}$

$$\begin{aligned} U_C(t) &= U \cdot \left( 1 - e^{-t/\tau} \right) \\ U_C(0.2 \text{ ms}) &= 10 \text{ V} \cdot \left( 1 - e^{-0.2 \text{ ms} / 8.0 \text{ ns}} \right) \end{aligned}$$

Again, the time constant is given by the equivalent circuit consisting of  $C$  in parallel with  $R_1 \parallel R_2$ . Without the parallel resistors, the current source would charge the capacitor "to infinity" ( $C \rightarrow \infty$ ). This is here limited by the parallel resistors  $R_1$  and  $R_2$ . Then, the source  $U$  drives the current through the series circuit given by  $R_3$ ,  $C$ ,  $R_1$  and  $R_2$ .

$$\begin{aligned} U_C &= U \cdot \frac{R_1 \parallel R_2}{R_3 + R_1 \parallel R_2 + R_1 \parallel R_2} = 10 \text{ V} \cdot \frac{8 \text{ k}\Omega \parallel 17 \text{ k}\Omega}{1 \text{ k}\Omega + 8 \text{ k}\Omega \parallel 17 \text{ k}\Omega + 8 \text{ k}\Omega \parallel 17 \text{ k}\Omega} \\ &= 10 \text{ V} \cdot \frac{5.71 \text{ k}\Omega}{1 \text{ k}\Omega + 5.71 \text{ k}\Omega + 5.71 \text{ k}\Omega} = 2.5 \text{ V} \end{aligned}$$

This is also the maximum voltage on the capacitor, since it is in parallel with the resistors.

Before  $t = 0$  all switches are switched as shown and the capacitor is fully discharged. At  $t = 0$  the switch  $S_1$  shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

## Solution

The time constant is generally given as:  $\tau = R \cdot C$

Once  $S_1$  is closed and  $S_2$  is open at  $t_0$ , the source  $U$  drives the current through the series circuit given by  $S_1$ ,  $C$ ,  $R_1$  and  $R_3$ .

Therefore,  $R = R_1 + R_3$   $\tau_1 = (R_1 + R_3) \cdot C \quad \&= (8 \cdot 10^3 + 7 \cdot 10^3) \cdot 0.2 \cdot 10^{-6} \text{ s} = 15 \cdot 10^{-3} \cdot 0.2 \cdot 10^{-6} \text{ s} = 3 \cdot 10^{-6} \text{ s}$

⚡...

## Solution

Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ( $\propto (1 - e^{-x})$ ).

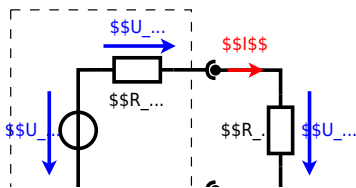
### Exercise E9 Impedances at Frequencies

(written test, approx. 14 % of a 60-minute written test, SS2023)

At a high frequency with  $C = 20 \text{ nF}$  following impedances  $Z$  are given for  $U = 10 \text{ V}$  the







### Exercise E11 Analyzing a Scope Plot (written test, approx. 12 % of a 60-minute written test, SS2023)

What is the phase difference between the voltage and current?

The measured current curve shall be visible as a dashed line.

**The continuous line shows the voltage.**

**Solution**

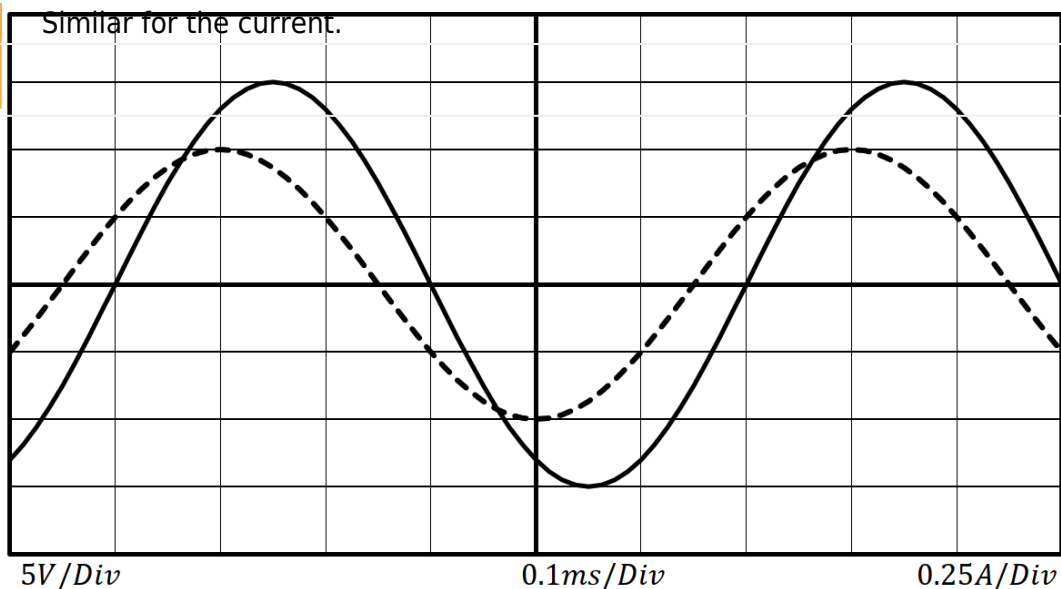
```
\begin{align*} f &= 15 \text{ kHz} \\ U_1 &= 10 \text{ V} \\ R_1 &= 10 \text{ } \Omega \\ R_2 &= 10 \text{ } \Omega \\ R_L &= 10 \text{ } \Omega \\ I_{\text{RMS}} &= 0.3535 \text{ A} \\ I_{\text{RMS}} &= 0.35 \text{ A} \end{align*}
```

The phase difference between the voltage and current is given by the RMS value of the voltage and current. The voltage is positive and the current is negative. The voltage lags the current.

A full period has an angle of  $360^\circ$  or  $2\pi$  in radians.

One period has an angle of  $360^\circ$  or  $2\pi$  in radians.

```
\begin{align*} I_{\text{RMS}} &= 0.35 \text{ A} \\ I_{\text{RMS}} &= 0.35 \text{ A} \\ I_{\text{RMS}} &= 0.35 \text{ A} \end{align*}
```



Use the correct symbols and units in your answers!

1. Calculate the frequency  $f$  of the periodic signals.

Solution

Frequency  $f$  is given by the period  $T$ . The period can be measured in the image of the scope.

1. The sine waves repeat after  $6 \sim \text{rm divisions}$  (e.g. from falling turning point to falling turning point of one curve)
2. The scale is  $0.1 \sim \text{rm ms/Div}$

$$\begin{aligned} f &= \frac{1}{T} \quad T = 6 \sim \text{rm Div} \cdot 0.1 \sim \text{ms/Div} \\ \rightarrow f &= \frac{1}{6 \sim \text{rm Div} \cdot 0.1 \sim \text{ms/Div}} \end{aligned}$$

## Exercise E12 Complex voltage dividers

(written test, approx. 16 % of a 60-minute written test, SS2023)

1. Calculate the two simplified impedances  $Z_1$  and  $Z_2$  by resulting phase for the total impedance  $Z$ . Choose an appropriate scaling factor and write it down.

- $R = 1.1 \sim \text{rm k}\Omega$

Solution  $L = 3.5 \sim \text{rm mH}$

Result  $\underline{U}_I = 5 \sim \text{rm V}$

$$\underline{U}_I = 5 \sim \text{rm V} \quad \underline{U}_O = 0.5 \sim \text{rm V} - j \cdot 1.5 \sim \text{V}$$

$$\underline{U}_O = 0.5 \sim \text{rm V} - j \cdot 1.5 \sim \text{V}$$

At cut off frequency the absolute values of impedances  $\underline{Z}_L$  is equal to  $\frac{R}{\sqrt{1 - \frac{R^2}{Z_L^2}}}$ . This leads to the complex voltage divider for cut off frequency:

$$\underline{U}_O = \underline{U}_I \cdot \frac{R}{R + j\omega L}$$


.. Calculate the impedance  $\underline{Z}_L$ .

Solution

$$\underline{Z}_L = j\omega L = j2\pi \cdot 150 \text{ kHz} \cdot 3.5 \text{ mH}$$

### Exercise E13 Pure Resistor Network Simplification (written test, approx. 12 % of a 60-minute written test, SS2023)

Calculate the voltage between  $\text{K}$ , when switch  $\text{S}$  is closed.

Result

The values in the circuit are

Solution

- $R_1 = 60 \text{ } \Omega$
- $R_2 = 40 \text{ } \Omega$
- $R_3 = 40 \text{ } \Omega$
- $R_4 = 10 \text{ } \Omega$
- The voltage divider for  $\text{K}$  has the same proportionality as the voltage divider for  $\text{A}$ . Therefore, the potential of  $\text{K}$  is the same as for  $\text{A}$ . There will be no current flow through  $R_3$ . The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node  $K$ , when switch  $S$  is open.  
It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch  $S$  is opened, the upper part is a parallel circuit. Therefore,  $R_{\text{eq}}$  is given as:

$$R_{\text{eq}} = (R_1 + R_2) \parallel (R_1 + R_2) + R_4 \parallel \frac{1}{2} \cdot (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (60 \, \Omega + 40 \, \Omega) + 100 \, \Omega$$

### Exercise E14 Pure Resistor Network Simplification I (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below should be given as  $U_{\text{AB}} = 60 \, \text{V}$ . What is the value for  $I_{\text{AB}}$  in the circuit?

Solution

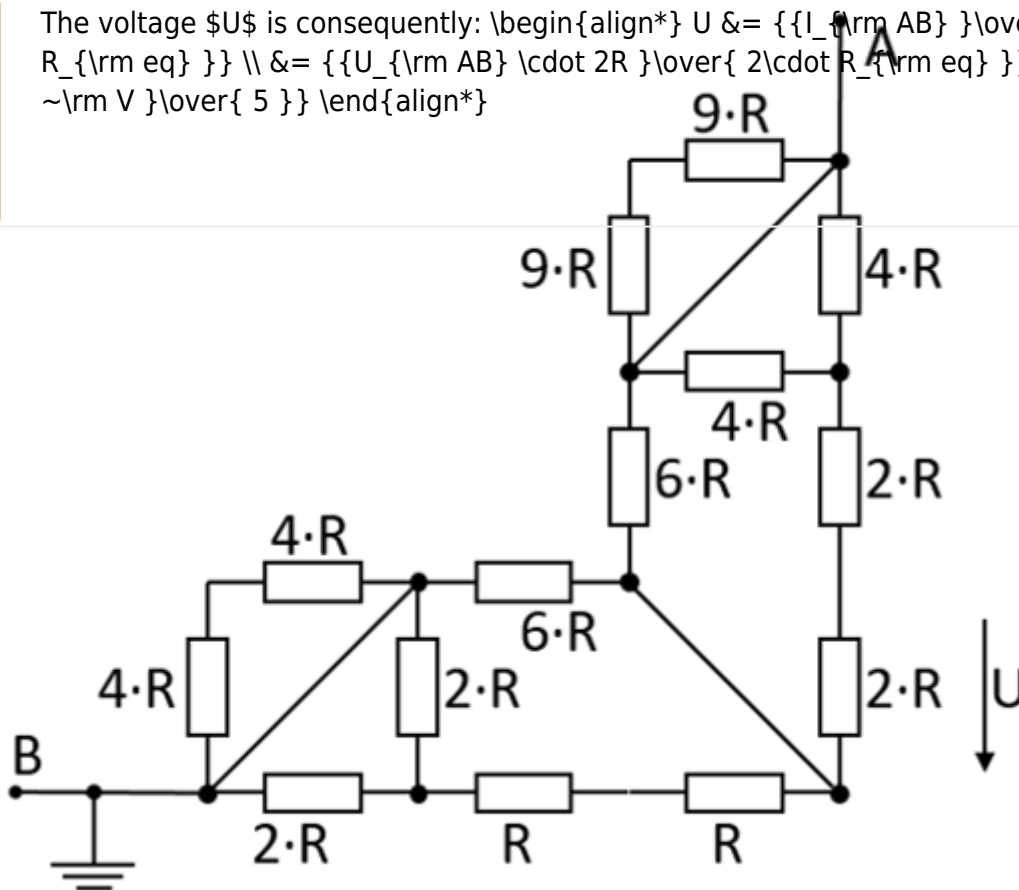
$$I_{\text{AB}} = 12 \, \text{A}$$

The current through the circuit is given as  $I_{\text{AB}} = U_{\text{AB}} \cdot R_{\text{eq}}$ .

This current has to flow in summary through parallel branches. The voltage  $U$  in question in the upper right branch given by  $(4R \parallel 4R) + 2R + 2R$ . Its resistance is just the same as the upper left branch  $6R$ .

Therefore, half of the current flows to the left half to the right side.

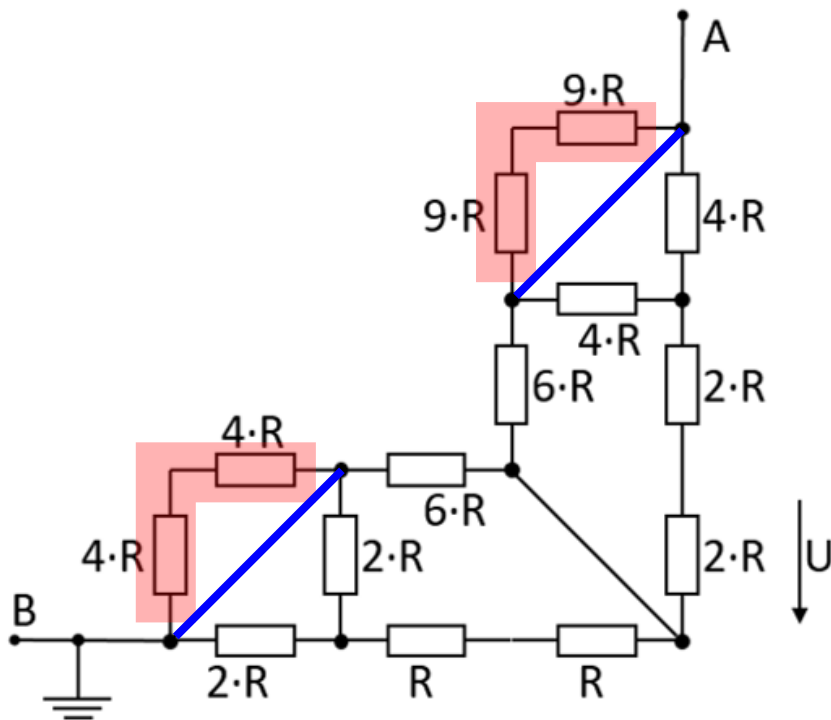
The voltage  $U$  is consequently: 
$$U = \frac{I_{\text{arm AB}}}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{U_{\text{AB}} \cdot 2R}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{60 \text{ V}}{5}$$



1. What is the equivalent resistance  $R_{\text{eq}}$ ?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

Therefore,  $R_{\text{eq}}$  is given as: 
$$R_{\text{eq}} = (2R \parallel 2R + R +$$



$$R) \parallel 6R \text{ \&\& } + 6R \parallel (2R + 2R + 4R \parallel 4R) \text{ \&\& } = (R + R + R) \parallel 6R \text{ \&\& } + 6R \parallel (2R + 2R + 2R) \text{ \&\& } \\ \&= 3R \parallel 6R \text{ \&\& } + 6R \parallel 6R \text{ \&\& } = \left\{ \frac{3R \cdot 6R}{3R + 6R} \right\} \text{ \&\& } + 3R \text{ \&\& } \end{align*}$$

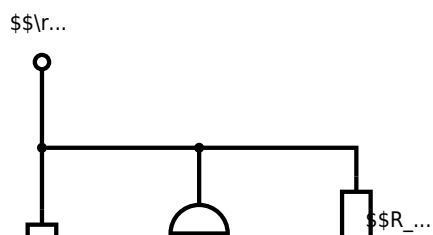
### Exercise E15 Equivalent Linear Source

(written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification.

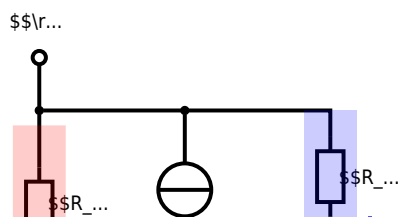
Calculate the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{s}}$  of an equivalent linear voltage source.

- $R_1 = 5 \text{ } \Omega$
  - $U_1 = 10 \text{ V}$
  - $R_2 = 5 \text{ } \Omega$
  - $I_3 = 0.5 \text{ A}$
  - $R_4 = 10 \text{ } \Omega$
  - $U_5 = 4 \text{ V}$
- $$U_{\text{AB}} = 1.11 \text{ V} \quad R_{\text{i}} = 5.55 \text{ } \Omega$$



### Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by  $I_x = \frac{U_x}{R_x}$   

$$I_0 = \frac{U_0}{R_1} = \frac{10 \text{ V}}{5 \text{ } \Omega} = 2 \text{ A}$$

$$I_5 = \frac{U_5}{R_4} = \frac{4 \text{ V}}{10 \text{ } \Omega} = 0.4 \text{ A}$$

$I_3$  and  $I_0$  can be combined to  $I_{03} = I_0 - I_3$  facing upwards:  

$$I_{03} = 1.5 \text{ A}$$

Then, the linear current source  $I_{03}$  with  $R_1$  gets transformed into a linear voltage source with  $U_{03} = R_1 \cdot I_{03}$  facing down.  

$$U_{03} = 7.5 \text{ V}$$

Then, the resistors  $R_1$  and  $R_2$  can be combined to  $R_{12} = R_1 + R_2$ .

After this, the next step is to make a linear current source out of  $U_{03}$  and  $R_{12}$ . The current will be  $I_{0123} = \frac{U_{03}}{R_{12}}$ , facing up again.  

$$I_{0123} = 0.6 \text{ A}$$

The second-last step is the sum up of the current sources  $I_{0123}$  and  $I_5$  as  $I_{01235} = I_{0123} - I_5$  and the resistors as  $R_{124} = R_{12} || R_4$ .  

$$I_{01235} = 0.2 \text{ A} \quad R_{124} = 5.55 \text{ } \Omega$$

The final step is the back-transformation to a linear voltage source, with  $U_{\text{AB}} = R_{124} \cdot I_{01235}$ .

The simplest and fastest (= for exams) is to work with interim results in the calculation.

Here, there is also a full final formula given:

$$\begin{aligned} U_{\text{AB}} &= U_{\text{AB}} = I_{01235} \cdot R_{124} \quad \&= (I_{0123} - I_5) \cdot (R_{12} \parallel R_4) \quad \&= \left( \frac{U_{03}}{R_{12}} - I_5 \right) \cdot (R_{12} \parallel R_4) \\ &= \left( \frac{R_1 \cdot I_{03}}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \quad \&= \left( \frac{R_1 \cdot \left( \frac{U_0}{R_1} - I_3 \right)}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \end{aligned}$$

### Exercise E16 (Dis)Charging Capacities (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit shown in the drawing consists of a DC voltage source  $U = 10 \text{ V}$ , a capacitor  $C = 200 \text{ nF}$ , and three resistors  $R_1 = 8 \text{ k}\Omega$ ,  $R_2 = 17 \text{ k}\Omega$ , and  $R_3 = 50 \text{ k}\Omega$ . The switch  $S_1$  is initially open and the capacitor is fully discharged. At  $t = 0$ , the switch  $S_1$  is closed. What is the new time constant?

- $C = 200 \text{ nF}$

Solution:  $R_1 = 8 \text{ k}\Omega$

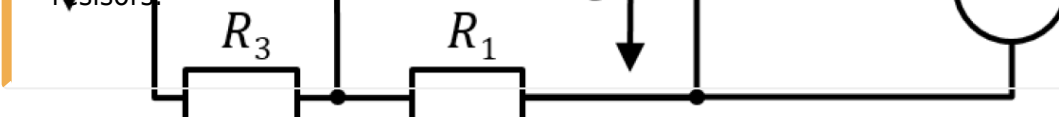
Solution:  $\tau = 8 \text{ k}\Omega \cdot 200 \text{ nF} = 1.6 \text{ ms}$

$$\begin{aligned} U_C(t) &= U \cdot \left( 1 - e^{-t/\tau} \right) \\ U_C(1.6 \text{ ms}) &= 10 \text{ V} \cdot \left( 1 - e^{-1.6 \text{ ms} / 1.6 \text{ ms}} \right) = 10 \text{ V} \cdot (1 - e^{-1}) \approx 6.32 \text{ V} \end{aligned}$$

Again, the time constant is given by the equivalent circuit consisting of  $C$  in parallel with  $R_1 \parallel R_2$ . Without the parallel resistors, the current source would charge the capacitor "to infinity" ( $C \rightarrow \infty$ ). This is here limited by the parallel resistors  $R_1$  and  $R_2$ . When  $S_2$  is closed, then, the source  $U$  drives the current through the series circuit given by  $U$ ,  $C$ ,  $R_1$  and  $R_2$ .

$$\begin{aligned} \text{There, the voltage on the branch with the capacitor is } U_C &= (8 \text{ k}\Omega + 17 \text{ k}\Omega) \cdot 0.2 \text{ mA} = 25 \text{ k}\Omega \cdot 0.2 \text{ mA} \\ U_C &= 5 \text{ V} \end{aligned}$$

This is also the maximum voltage on the capacitor, since it is in parallel with the resistors.



Before  $t_0$  all switches are switched as shown and the capacitor is fully discharged. At  $t_0 = 0 \text{ s}$  the switch  $S_1$  shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

## Solution

The time constant is generally given as:  $\tau = R \cdot C$

Once  $S_1$  is closed and  $S_2$  is open at  $t_0$ , the source  $U$  drives the current through the series circuit given by  $S_1$ ,  $C$ ,  $R_1$  and  $R_3$ .

Therefore,  $R = R_1 + R_3$   $\tau_1 = (R_1 + R_3) \cdot C \quad \&= (8 \cdot 10^3 + 7 \cdot 10^3) \cdot 0.2 \cdot 10^{-6} \text{ s} = 15 \cdot 10^{-3} \cdot 0.2 \cdot 10^{-6} \text{ s} = 3 \cdot 10^{-6} \text{ s}$

⚡...

## Solution

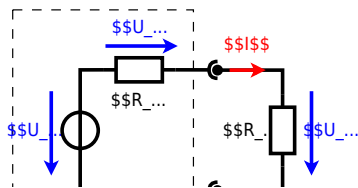
Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ( $1 - e^{-x}$ ).

**Exercise E17 Impedances at Frequencies**

**(written test, approx. 14 % of a 60-minute written test, SS2023)**

At a high frequency with  $S_2$  open and  $S_1$  closed, the voltage across  $C$  is  $U_C = U \cdot \frac{R_3}{R_1 + R_3}$ . At a low frequency with  $S_2$  open and  $S_1$  closed, the voltage across  $C$  is  $U_C = U \cdot \frac{R_3}{R_1 + R_3}$ .





From:

<https://first.mexle.te.hs-heilbronn.de/> - MEXLE Wiki

Permanent link:

[https://first.mexle.te.hs-heilbronn.de/electrical\\_engineering\\_1/ss2023\\_exam](https://first.mexle.te.hs-heilbronn.de/electrical_engineering_1/ss2023_exam)

Last update: **2023/10/05 08:58**

