task_yh4srwxu1bo1rdy4_with_calculation

Student Group

First Name	Surname	Matrikel Nr.

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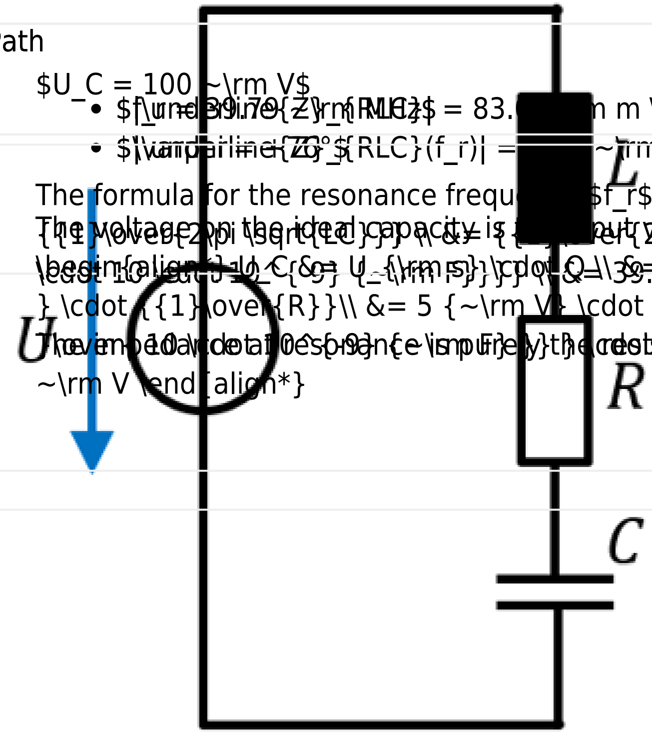
resonance, impedance, resonant circuit, exam ee2 SS2024

Exercise E10 Magnetic Circuit
(written test, approx. 10 % of a 120-minute written test, SS2024)

2. A real capacitor ... the components RLC ... resonance ... equivalent series ... Result ... in the shown circuit?



Path

$U_C = 100 ~\rm V$

- $f_r = 39.79 ~\rm MHz$ / $|\underline{Z}_{RLC}|$ = 83.0 m \Omega$
- $\varphi = 0°$ / $|\underline{Z}_{RLC}(f_r)|$ = ~\rm m \Omega$

The formula for the resonance frequency f_r is: \begin{align*} f_r &= {{1}\over{2\pi \sqrt{LC}}} \\ &= {{1}\over{2\pi \sqrt{1.6 \cdot 10^{-9} {~\rm H} \cdot 10 \cdot 10^{-9} {~\rm F}}}} \\ &= 39.78... ~\rm MHz \\ \end{align*}

The voltage on the ideal capacity is the input voltage by the Q-factor increased. \begin{align*} U_C &= U_{\rm s} \cdot Q \\ &= U_{\rm s} \cdot \sqrt{{L}\over{C}} \cdot {{1}\over{R}}\\ &= 5 {~\rm V} \cdot \sqrt{ {{ 1.6 \cdot 10^{-9} {~\rm H} }\over{10 \cdot 10^{-9} {~\rm F}}} } \cdot {{1}\over{0.020~\Omega}}\\ &= 100 ~\rm V \end{align*}

The impedance at resonance is purely the resistance.

A given capacitor shall have the following values:

- $C=10 ~\rm nF$
- $R=20 ~\rm m\Omega$
- $L=1.6 ~\rm nH$

1. What is the impedance Z_{RLC} of this real capacitor for $f_0=44 ~\rm MHz$? (Phase and magnitude)

Path

The impedance is based on the resistance R and the reactance $X_{LC}= {\rm j}\cdot (X_L - X_C)$: \begin{align*} \underline{Z}_{RLC} &= R + {\rm j}\cdot (X_L - X_C) \\ &= R + {\rm j}\cdot (\omega L - {{1}\over{\omega C}}) \\ &= R + {\rm j}\cdot (2\pi f \cdot L - {{1}\over{2\pi f \cdot C}}) \\ \end{align*}

The reactive part is \begin{align*} X_{LC} &= 2\pi f \cdot L - {{1}\over{2\pi f \cdot C}

C}} \\ &= 2\pi 44 \cdot 10^{6} {~\rm MHz} \cdot 1.6 \cdot 10^{-9} {~\rm H} - {{1}\over{2\pi \cdot 10^{6} {~\rm MHz} \cdot 10 \cdot 10^{-9} {~\rm F}}} \\ &= +0.08062... ~\Omega \\ \end{align*}

To get the magnitude of the impedance $|\underline{Z}_{RLC}|$ one can use the Pythagorean Theorem: \begin{align*} |\underline{Z}_{RLC}| &= \sqrt{R^2 + X_{LC}^2} \\ &= \sqrt{(0.020~\Omega)^2 + (0.08062... ~\Omega)^2} \\ &= 0.0830 ... ~\Omega \\ \end{align*}

For the phase φ the $\arctan$ can be applied: \begin{align*} \varphi &= \arctan \left({{X_{LC}}\over{R}} \right) \\ &= \arctan \left({{0.08062... ~\Omega}\over{0.020 ~\Omega}} \right) \\ &= 1.3276 ... \hat{=} +76° \\ \end{align*}