Block 20 — Inductance and Energy

Student Group

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20.0 Intro

20.0.1 Learning objectives

> After this 90-minute block, you can
>
> - …

20.0.2 Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

20.0.3 90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

20.0.4 Conceptual overview

> 1. …

20.1 Core content

20.1.1 Self-Induction

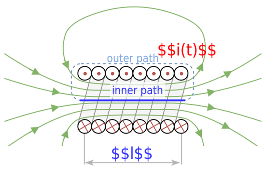
Up to now, we investigated the induction of electric voltages and currents based on the change of an external flux ${\rm d}\Psi / {\rm d}t$. For the induced current $i_{\rm ind}$, we found that it counteracts the change of the external flux (Lenz law).

But what happens, when there is no external field - only a coil which creates the flux change itself (see figure 1)?

Fig. 1: Induction Phenomenons

To understand this, we will investigate the situation for a long coil (figure 2).

Fig. 2: Self-Induction of a Coil



Given by the Recap of the fieldline images in Block16, we know that the H-field is given by magnetic voltage $\theta(t) = N \cdot i$ as: \begin{align*} {H}(t) &= {{N \cdot i }\over {l}} \\ \end{align*}

This lead to the magnetic flux density $B(t)$

\begin{align*} {B}(t) &= \mu_0 \mu_{\rm r} \cdot {{N \cdot i }\over {l}} \\ \end{align*}

Based on the magnetic flux density $B(t)$ it is possible to calculate the flux $\Phi(t)$:

\begin{align*} \Phi(t) &= \iint_A \vec{B}(t) \cdot {\rm d}\vec{A} \\ &= \iint_A \mu_0 \mu_{\rm r} \cdot {{N \cdot i }\over {l}} \cdot {\rm d}A \\ &= \mu_0 \mu_{\rm r} \cdot {{N \cdot i }\over {l}} \cdot A \\ \end{align*}

The changing flux Φ is now creating an induced electric voltage and current, which counteracts the initial change of the current.
This effect is called **Self Induction**. The induced electric voltage $u_{\rm ind}$ is given by:

\begin{align*} u_{\rm ind} &= - N \cdot {{{\rm d} \Phi(t)}\over{{\rm d}t}} \\ &= - N \cdot {{{\rm d} (\mu_0 \mu_{\rm r} \cdot {{N \cdot i }\over {l}} \cdot A)}\over{{\rm d}t}} \\ &= - N \cdot \mu_0 \mu_{\rm r} \cdot {{N \cdot A }\over {l}} \cdot {{{\rm d}i}\over{{\rm d}t}} \\ \end{align*}

\begin{align*} \boxed{ u_{\rm ind} = - \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{A }\over {l}} \cdot

{{{\rm d}i}\over{{\rm d}t}} \\ } \\ \text{for a long coil} \end{align*}

The result means that the induced electric voltage $u_{\rm ind}$ is proportional to the change of the current ${{\rm d}\over{{\rm d}t}}i$.
The proportionality factor is also called **Self-inductance** L (or often simply called inductance).

20.1.2 Inductance

The inductance is another passive basic component of the electric circuit. Besides the ohmic resistor R and the capacitor C, the inductor L is the lump component entailing the inductance.

Generally, the inductance is defined by: \begin{align*} \boxed{ L = \left|{{u_{\rm ind}}\over{{\rm d}i / {\rm d}t}}\right| \\ } \end{align*}

The inductance L can also be described differently based on Lenz law $u_{\rm ind} = - {{\rm d}\over{{\rm d}t}}\Psi(t)$:

\begin{align*} L &= \left|{{u_{\rm ind}}\over{{\rm d}i / {\rm d}t}}\right| \\ &= {{{d \Psi(t)}/{dt}}\over{{\rm d}i / {\rm d}t}} \\ \end{align*}

\begin{align*} \boxed{ L = {{ \Psi(t)}\over{i}} } \end{align*}

One can also consider an inductor a “conservative person”: it does not like to see abrupt changes in the passing current. It reacts to any change in the current with a counteracting voltage since the current change leads to a changing flux and - therefore - an induced voltage. The figure 3 shows an inductor in series with a resistor and a switch (any real switch also behaves as a capacitor, when open). Once the simulation is started, the inductor directly counteracts the current, which is why the current only slowly increases.

The unit of the inductance is $\rm 1 ~Henry = 1 ~H = 1 {{Vs}\over{A}} = 1{{Wb}\over{A}} $

Fig. 3: Example of a Circuit with an Inductor

Mathematically the voltages can be described in the following way:

\begin{align*} u_0 &= u_R &+ &u_L \\ &= i \cdot R & + & {{{\rm d}\Psi}\over{{\rm d}t}} \\ &= i \cdot R & + &L \cdot {{{\rm d}i}\over{{\rm d}t}} \\ \end{align*}

20.1.3 Inductance of different Components

Long Coil

In the last sub-chapter, the formula of a long coil was already investigated. By these, the inductance of a long coil is

\begin{align*} \boxed{L_{\rm long \; coil} = \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{A }\over {l}}} \end{align*}

Toroidal Coil

The toroidal coil was analyzed in the last chapter(see magnetic Field Strength Part 1: Toroidal Coil). Here, a rectangular intersection a assumed (see figure 4).

Fig. 4: Self-Induction of a toroidal Coil

This leads to

\begin{align*} H(t) = {{N \cdot i}\over {l}} \end{align*}

with the mean magnetic path length (= length of the average field line) $l = \pi(r_{\rm o} + r_{\rm i})$:

\begin{align*} H(t) = {{N \cdot i}\over { \pi(r_{\rm o} + r_{\rm i})}} \end{align*}

The inductance L can be calculated by

\begin{align*} L_{\rm toroidal \; coil} &= {{ \Psi(t)}\over{i}} \\ &= {{ N \cdot \Phi(t)}\over{i}} \\ \end{align*}

With the magnetic flux density $B(t) = \mu_0 \mu_{\rm r} H(t) = \mu_0 \mu_{\rm r} {{i \cdot N }\over {l}}$ and the cross section $A = h (r_{\rm o} - r_{\rm i})$, we get:

\begin{align*} \quad \quad L_{\rm toroidal \; coil} &= {{ N \cdot \mu_0 \mu_{\rm r} {{i \cdot N } \over { \pi(r_{\rm o} + r_{\rm i})}} \cdot h(r_{\rm o} - r_{\rm i})}\over{i}} \\ &= {{ N^2 \cdot \mu_0 \mu_{\rm r} \cdot h(r_{\rm o} - r_{\rm i})}\over{ \pi(r_{\rm o} + r_{\rm i})}} \\ \end{align*}

\begin{align*} \boxed{ L_{\rm toroidal \; coil} = \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{ h(r_{\rm o} - r_{\rm i})}\over{ \pi(r_{\rm o} + r_{\rm i})}} } \end{align*}

20.1.4 Inductances in Circuits

Focus here: uncoupled inductors!

Series Circuits

Based on $L = {{ \Psi(t)}\over{i}}$ and Kirchhoff's mesh law ($i=\rm const$) the series circuit of inductions can be interpreted as a single current i which generates multiple linked fluxes Ψ. Since the current must stay constant in the series circuit, the following applies for the equivalent inductor of a series connection of single ones:

\begin{align*} L_{\rm eq} &= {{\sum_i \Psi_i}\over{I}} = \sum_i L_i \end{align*}

A similar result can be derived from the induced voltage $u_{ind}= L {{{\rm d}i}\over{{\rm d}t}}$, when taking the situation of a series circuit (i.e. $i_1 = i_2 = i_1 = ... = i_{\rm eq}$ and $u_{\rm eq}= u_1 + u_2 + ...$):

\begin{align*} & u_{\rm eq} & = &u_1 & + &u_2 &+ ... \\ & L_{\rm eq} {{{\rm d}i_{\rm eq} }\over{{\rm d}t}} & = &L_{1} {{{\rm d}i_{1} }\over{{\rm d}t}} & + &L_{2} {{di_{2} }\over{dt}} &+ ... \\ & L_{\rm eq} {{{\rm d}i }\over{{\rm d}t}} & = &L_{1} {{{\rm d}i }\over{{\rm d}t}} & + &L_{2} {{di }\over{dt}} &+ ... \\ & L_{\rm eq} & = &L_{1} & + &L_{2} &+ ... \\ \end{align*}

Parallel Circuits

For parallel circuits, one can also start with the principles based on Kirchhoff's mesh law:

\begin{align*} u_{\rm eq}= u_1 = u_2 = ... \\ \end{align*}

and Kirchhoff's nodal law:

\begin{align*} i_{\rm eq}= i_1 + i_2 + ... \\ \end{align*}

Here, the formula for the induced voltage has to be rearranged:

\begin{align*} u_{\rm ind} &= L {{{\rm d}i}\over{{\rm d}t}} \quad \quad \quad \quad \bigg| \int(){\rm d}t \\ \int u_{\rm ind} {\rm d}t &= L \cdot i \\ i &= {{1}\over{L}} \cdot \int u_{\rm ind} {\rm d}t \\ \end{align*}

By this, we get:

\begin{align*} i_{\rm eq} &=& i_1 &+& i_2 &+& ... \\ {{1}\over{L_{\rm eq}}} \cdot \int u_{\rm eq} {\rm d}t &=& {{1}\over{L_1}} \cdot \int u_{1} {\rm d}t &+& {{1}\over{L_2}} \cdot \int u_{2} {\rm d}t &+& ... \\ {{1}\over{L_{\rm eq}}} \cdot \int u {\rm d}t &=& {{1}\over{L_1}} \cdot \int u {\rm d}t &+& {{1}\over{L_2}} \cdot \int u {\rm d}t &+& ... \\ {{1}\over{L_{\rm eq}}} &=& {{1}\over{L_1}} &+& {{1}\over{L_2}} &+& ... \\ \end{align*}

Notice:

The inductor behaves in the parallel and series circuit similar to the resistor.

20.1.5 Energy of the magnetic Field

not covered

20.2 Common pitfalls

- …

20.3 Exercises

Exercise E16 Self-Induction
(written test, approx. 8 % of a 120-minute written test, SS2024)

A coil with a length of $20 ~\rm cm$ and a radius of $2 ~\rm cm$ has 500 turns.
The current through the coil changes linearly from 0 to $3 ~\rm A$ in $0.02 ~\rm ms$.
The arrangement is located in air ($\mu_{\rm r}=1$).

1. Calculate the (self-)inductance of the coil.

Path

The formula for the induction of a long coil is: \begin{align*} L &= \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{A}\over{l}} \\ &= 4\pi \cdot 10^{-7} {~\rm Vs/Am} \cdot (500)^2 \cdot {{\pi \cdot (2\cdot 10^{-2} ~\rm m)^2}\over{ 2 \cdot 10^{-2} ~\rm m}} \\ \end{align*}

$\mu_0= 4\pi \cdot 10^{-7} ~\rm Vs/Am$

Result

$ L = 1.32 ~\rm mH$

2. Determine the induced voltage in the coil during the change in current.

Path

For the linear change of the current the formula of the induced voltage can also be linearized: \begin{align*} u_{\rm ind} &= - L \cdot {{ {\rm d} i }\over{ {\rm d} t }} \\ &\rightarrow - L \cdot {{ {\Delta} i }\over{ {\Delta} t }} \\ &= - 1.32 \cdot 10^{-3} \cdot {{3 A}\over{0.02 \cdot 10^{-3} s}} \end{align*}

Result

$ u_{\rm ind} = -197 ~\rm V$

Exercise E12 Self Induction
(written test, approx. 8 % of a 120-minute written test, SS2022)

Accidentally, a motor with an inductance of $L=50 ~\rm H$ is connected to a DC voltage source, which is fused with a circuit breaker.

2. What is the maximum magnitude of the voltage, which the circuit breaker has to withstand?

Result

Sketch the diagram $i(t)$ and $u_{\rm ind}(t)$ with the calculated value of the induced voltage $u_{\rm ind}(t) = 3150 ~\rm V$

Circuit breaker opens the circuit at $t_0=0$ with a current of $I=63 ~\rm A$. The current drops linearly down to $0 ~\rm A$ within $1 ~\rm \mu s$.

The inner resistance of the motor shall be neglected.)

Path

1. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

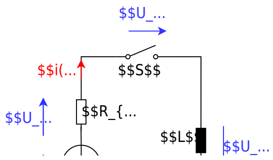
Result

- external voltage of the voltage source $U_\rm s$
- voltage $u_{\rm ind} (t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

The induced voltage can be calculated by linearizing the following: \begin{align*} u_{\rm ind} (t) &= - L {{ {\rm d} i}\over{ {\rm d} t }}\\ \rightarrow u_{\rm ind} (t) &= - L {{ \Delta i}\over{ \Delta t }}\\ \end{align*}

With the given details: \begin{align*} u_{\rm ind} (t) &= - L {{ 0 - I}\over{ t_1 - t_0 }}\\ &= 50 \cdot 10^{-6} {~\rm H} {{ 63 ~\rm A}\over{ 1 \cdot 10^{-6} ~\rm s}}\\ &= 3150 {{ ~\rm Vs}\over{ ~\rm A}} \; {{ ~\rm A}\over{ ~\rm s}}\\ \end{align*}



Exercise 4.5.1 Self Induction I

Calculate the inductance for the following settings ... 4. Same coil geometry as in 1, but with an iron core ($\mu_{\rm r}=1000$)

Result

1. Cylindrical long air coil with $N=390$, winding diameter $d=3.0 ~\rm cm$ and length $l=18 ~\rm cm$

Solution

Solution

Solution

\begin{align*} L_1 &= 0.75 ~\rm mH \end{align*}

\begin{align*} L_2 &= \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{A }\over {l}} \\ &= ... \end{align*}

\begin{align*} L_4 &= \mu_0 \cdot 1000 \cdot N^2 \cdot {{A}\over {l}} \\ &= 1000 \cdot L_1 \\ \end{align*}

\begin{align*} L_1 &= \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{A }\over {l}} \\ &= 4\pi

\cdot 10^{-7} {\rm {{H}\over{m}}} \cdot 1 \cdot (390)^2 \cdot {{\pi \cdot ({{0.03~\rm m}\over{2}})^2 }\over {0.18 ~\rm m}} \end{align*}

Exercise 4.5.2 Self Induction II

A cylindrical air coil (length $l=40 ~\rm cm$, radius $r=5.0 ~\rm cm$, and a number of windings $N=300$) passes a current of $30 ~\rm A$. The current shall be reduced linearly in $2.0 ~\rm ms$ down to $0.0 ~\rm A$.

Result

What is the amount of the induced voltage $u_{\rm ind}$?
\begin{align*} \left|u_{\rm ind}\right| &= 33 ~\rm V\end{align*}

Solution

The requested induced voltage can be derived by:

\begin{align*} L &= \left|{{u_{\rm ind}}\over{{\rm d}i / {\rm d}t}}\right| \\ \rightarrow \left|u_{\rm ind}\right| &= L \cdot \left|{{{\rm d}i}\over{{\rm d}t}}\right| \\ &= L \cdot \left|{{\Delta i}\over{\Delta t}}\right| \\ \end{align*}

Therefore, we just need the inductance L, since ${{\Delta i}\over{\Delta t}}$ is defined as $30 ~\rm A$ per $2 ~\rm ms$:

\begin{align*} L &= \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{A }\over {l}} \\ \end{align*}

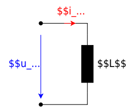
So, the result can be derived as: \begin{align*} \left|u_{\rm ind}\right| &= \mu_0 \mu_{\rm r} \cdot N^2 \cdot {{A }\over {l}} \cdot \left|{{\Delta i}\over{\Delta t}}\right| \\ &= 4\pi \cdot 10^{-7} {\rm {{H}\over{m}}} \cdot 1 \cdot (300)^2 \cdot {{\pi \cdot (0.05~\rm m)^2 }\over {0.40 ~\rm m}} \cdot {{30 ~\rm A}\over{2 ~\rm ms}} \end{align*}

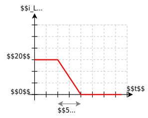
Exercise 4.5.3 Self Induction III

A coil with the inductance $L=20 ~\rm µH$ passes a current of $40 ~\rm A$. The current shall be reduced linearly in $5 ~\rm µs$ down to $0 ~\rm A$ (see figure 5).

- What is the amount of the induced voltage $u_{\rm ind}$?
- Sketch the course of $u_{\rm ind}(t)$!

Fig. 5: Circuit and timing Diagram





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Embedded resources

Explanation (video): …