dummy

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Exercise E1 Machine-Vision Strobe: Capacitor Charging and Safe Discharge

A. ... charged ... capacitor ... 1 ... / must ... safely discharged before maintenance.

Solution

Solution

Data: \begin{align*} C &= 1~{\rm \mu F} \\ W &= 0.1~{\rm J} \\ I_{\rm max} &= 100~{\rm mA} \end{align*}

Assume the DC charging source is adjusted to the required final capacitor voltage.

\begin{align*} I_{\rm max} &= 4.72~{\rm k\Omega} ... \end{align*}

What voltage must be present across the capacitor so that it stores the required energy?

with

\end{align*} A practical standard value would be about $4.7~{\rm k\Omega}$.

Energy capacitor ... fully charged after about 5τ.

\begin{align*} U_0 &= 447.2~{\rm V} \\ RC &= 4.47~{\rm ms} \end{align*}

with

\begin{align*} W(t) &= \frac{1}{2} C u_C^2(t) \end{align*}

Initial and final values:

\begin{align*} t_{\rm charge} &\approx 5\tau \approx 5 \cdot 4.47~{\rm ms} \approx 22.4~{\rm ms} \end{align*}

\begin{align*} \tau &= RC \approx 4.47~{\rm ms} \end{align*}

\begin{align*} u_C(0) &= 0 & u_C(\infty) &= U_0 \\ u_R(0) &= U_0 & u_R(\infty) &= 0 \end{align*}

\begin{align*} W &= \frac{1}{2} C U^2 \\ U &= \sqrt{\frac{2W}{C}} = \sqrt{\frac{2 \cdot 0.1~{\rm J}}{1~{\rm \mu F}}} = \sqrt{200000}~{\rm V} \approx 447.2~{\rm V} \end{align*}

Thus, the capacitor voltage rises exponentially, while the resistor voltage falls exponentially.

The energy stored initially in the capacitor is

\begin{align*} \frac{1}{2} &= e^{-2t/(R_i C)} \\ t &= \frac{R_i C}{2}\ln(2) \end{align*}

\begin{align*} W_0 &= 0.1~{\rm J} \end{align*}

With

After full discharge, the capacitor energy is zero, so the entire initial energy is converted into heat in the resistor:

\begin{align*} R_i C &= 10~{\rm M\Omega} \cdot 1~{\rm \mu F} = 10~{\rm s} \end{align*}

\begin{align*} W_R &= W_0 = 0.1~{\rm J} \end{align*}

we get

In a real design, the resistor must therefore be checked for pulse-load capability.

\begin{align*} t &= \frac{10~{\rm s}}{2}\ln(2) \approx 3.47~{\rm s} \end{align*}

The voltage at this instant is

\begin{align*} u_C &= U_0 \cdot \frac{1}{\sqrt{2}} = \frac{447.2~{\rm V}}{\sqrt{2}} \approx 316.2~{\rm V} \end{align*}

Exercise E2 Industrial Sensor Interface: Buffered Measurement Node

... network. The capacitor at the output node is used to smooth the signal and to provide a stable voltage for a short measurement cycle. At first, the measurement electronics are disconnected. Once the capacitor is fully charged, a switch closes and the measurement load is connected.

Result

Solution

Solution

\begin{align*} U_C ... \end{align*}

Data: $U = 12~\text{V}$, $R_1 = 2~\text{k}\Omega$, $R_2 = 10~\text{k}\Omega$, $R_3 = 3~\text{k}\Omega$, $R_L = 5~\text{k}\Omega$

Required: load voltage

$$R_{\text{th}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 = \frac{2~\text{k}\Omega \cdot 10~\text{k}\Omega}{12~\text{k}\Omega} + 3~\text{k}\Omega \approx 4.67~\text{k}\Omega$$

$$U_L = U \cdot \frac{R_L}{R_{\text{th}} + R_L} = 12~\text{V} \cdot \frac{5}{9.67} \approx 6.21~\text{V}$$

With $R_1 = 2~\text{k}\Omega$, $R_2 = 10~\text{k}\Omega$, $R_3 = 3~\text{k}\Omega$, $R_L = 5~\text{k}\Omega$, $U = 12~\text{V}$ and $C = 5~\mu\text{F}$ the load voltage is $6.21~\text{V}$.

Data: $U = 12~\text{V}$, $R_1 = 2~\text{k}\Omega$, $R_2 = 10~\text{k}\Omega$, $R_3 = 3~\text{k}\Omega$, $R_L = 5~\text{k}\Omega$, $C = 5~\mu\text{F}$

Required: time-dependent capacitor voltage

Thus, the transient load voltage is

$$u_L(t) = 10\left(1 - e^{-t/(10~\text{ms})}\right) - 5\left(1 - e^{-t/(10~\text{ms})}\right) = 5\left(1 - e^{-t/(10~\text{ms})}\right)~\text{V}$$

It starts at $0~\text{V}$ and exponentially approaches $5~\text{V}$.

Data: $U = 12~\text{V}$, $R_1 = 2~\text{k}\Omega$, $R_2 = 10~\text{k}\Omega$, $R_3 = 3~\text{k}\Omega$, $R_L = 5~\text{k}\Omega$

Required: load voltage

The charging time constant is R_1 and R_2:

So the load voltage starts at $10~\text{V}$ and decays exponentially to $5~\text{V}$.

$$\tau_1 = \frac{R_1 R_2}{R_1 + R_2} C = \frac{20~\text{k}\Omega}{12~\text{k}\Omega} \cdot 5~\mu\text{F} = 8.33~\text{ms}$$

$$\tau_2 = (R_3 + R_L) C = 8~\text{k}\Omega \cdot 5~\mu\text{F} = 40~\text{ms}$$

$$u_L(t) = 5~\text{V} + 5~\text{V} \cdot e^{-t/(40~\text{ms})}$$

In practice, the capacitor is considered fully charged after about $5\tau_1$:

$$t_{\text{charge}} \approx 5\tau_1 = 5 \cdot 10~\text{ms} = 50~\text{ms}$$

Exercise E3 Hall-Sensor Test Bench: Air-Core Calibration Coil

A Hall-sensor test bench is to be calibrated with a magnetic field. An air-core coil is chosen because it avoids hysteresis and remanence effects of iron cores. The coil is wound as a short, single-layer cylindrical coil.

Data: $l = 22~\text{mm}$, $d = 20~\text{mm}$, $d_{\text{Cu}} = 0.8~\text{mm}$, $N = 275$, $\rho_{\text{Cu}} = 1.71 \cdot 10^{-8}~\Omega\text{m}$

Required: coil resistance R, inductance L, time constant τ when the coil is connected to a $12~\text{V}$ supply

Solution

Data: $l = 22~\text{mm}$, $d = 20~\text{mm}$, $d_{\text{Cu}} = 0.8~\text{mm}$, $N = 275$, $\rho_{\text{Cu}} = 1.71 \cdot 10^{-8}~\Omega\text{m}$

Required: coil resistance R, inductance L, time constant τ when the coil is connected to a $12~\text{V}$ supply

Solution

$$A = \frac{\pi d^2}{4} = \frac{\pi \cdot (20~\text{mm})^2}{4} \approx 314~\text{mm}^2$$

$$R = \rho \frac{N \cdot \pi d}{A_{\text{Cu}}} = 1.71 \cdot 10^{-8}~\Omega\text{m} \cdot \frac{275 \cdot \pi \cdot 20~\text{mm}}{0.5~\text{mm}^2} \approx 7.94~\Omega$$

$$L = \frac{\mu_0 N^2 A}{l} = 4\pi \cdot 10^{-7}~\frac{\text{Vs}}{\text{Am}} \cdot \frac{275^2 \cdot 314~\text{mm}^2}{22~\text{mm}} \approx 1.36~\text{mH}$$

$$\tau = \frac{L}{R} = \frac{1.36~\text{mH}}{7.94~\Omega} \approx 0.171~\text{ms}$$

$$i(t) = 1.51~\text{A} \cdot \left(1 - e^{-t/0.171~\text{ms}}\right)$$

$$i(5\tau) = 1.51~\text{A} \cdot \left(1 - e^{-5}\right) \approx 1.51~\text{A} \cdot 0.993 \approx 1.50~\text{A}$$

$$B = \mu_0 \frac{N}{l} I = 4\pi \cdot 10^{-7}~\frac{\text{Vs}}{\text{Am}} \cdot \frac{275}{22~\text{mm}} \cdot 1.5~\text{A} \approx 23.6~\text{mT}$$

$$\frac{B}{I} = \mu_0 \frac{N}{l} = 4\pi \cdot 10^{-7}~\frac{\text{Vs}}{\text{Am}} \cdot \frac{275}{22~\text{mm}} \approx 15.7~\frac{\text{mT}}{\text{A}}$$

$$\frac{B}{I} = \frac{\mu_0 N^2 A}{l^2} = \frac{L}{l^2} = \frac{1.36~\text{mH}}{(22~\text{mm})^2} \approx 2.81~\frac{\text{mT}}{\text{A}}$$

$$\frac{B}{I} = \frac{\mu_0 N^2 A}{l^2} = \frac{L}{l^2} = \frac{1.36~\text{mH}}{(22~\text{mm})^2} \approx 1.99~\frac{\text{mT}}{\text{A}}$$

First, determine the copper cross-sectional area:

$$\begin{align*} A_{\rm Cu} &= \frac{\pi}{4} d_{\rm Cu}^2 = \frac{\pi}{4}(0.8~{\rm mm})^2 \\ &= 0.503~{\rm mm^2} \end{align*}$$

The mean length of one turn is approximately the circumference:

$$\begin{align*} l_{\rm turn} &\approx \pi d = \pi \cdot 20~{\rm mm} = 62.83~{\rm mm} \end{align*}$$

Thus, the total wire length is

$$\begin{align*} l_{\rm Cu} &= N \cdot l_{\rm turn} = 25 \cdot 62.83~{\rm mm} \\ &= 1570.8~{\rm mm} = 1.571~{\rm m} \end{align*}$$

Now calculate the resistance:

$$\begin{align*} R &= \rho_{\rm Cu}\frac{l_{\rm Cu}}{A_{\rm Cu}} \\ &= 0.0178~{\rm \Omega\,mm^2/m}\cdot\frac{1.571~{\rm m}}{0.503~{\rm mm^2}} \\ &\approx 0.0556~{\rm \Omega} \end{align*}$$