task_dtoqvpvrbdtcozfk_with_calculation

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

electrostatic, field lines, exam ee2 SS2022

Exercise E1 Electrostatics I
(written test, approx. 10 % of a 120-minute written test, SS2022)

Given is the arrangement of electric charges in the picture below. The values of the point charges are:

- $q_0=-1 ~\rm nC$
- $q_1=-2 ~\rm nC$

In the beginning, the area charge is $q_4=0 ~\rm nC$.

The permittivity is $\varepsilon_0 = 8.854 \cdot 10^{-12} ~\rm As/Vm$

2. What is the magnitude of the resulting force?

Path

With all the x and y components, we can calculate the resulting magnitude of the force with the Pythagorean Theorem:

\begin{align*} F = |\vec{F}| &= \sqrt{\left(\sum_i F_{i,x} \right)^2 + \left(\sum_i F_{i,y} \right)^2} \\ &= \sqrt{\left(19.97 {~\rm \mu N} + 28.09 {~\rm \mu N} - 10.05 {~\rm \mu N} \right)^2 + \left(20.10 {~\rm \mu N} \right)^2} \\ &= 42.99... {~\rm \mu N} \\ \end{align*}

Result

3. Now the charge $q_2=0$ is set to 0. The area charge q_4 generates a homogeneous field E_4. Which value needs E_4 to have to get a resulting force of $0 ~\rm N$ on q_0?

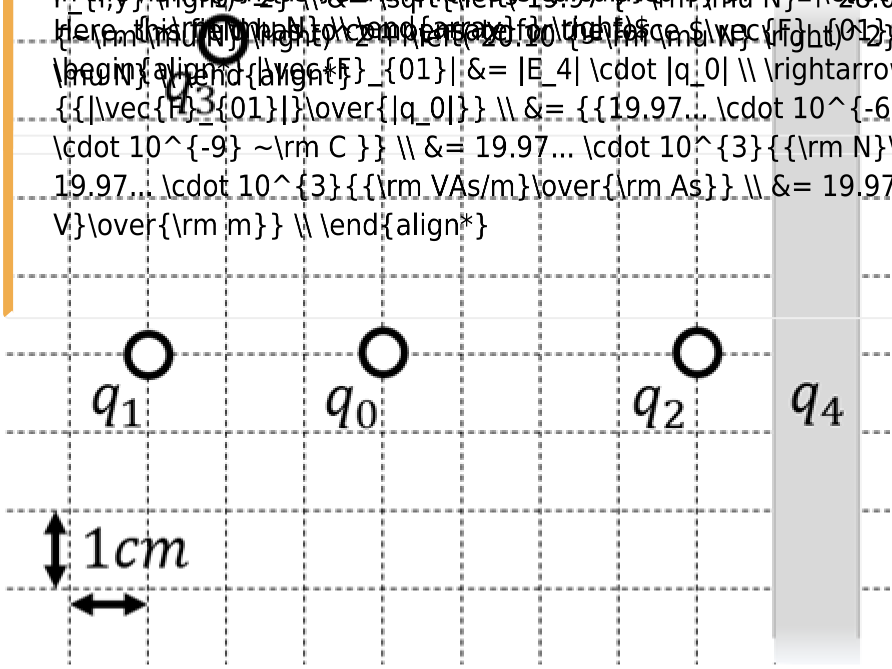
Path

In the homogeneous field the force is calculated by $F=E\cdot q$.
Here, this field has to compensate for the force $\vec{F}_{01}$ from q_1 on q_0:
\begin{align*} |\vec{F}_{01}| &= |E_4| \cdot |q_0| \\ \rightarrow E_4 &= {{|\vec{F}_{01}|}\over{|q_0|}} \\ &= {{19.97... \cdot 10^{-6} {~\rm N}}\over{1 \cdot 10^{-9} ~\rm C }} \\ &= 19.97... \cdot 10^{3}{{\rm N}\over{\rm C}} \\ &= 19.97... \cdot 10^{3}{{\rm VAs/m}\over{\rm As}} \\ &= 19.97... \cdot 10^{3}{{\rm V}\over{\rm m}} \\ \end{align*}

Result

$E_4 = 19.97 {{\rm kV}\over{\rm m}}$

- $\vec{F}_{01} = \left({\begin{array}{cccc} 19.97 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{02} = \left({\begin{array}{cccc} 28.09 {~\rm \mu N} \\ 0 \\ \end{array} } \right)$
- $\vec{F}_{03} = \left({\begin{array}{cccc} -10.05 {~\rm \mu N} \\ 20.10 {~\rm \mu N} \\ \end{array} } \right)$



1. Calculate the single forces $\vec{F}_{01}$, $\vec{F}_{02}$, $\vec{F}_{03}$, on the charge q_0!

Path

First, calculate the magnitude of the forces, like $\vec{F}_{01}$.
The force $\vec{F}_{01}$ is purely on the x-axis and therefore equal to $F_{01,x}$. \begin{align*} \vec{F}_{01} = F_{01,x} &= {{1}\over{4\pi\varepsilon}}\cdot {{q_1\cdot q_0}\over{r^2_{01}}} \\ &= {{1}\over{4\pi\cdot 8.854 \cdot 10^{-12} ~\rm As/Vm}}\cdot {{1 \cdot 10^{-9} ~\rm C \cdot 2 \cdot 10^{-9} ~\rm C}\over{(3 \cdot 10^{2} ~\rm m)^2}} \\ &= 19.97... \cdot 10^{-6} {\rm {{(As)^2 \cdot Vm}\over{As \cdot m^2}}} = 19.97...

\cdot 10^{-6} {\rm {{VAs}\over{m}}} = 19.97... \cdot 10^{-6} {\rm {{Ws}\over{m}}} \\ &= 19.97... {\rm \mu N} \quad \text{(to the right)} \end{align*}

Similarly, we get for $\vec{F}_{02}$ and $\vec{F}_{03}$ \begin{align*} \vec{F}_{02} = F_{02,x} &= -28.09... {\rm \mu N} \quad \text{(to the right)} \\ \vec{F}_{03} &= -22.47... {\rm \mu N} \quad \text{(to the top left)} \\ \end{align*}

For $\vec{F}_{03}$, we have to calculate the x- and y-component.
This is possible, by using the angle α between the line through q_0 and q_3 and the positive x-axis (pointing to the right).
So, α has to be between $90°$ and $180°$. It can be calculated by: \begin{align*} \alpha = \arctan(\rm {{-4~cm}\over{+2~cm}}) = \pi - 1.1071... = 180° - 63.4...° = 116.6...° \end{align*}

Based on this, the x- and y-component is: \begin{align*} F_{03,x} &= |\vec{F}_{03}| \cdot \cos \alpha = 10.05... {~\rm \mu N} \text{(to the left)} \\ F_{03,y} &= |\vec{F}_{03}| \cdot \sin \alpha = 20.10... {~\rm \mu N} \text{(to the top)} \\ \end{align*}