aufgabe_7.2.6_mit_rechnung

Student Group

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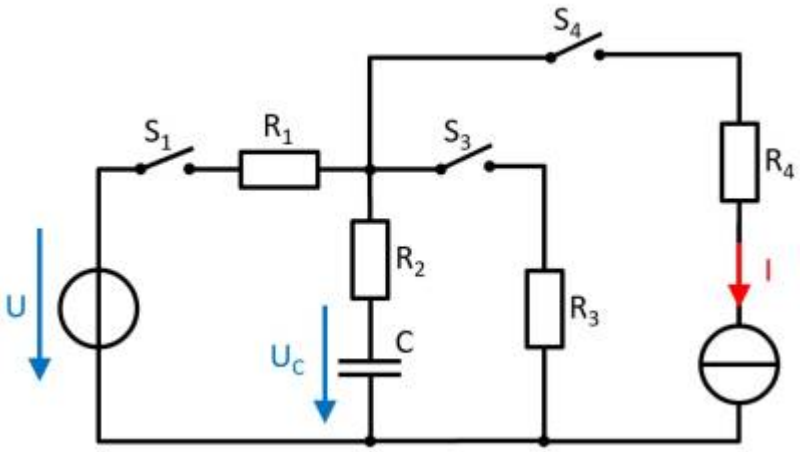

The circuit shown right is given with the following data:

- $U = 10 ~{\rm V}$
- $I = 4 ~{\rm mA}$
- $R_1 = 100 ~\Omega, R_2 = 80 ~\Omega, R_3 = 50 ~\Omega, R_4 = 10 ~\Omega$
- $C = 40 ~{\rm nF}$

At first, the voltage drop on the capacitor $u_C = 0$, and all switches are open. The switch S1 will be closed at $t = 0$.

Simulation

1. Determine the time constant τ for this charging process.

Tips

- What equivalent circuit can be found for the mentioned states of the switches?
- What parameter do you need to determine τ?
- The charging current is flown through which component?

Solution

The electrical components R_1, R_2, and C are connected in series with a source U. The time constant τ is therefore: \begin{align*} \tau &= (R_1 + R_2) \cdot C \\ \tau &= 180 ~\Omega \cdot 40 ~{\rm nF} \end{align*}

Final value

\begin{align*} \tau = 7.2 ~{\rm µs} \end{align*}

2. What is the value of the voltage $u_C(t)$ drop over the capacitor C at $t=10 ~{\rm

µs}$?

Solution

\begin{align*} U_C(t) = U \cdot (1 - e^{-t/\tau}) \\ U_C(t) = 10 ~{\rm V} \cdot (1 - e^{-10 ~{\rm µs}/7.2 ~{\rm µs}}) \end{align*}

Final value

\begin{align*} U_C(t) = 7.506 ~{\rm V} \rightarrow 7.5 ~{\rm V} \end{align*}

3. What is the value of the stored energy in the capacitor, when it is fully charged?

Solution

\begin{align*} W_C &= \frac{1}{2} C U^2 \\ &= \frac{1}{2} \cdot 40~{\rm nF} \cdot (10~{\rm V})^2 \end{align*}

Final value

\begin{align*} W_C = 2 ~{\rm µJ} \end{align*}

4. Determine the new time constant when the switch S_1 will be opened and the switch S_3 will be closed simultaneously.

Solution

The capacitor C discharges by the series connected resistors R_2 und R_3.
\begin{align*} \tau &= (R_2 + R_3) \cdot C \\ &= 130 ~\Omega \cdot 40 ~{\rm nF} \end{align*}

Final value

\begin{align*} \tau = 5.2 ~{\rm µs} \end{align*}

5. When the capacitor is completely discharged, all switches will be opened. The switch S_4 will be closed at $t= 0$.
What is the voltage u_C at the capacitor C after $t = 1 ~ {\rm µs}$?

Tips

- Through the current source there is a continuous flow of electric charge into the capacitor.
- The resistors passed by the current on the way to the capacitor are irrelevant. They only increase the voltage of an ideal current source to guarantee the current.

Solution

The voltage U_C is in general: $U_C = \frac{Q}{C}$. In this case, the constant current I results in $Q = \int I {\rm d}t = I \cdot t$ \begin{align*} U_C(t) &= \frac{Q}{C} \\ U_C(t) &= \frac{I \cdot t}{C} \\ U_C(1µs) &= \frac{4~{\rm mA} \cdot 1~{\rm µs}}{40~{\rm nF}} = \frac{4 \cdot 10^{-3}~{\rm A} \cdot 1\cdot 10^{-6}~{\rm s}}{40\cdot 10^{-9}~{\rm F}} \\ \end{align*}

Final value

$$\begin{align*} U_C(1~{\rm µs}) &= 1~{\rm V} \\ \end{align*}$$