6 Introduction to Alternating Current Technology

Student Group

First Name	Surname	Matrikel Nr.

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6. Introduction into Alternating Current Technology

Up to now we had analysed DC signals (chapters 1. - 4.) and abrupt voltage changes for (dis)charging capacitors (chapter 5.). In households we use instead of a constant voltage (DC) alternating voltage (AC). This is due to at least three main facts

1. Often the voltage given by the **power plant is AC**. This is true for example in all power plants which use electric generators. In these, an mechanic energy of a rotating system is transformed into electric energy by means of moving magnets, which induce an alternating electic voltage. Some modern plants, like photovoltaic plants do not primary generate AC voltages.
2. For long-range power transfer the power losses P_{loss} can be reduced by reducing the currents I since $P_{loss}=R\cdot I^2$. Therefore, for constant power transfer the voltage have to be increased. This is much easier done with AC voltages: **AC enables to transform lower voltages to higher** by the use of alternating magnetic fields in a transformer.
3. AC signals have **at least one more value** which can be used for understanding the situation of the source or load. This simplifies the power and load management in a complex power network.

This does not mean that DC power lines are useless or only full of disadvantages:

- A lot of modern loads need DC voltages, like battery based systems (laptops, electric cars, smartphones). Others can simpliy be changed into DC loads like systems with electric motors (refridgerators, oven, lighting, heating).
- Long-range power transfer with DC voltages show often much lower power losses.

Besides the applications in power systems AC values are also important in communication engineering. Acoustic and visual signals like sound and images can often be considered as wavelike AC signals. Additionally, also for signal transfer like Bluetooth, RFID, antenna design AC signals are important.

In order to understand these systems a bit more, we will start in this chapter with a first introduction into AC systems.

6.1 Description of ime-dependent Signals

Goals

After this lesson, you should:

1. know which types of time-dependent waveforms there are and be able to assign them
2. Know the relationship between amplitude and peak-to-peak value.
3. Know the relationship between period, frequency and angular frequency.
4. Know the difference between zero phase angle and phase shift angle.
5. Know the direction of the phase shift angle.
6. know the formula symbols of the above-mentioned quantities.

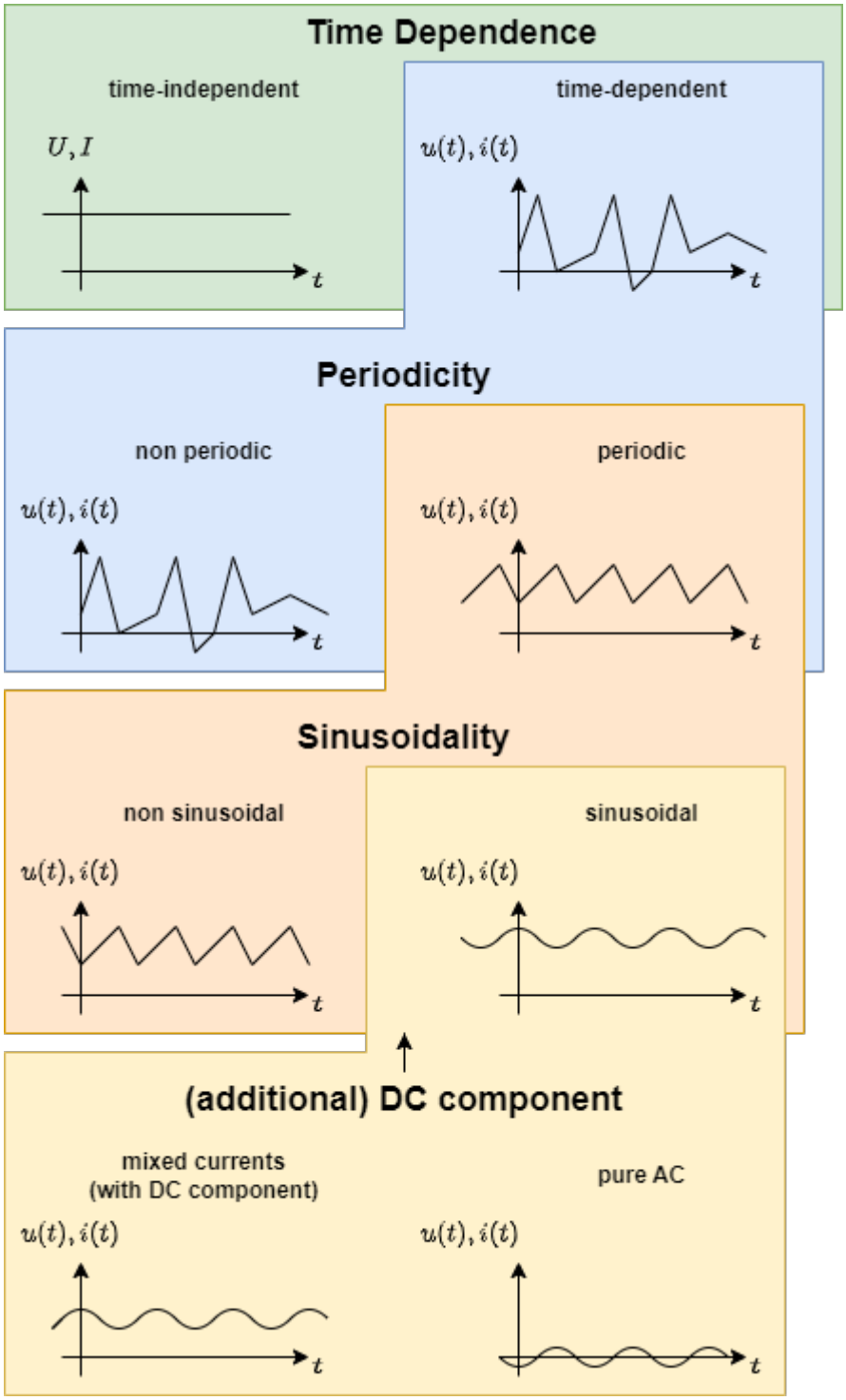
6.1.1 Description of Classification of time-dependent Signals

Voltages and currents in the following chapters will be time-dependent values. As already used in chapter 5. for the time-dependent values lowercase letters will be written.

By this time-dependent values any temporal form of the voltage / current curves are possible (see figure 1).

- We distinguish periodic and non periodic signals
- One important family of periodic signals are sinusoidal signals
- Sinusoidal signals can be mixed with DC signals

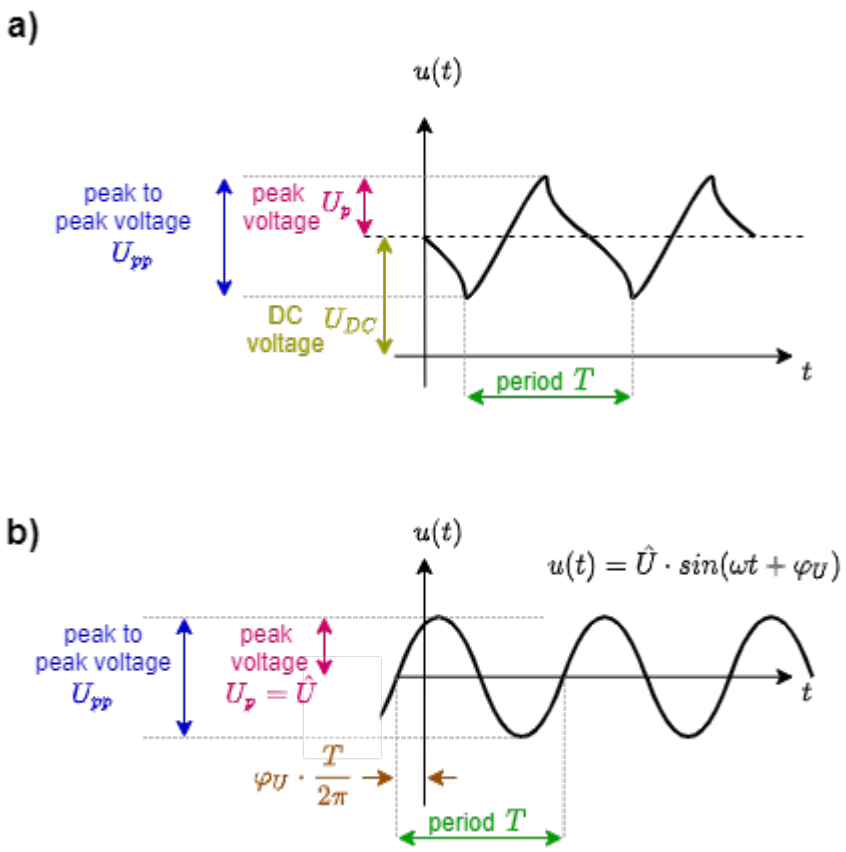
Fig. 1: Classification of time-dependent values



In the following we will investigate mainly pure AC signals.

6.1.2 Descriptive Values of AC Signals

Fig. 2: Values of AC signals



There are some important characteristic values when investigating AC signals (figure 2). For the singal itself these are:

- The **DC voltage** or DC offset is given by the value U_{DC} of V_{DC} (in German: Gleichanteil). The DC component also defines the average value of an AC signal.
- The maximum deviation from the DC value is called **peak voltage** U_p (in German : Spitzespannung). Specifically for sinusidal signals the **peak voltage** U_p is also called **amplitude** $\hat{U}$ (in German: Scheitelwert or Amplitude).
- The voltage difference between maximum and minimum deviation is called **peak-to-peak voltage** U_{pp} (in German: Spitze-Spitze-Spannung).
  Be aware, that in English texts often amplitude is also used for (non sinusidal) U_{pp} - based on German DIN standards the term amplitude is only valid for the sinusidal peak voltage.

Additionally, there are also characteristic values related to the time:

- The shortest time difference for the signal to repeat is called **period** T.
- Based on the period T the frequency $f = {{1}\over{T}}$ can be derived. The unit of the frequency is $1 Hz = 1 Hertz$.
- For calculation, often the **angular frequency** ω is used. The angular frequency is given by $\omega = {{2\pi}\over{T}}$ with the unit ${{1}\over{s}}$.
  The angular frequency represents the angle which is covered in one second.
- Another handy value is the time offset between the start of the sinus wave ($u(t)=0V$ and rising) and $t=0s$. This difference is often written based on an angular difference and is called the **phase angle** or **initial phase** φ_U (in German: Nullphasenwinkel). This then has to be calculated back to a time value: $\Delta t= {{\varphi_U}\over{\omega}}= \varphi_U\cdot{{T}\over{2\pi}}$

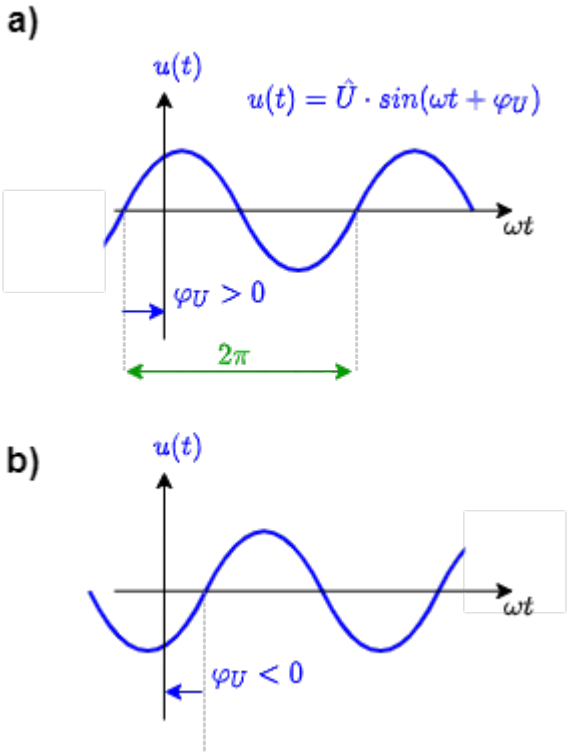
Mathematically, the AC voltages and currents can be written as: $$u(t)=\hat{U}\cdot sin(\omega t +

\varphi_U)$$ $$i(t)=\hat{I}\cdot sin(\omega t + \varphi_I)$$

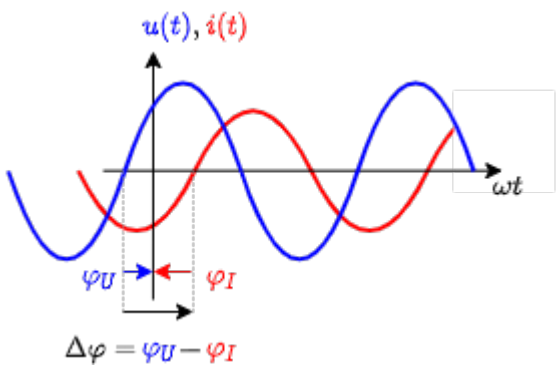
Between the AC voltages and currents there is also another important characteristic: The **phase difference** $\Delta \varphi$ is given by $\Delta \varphi = \varphi_U + \varphi_I$. The phase difference shows how far the momentary value of the current is ahead of the momentary value of the voltage.

Notice:

The initial phase φ_0 has an direction / sign which have to be considered. In the case **a)** in the picture the zero-crossing of the sinusidal signal is before $t=0$ or $\omega t =0$. Therefore, the initial phase φ_0 is positive.



Similarly also for the phase difference $\Delta \varphi$ the direction has to be taken into account. In the following image the zero-crossing of the voltage curve is before the zero-crossing of the current. This leads to a positive phase difference $\Delta \varphi$.



6.2 Averaging of AC Signals

Goals

After this lesson, you should:

1. be able to calculate the arithmetic mean, the rectified value and the rms value.
2. know these mean values for sinusoidal quantities.
3. know the reason for using the rms value.

In order to analyse AC signals more, often different types of averages are taken into account. The most important values are:

1. the arithmetic mean $\overline{X}$
2. the recified value $\overline{|X|}$
3. the rms value X

These shall be discussed in the following. The video “Alternating Current AC Basics - Part 1” of EEVblog explains the ideas behind these values alternatively to the following subchapter:

6.2.1 The Arithmetic Mean

The arithmetic mean is given by the (equally weighted) averaging of the signed measuring points. For finite values the arithmetic mean is given by: $$\overline{X}={{1}\over{n}}\cdot \sum_{i=1}^n x_i$$

For functions it is given by: $$\boxed{\overline{X}={{1}\over{T}}\cdot \int_{t=t_0}^{t_0 + T} x(t) dt}$$

For pure AC signals, the arithmetic mean $\overline{X}=0$, since the unsigned value of the integral between upper half-wave and 0 is equal to the unsigned value of the integral between lower half-wave and 0.

6.2.2 The Rectified Value

Since the arithmetic mean of pure AC signals with $\overline{X}=0$ does not really give an insight into the signal, different other (weighted) average can be used.
One of them is the rectified value. For this the signal is first recified (visually: negaive values are folded up onto the x-axis) and then averaged.
For finite values the rectified value is given by: $$\overline{|X|}={{1}\over{n}}\cdot \sum_{i=1}^n |x_i|$$

For functions it is given by: $$\boxed{\overline{|X|}={{1}\over{T}}\cdot \int_{t=t_0}^{t_0 + T} |x(t)| dt}$$

> For pure AC signals this results in:
>
> \begin{align*} \overline{|X|} &= {{1}\over{T}}\cdot \int_{t=t_0}^{t_0 + T} |\hat{X}\cdot sin(\omega t + \varphi) | dt \\ \end{align*}
>
> Without limiting the generality, we use $\varphi=0$ and $t_0 = 0$ \begin{align*} \overline{|X|} &= {{1}\over{T}}\cdot \int_{t=0 }^{T } |\hat{X}\cdot sin(\omega t) | dt \\ \end{align*}
>
> Since $sin(\omega t)\geq0$ for $t\in [0,\pi]$, the integral can changed and the absolute value bars can be excluded like the following
> \begin{align*} \overline{|X|} &= {{1}\over{T}}\cdot 2 \cdot \int_{t=0}^{T/2} \hat{X}\cdot sin({{2\pi}\over{T}} t) dt \\ &= 2 \cdot {{1}\over{T}}\cdot [-\hat{X}\cdot {{T}\over{2\pi}}\cdot cos({{2\pi}\over{T}} t)]_{t=0}^{T/2} \\ &= 2 \cdot {{1}\over{T}}\cdot {{T}\over{2\pi}}\cdot \hat{X}\cdot [-cos({{2\pi}\over{T}} t)]_{t=0}^{T/2} \\ &= {{1}\over{\pi}}\cdot \hat{X} \cdot [1+1] \\ \boxed{\overline{|X|} = {{2}\over{\pi}}\cdot \hat{X} \approx 0.6366\cdot \hat{X}}\\ \end{align*}

Exercise 6.3.1 The Rectified Value of rectangular and triangular signals

> Calculate the rectified value of rectangular and triangular signals! Use similar symmetry simplificatinos like shown for AC signals. Compare it to the values shown in figure 3.

6.2.3 The RMS Value

Often it is important be able to compare AC signals to DC signals by having equivalent values. But what does equivalent mean?
Most importantly, these “equivalent values” are used to compare the output power of a system. One of this equivalent value is supply voltage value of $230V$ (or in some countries $110V$).
How do we come to this values?

We want to find the voltage U_{DC} and I_{DC} of a DC source, that the output power P_{DC} on a resistor R is similar to the output power P_{AC} of an AC source with the instantaneous values $u(t)$ and $i(t)$. For this, we have to consider the instantaneous power $p(t)$ for a distinct time t and integrate this over one period T.

\begin{align*} P_{DC} &= P_{AC} \\ U_{DC} \cdot I_{DC} &= {{1}\over{T}} \int_{0}^{T} u(t) \cdot i(t) dt \\ R \cdot I_{DC}^2 &= {{1}\over{T}} \int_{0}^{T} R \cdot i^2(t) dt \\ I_{DC}^2 &= {{1}\over{T}} \int_{0}^{T} i^2(t) dt \\ \rightarrow I_{DC} &= \sqrt{{{1}\over{T}} \int_{0}^{T} i^2(t) dt} \end{align*}

The similar approach can be used on instantaneous voltage $u(t)$. Generally, the RMS values is given by \begin{align*} \boxed{X_{RMS} = \sqrt{{{1}\over{T}} \int_{0}^{T} x^2(t) dt}} \end{align*}

$$X_{RMS} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}$$

What is the meaning of RMS? Simple: $Root\ Mean\ Squared$

By this abbreviation, one can also not forget in which order the fromula cas to be written... Often the rms value is also called effective value (in German: Effektivwert).

Note:

- The heat dissipation on an resistor R of an AC current with the rms value of $I_{rms}=1A$ is equal to the heat dissipation of an DC current with $I_{DC}=1A$.
- To shorten writing formulas, the values of AC signals given with uppercase letters will represent the RMS value in the following: $U = U_{RMS}$, $I = I_{RMS}$.
- It holds for AC signals ant their RMS values:
  - The resistance is $R={{U}\over{I}}$
  - The power dissipation on a resistor is $P=U\cdot I$

For pure AC signals this results in:

\begin{align*} X &= \sqrt{{{1}\over{T}}\cdot \int_{t=t_0}^{t_0 + T} x^2(t) dt} \\ &= \sqrt{{{1}\over{T}}\cdot \int_{t=0} ^{T} \hat{X}^2\cdot sin^2(\omega t) dt} \\ &= \sqrt{{{1}\over{T}}\cdot \int_{t=0} ^{T} \hat{X}^2\cdot {{1}\over{2}}\cdot (1- cos(2\cdot \omega t)) dt} \\ &= \sqrt{{{1}\over{T}}\cdot \hat{X}^2\cdot {{1}\over{2}}\cdot [t + {{1}\over{2\omega }}\cdot sin(2\cdot \omega t)]_{0}^{T}} \\ &= \sqrt{{{1}\over{T}}\cdot \hat{X}^2\cdot {{1}\over{2}}\cdot (T - 0 + 0 - 0)} \\ &= \sqrt{{{1}\over{2}}\cdot \hat{X}^2} \\ \boxed{X = {{1}\over{\sqrt{2}}}\cdot \hat{X} \approx 0.707\cdot \hat{X}}\\ \end{align*}

This can also be seen in this youtube video.

<callout icon="fa fa-exclamation" color="red" title="Note:"> In the following we will often ose $\sqrt{2}X$ instead of $\hat{X}$, e.g.: $u(t)=\hat{X}\cdot sin(\omega t + \varphi_u) \rightarrow u(t)=\sqrt{2}{U}\cdot sin(\omega t + \varphi_u) $

</callout>

Exercise 6.3.2 The RMS Value of rectangular and triangular signals

Calculate the RMS value of rectangular and triangular signals! Use similar symmetry simplificatinos like shown for AC signals. Compare it to the values shown in figure 3.

6.2.4 Comparison of the different averages

Fig. 3: The averages of different signals

6.3 AC Two-Poles

Goals

After this lesson, you should:

1. know that real, lossy components are described by equivalent circuits of ideal components.
2. know and be able to apply the definition of apparent resistance, apparent conductance, impedance, and admittance.

In the chapters 2. Simple Circuits and 3 Non-ideal Sources and Two Pole Networks we already have seen, that it is possible to reduce complex circuitries down to equivalent resistors (and ideal sources). This we will try to adopt for AC components, too.

We want to analyze how relationship between the current through a component and the voltage drow on this component behaves, when an AC current is applied.

6.3.1 Resistance

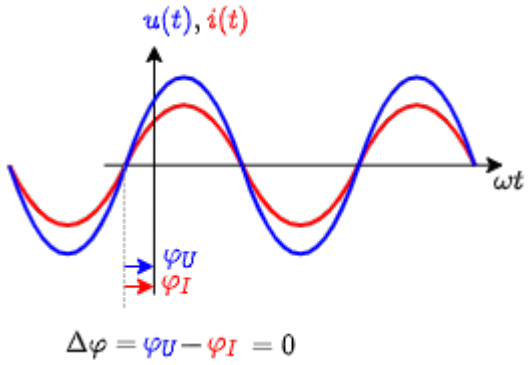
We start with Ohms law, which states, that the instantaneous voltage $u(t)$ is proportional to the instantaneous current $i(t)$ by the factor R. $$u(t) = R \cdot i(t)$$

Then we insert the functions representing the instantaneous signals: $x(t)= \sqrt{2}{X}\cdot sin(\omega t + \varphi_x)$: $$\sqrt{2}{U}\cdot sin(\omega t + \varphi_u) = R \cdot \sqrt{2}{I}\cdot sin(\omega t + \varphi_i)$$

Since we know, that $u(t)$ must be proportional to $i(t)$ we conclude that for a resistor $\varphi_u=\varphi_i$!

\begin{align*} R &= {{\sqrt{2}{I}\cdot sin(\omega t + \varphi_i)}\over{\sqrt{2}{U}\cdot sin(\omega t + \varphi_i) }} \\ &= {{U}\over{I}} \end{align*}

Fig. 4: time course of instantaneous voltage and current on a resistance



This was not too hard und quite obvious. But, what about the other types of passive two-poles - namely the capacitance and inductance?

6.3.2 Capacitance

For the capacitance we have the basis formula: $$C={{Q}\over{U}}$$ This formula ia also true for the instantaneous values: $$C={{q(t)}\over{u(t)}}$$ Additionally we know, that the instantaneous current is defined by $i(t)={{dq(t)}\over{dt}}$.

By this we can set up the formula: \begin{align*} i(t) &= {{dq(t)}\over{dt}} \\ &= {{d}\over{dt}}\left(C \cdot u(t) \right) \end{align*}

Now, we insert the functions representing the instantaneous signals and calculate the derivative: \begin{align*} \sqrt{2}{I}\cdot sin(\omega t + \varphi_i) &= {{d}\over{dt}}\left(C \cdot \sqrt{2}{U}\cdot sin(\omega t + \varphi_u) \right) \\ &= C \cdot \sqrt{2}{U}\cdot \omega \cdot cos(\omega t + \varphi_u) \\ \\ {I}\cdot sin(\omega t + \varphi_i) &= C \cdot {U}\cdot \omega \cdot sin(\omega t + \varphi_u + {{1}\over{2}}\pi) \tag{6.3.1} \end{align*}

Equating coefficients in $(6.3.1)$ leads to: \begin{align*} I &= C \cdot U \cdot \omega \\ {{U}\over{I}} &= {{1}\over{\omega \cdot C}} \end{align*} and: \begin{align*} \omega t + \varphi_i &= \omega t + \varphi_u + {{1}\over{2}}\pi \\ \varphi_i &= \varphi_u + {{1}\over{2}}\pi \\ \varphi_u -\varphi_i &= - {{1}\over{2}}\pi \end{align*}

The phase shift of $- {{1}\over{2}}\pi$ can also be seen in figure 6 and figure 5.

Notice:

In order not to mix up the definitions, for AC signals the fraction of rms voltage by rms current is called **(apparent) impedance** Z (in German: Scheinwiderstand or Impedanz).
The impedance is generally defined as $$Z = {{U}\over{I}}$$

Only for pure resistor two pole the impedance Z_R is equal to the value of the resistance: $Z_R=R$.

For the pure capacitive two pole the impedance Z_C is $Z_C={{1}\over{\omega \cdot C}}$.

Fig. 5: time course of instantaneous voltage and current on a capacitance

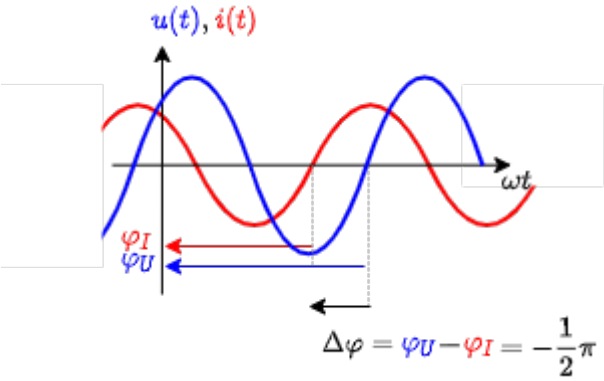


Fig. 6: time course of instantaneous voltage and current on a capacitance

6.3.3 Inductance

The inductance will here be introduced shortly - the detailed introduction is part of electrical engineering 2.
For the capacitance C we had the situation, that it reacts to a voltage change ${{d}\over{dt}}u(t)$ with a counteracting current: $$i(t)= C \cdot {{d}\over{dt}}u(t)$$ This is due to the fact, that the capacity stores charge carriers q. It appears that “the capacitance does not like voltage changes and reacts with a compensating current”. When the voltage on a capacity drops, the capacity supplies a current - when the voltage rises the capacity drains a current.

For an inductance L it is just the other way around: “the inductance does not like current changes and reacts with a compensating voltage drop”. Once the current changes the inductance will create a voltage drop that counteracts and continues the current: A current change ${{d}\over{dt}}i(t)$ lead to a voltage drop $u(t)$: $$u(t)= L \cdot {{d}\over{dt}}i(t)$$ The proportionality factor here is L, the value of the inductance, and it is measured in $[L] = 1H = 1\; Henry$.

We can now again insert the functions representing the instantaneous signals and calculate the derivative: \begin{align*} \sqrt{2}{U}\cdot sin(\omega t + \varphi_u) &= L \cdot {{d}\over{dt}}\left(\sqrt{2}{I}\cdot sin(\omega t + \varphi_i) \right) \\ &= L \cdot \sqrt{2}{I}\cdot \omega \cdot cos(\omega t + \varphi_i) \\ \\ {U}\cdot sin(\omega t + \varphi_u) &= L \cdot {I}\cdot \omega \cdot sin(\omega t + \varphi_i + {{1}\over{2}}\pi) \tag{6.3.2} \end{align*}

Equating coefficients in $(6.3.2)$ leads to: \begin{align*} U &= L \cdot {I}\cdot \omega \\ \boxed{Z_L = {{U}\over{I}} = \omega \cdot L} \end{align*} and: \begin{align*} \omega t + \varphi_u &= \omega t + \varphi_i + {{1}\over{2}}\pi \\ \varphi_u &= \varphi_i + {{1}\over{2}}\pi \\ \boxed{\varphi = \varphi_u -\varphi_i = + {{1}\over{2}}\pi } \end{align*}

The phase shift of $+ {{1}\over{2}}\pi$ can also be seen in figure 8 and figure 7.

Fig. 7: time course of instantaneous voltage and current on a inductance

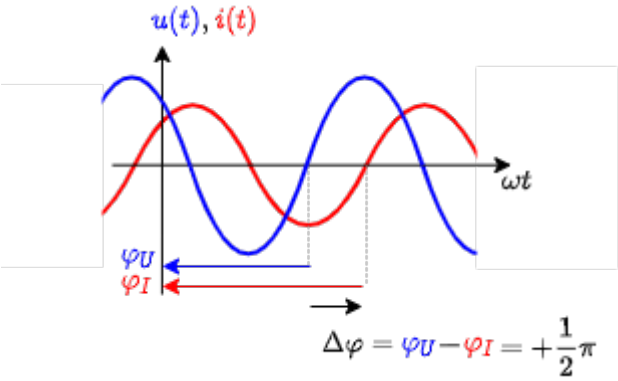


Fig. 8: time course of instantaneous voltage and current on a inductance

Notice:

Remember the formulas for the different pure loads:

Load		**impedance $Z={{U}\over{I}}$**	**phase φ**
Resistance	R	$Z_R = R $	$\varphi_R = 0$
Capacitance	C	$Z_C = {{1}\over{\omega \cdot C}}$	$\varphi_C = - {{1}\over{2}}\pi $
Impedance	L	$Z_L = \omega \cdot L $	$\varphi_L = + {{1}\over{2}}\pi $

Tab. 1: Formulas for the different pure loads

One way to memorize the phase shift is given bei the word **CIVIL**:

- **CIVIL**: for a capacitance C the current I leads the voltage V.
  Therefore the phase angle φ_I of the current is larger than the phase angle φ_U of the voltage: $\rightarrow \varphi = \varphi_U - \varphi_I < 0 $.
- **CIVIL**: for an inductance L the voltage V leads the current I.
  Therefore the phase angle φ_U of the voltge is larger than the phase angle φ_I of the current: $\rightarrow \varphi = \varphi_U - \varphi_I > 0 $.

By the concept of AC two-poles we are also able to use the DC methods of network analysis in order to solve AC networks.

6.4 Complex Values in Electical Engineering

Goals

After this lesson, you should:

1. know how sine variables can be symbolized by a vector.
2. know which parameters can determine a sinusoidal quantity.
3. be able to graphically derive a pointer diagram for several existing sine variables.
4. be able to plot the phase shift on the vector and time plots.
5. Be able to add sinusoidal quantities in vector and time representation.

6. know and be able to apply the impedance of components.
7. know the frequency dependence of the impedance of the components. In particular, you should know the effect of the ideal components at very high and very low frequencies and be able to apply it for plausibility checks.

6.4.1 Representation and Interpretation

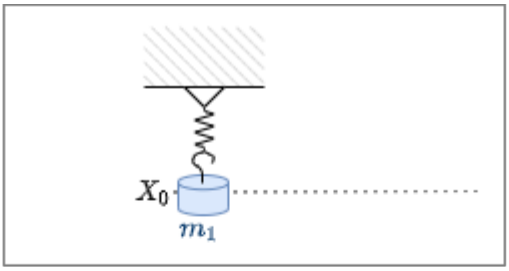
Up to now, we used for the AC signals the formula $x(t)= {{1}\over{\sqrt{2}}} X \cdot sin (\omega t + \varphi_x)$ - which was quite obvious.

However, there is an alternative way to look onto the alternating sinusidal signals. For this, we look first on a different, but already familiar problem (see figure 9).

1. A mechanical, linear spring with the characteristic constant D is displaced due a mass m in the Earth's gravitational field. The deflection only based on the gravitational field is X_0.
2. At the time $t_0=0$, we deflect this spring a bit more to $X_0 + x(t_0)=X_0 + \hat{X}$ and therefore induce energy into the system.
3. When the mass is released, the mass will spring up and down for $t>0$. The signal can be shown as shadow when the mass is illuminated sideways.
   For $t>0$, the energy is continuously shifted between poential energy (deflection $x(t)$ around X_0) and kinetic energy (${{d}\over{dt}}x(t)$)
4. When looking onto the course of time of $x(t)$, the signal will behave as: $x(t)= \hat{X} \cdot sin (\omega t + \varphi_x)$
5. The movement of the shadow can also be created by a the sideways shadow of a stick on a rotating disc.
   This means, that a two dimensional rotation is reduced down to a single dimension.

Fig. 9: interpretation of sinusidal deflection of a spring

1.



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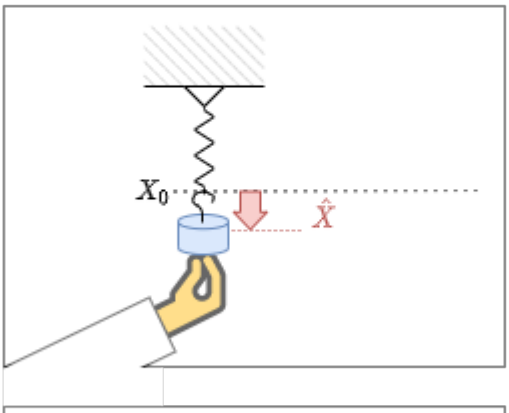

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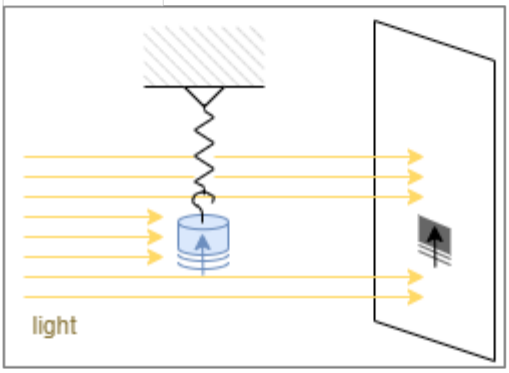

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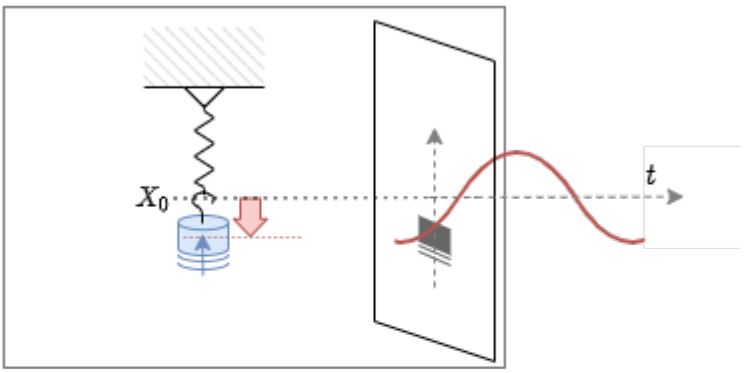
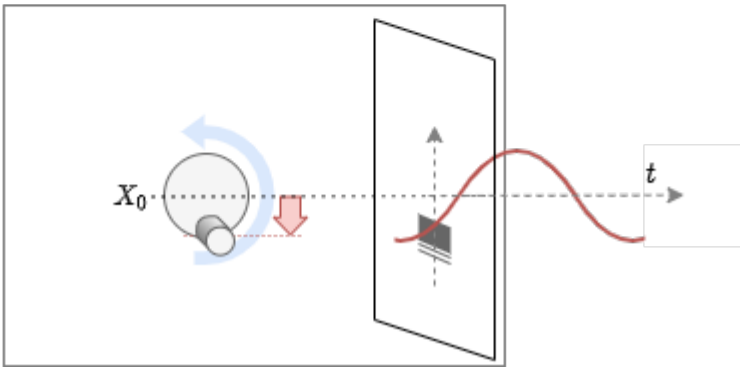


5.



The transformation of the two dimensional rotation to a one dimensional sinusidal signal is also shown

in figure 10.

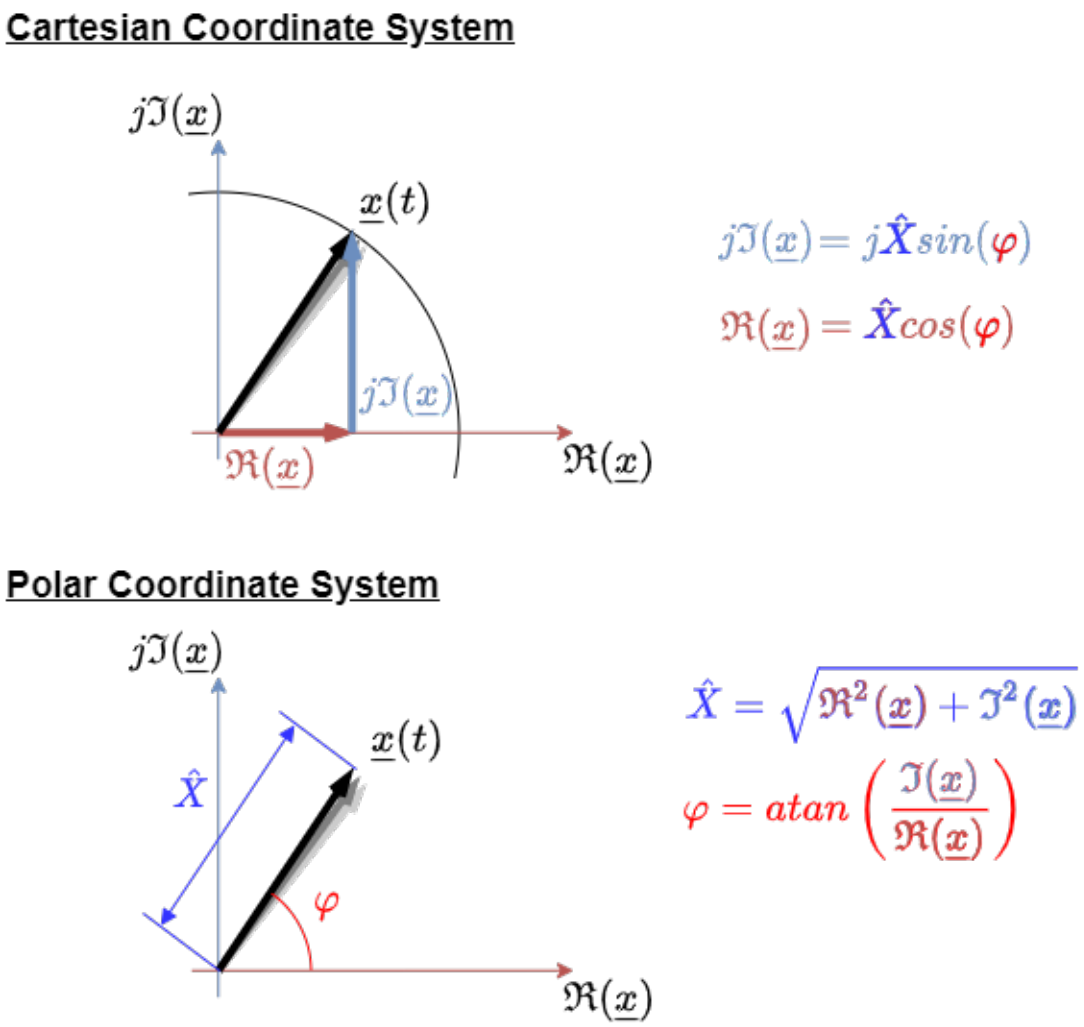
Fig. 10: Creation of the sinusidal signal from a rotational movement

Klick on the box “animate?”
press here for the animation

The two dimensional rotation can be represented with a complex number in Euler's formula. It combines the exponential representation with real part $\Re$ and imaginary part $\Im$ of a complex value: $$ \underline{x}(t)=\hat{X}\cdot e^{j(\omega t + \varphi_x)} = \Re(\underline{x}) + j\cdot \Im(\underline{x})$$

For the imaginary unit i the letter j is used in electical engineering, since the letter i is already taken for currents.

Fig. 11: represenation of a phasor on the complex plane



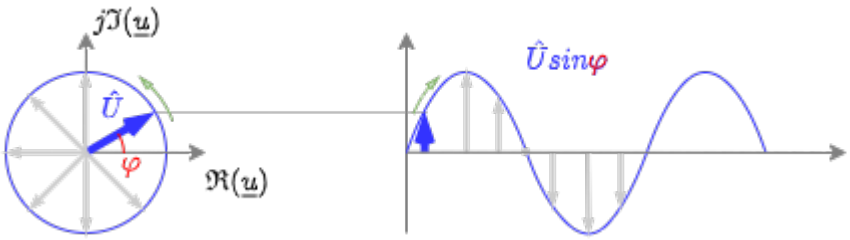
6.4.2 Complex Current and Voltage

The concepts of complex numbers shall now be applied to voltages and currents. Up to now we used the following formula to represent alternating voltages:

$$U(t)= {{1}\over{\sqrt{2}}} U \cdot sin (\varphi)$$

This is now interpreted as the instantaneuos value of a complex vector $\underline{u}(t)$, which rotates given by the time dependent angle $\varphi = \omega t + \varphi_u$.

Fig. 12: represenation of a voltage phasor on the complex plane



The parts on the complex plane are then given by:

1. The real part $\Re{(\underline{u}(t))} = \sqrt{2}U \cdot cos (\omega t + \varphi_u)$
2. The imaginary part $\Im{(\underline{u}(t))} = \sqrt{2}U \cdot sin (\omega t + \varphi_u)$

This is equivalent to the complex phasor $\underline{u}(t)=\sqrt{2}U \cdot e ^{j (\omega t + \varphi_u)}$

The complex phasor can be separated: \begin{align*} \underline{u}(t) &=\sqrt{2}I \cdot e ^{j (\omega t + \varphi_u)} \\ &=\sqrt{2}\color{blue}{U \cdot e ^{j \varphi_u}} \cdot e ^{j \omega t } \\ &=\sqrt{2}\color{blue}{\underline{U}} \cdot e ^{j \omega t} \\ \end{align*}

The **fixed phasor** (in German: komplexer Festzeiger) of the voltage is given by $\color{blue}{\underline{U}}= \color{blue}{U \cdot e ^{j \varphi_u}} $

Generally, from now on not only the voltage will be considered as a phasor, but also the current $\underline{I}$ and derived quantities like the impedance $\underline{X}$.
Therefore, the from Mathematics 101 known properties of complex numbers can be applied:

- A multiplication with j equals a phase shift of $+90°$
- A multiplication with $-j$ equals a phase shift of $-90°$

6.5 Complex Impedance

Goals

After this lesson you should:

1. Be able to draw and read pointer diagrams.
2. Know and be able to apply the complex value formulas of impedance, reactance, resistance.

6.5.1 Introduction into Complex Impedance

The complex impedance is “nearly” similar calculated like the resistance. In the subchapters before, that impedance Z was calculated by $Z=\frac{U}{I}$.
Now the complex impedance is:

$$\begin{align*} \underline{Z}&=\frac{\underline{U}}{\underline{I}} \\ &= \Re{(\underline{Z})} + j \cdot \Im{(\underline{Z})} \\ &= R + j \cdot X \\ &= Z \cdot e^{j \varphi} \\ &= Z \cdot (\cos \varphi + j \cdot \sin \varphi) \end{align*}$$

With

- the resistance R (in German: Widerstand) as the pure real part
- the reactance X (in German: Blindwiderstand) as the pure imaginary part
- the impedance Z (in German: Scheinwiderstand) as the complex number given by the complex addition of resistance and the reactance as a complex number

The impedance can be transformed from cartesian to polar by:

- $Z=\sqrt{R^2 + X^2}$
- $\varphi = \arctan \frac{X}{R} $

The other way around it is possible to transform by:

- $R = Z \cos \varphi$
- $X = Z \sin \varphi$

6.5.2 Application on pure Loads

With the complex impedance in mind, the table 1 can be expanded to:

<tabcaption tab02 | Formulas for the different pure loads>

Load 		**integral representation **	**complex impedance $\underline{Z}={{\underline{U}}\over{\underline{I}}}$**	**impedance $Z $**	**phase $\varphi $**
Resistance	R	$\underline{u} = R \cdot \underline{i}$	$Z_R = R $	$Z_R = R $	$\varphi_R = 0$
Capacitance	C	$\underline{u} ={{1}\over{C}}\cdot \int \underline{i} dt$	$Z_C = {{1}\over{j\omega \cdot C}} = {{-j}\over{\omega \cdot C}}$	$Z_C = {{1}\over{\omega \cdot C}}$	$\varphi_C = -{{1}\over{2}}\pi \hat{=} -90°$
Impedance	L	$\underline{u} = L \cdot {{d}\over{dt}} \underline{i}$	$Z_L = j \omega \cdot L $	$Z_L = \omega \cdot L $	$\varphi_L = +{{1}\over{2}}\pi \hat{=} +90°$

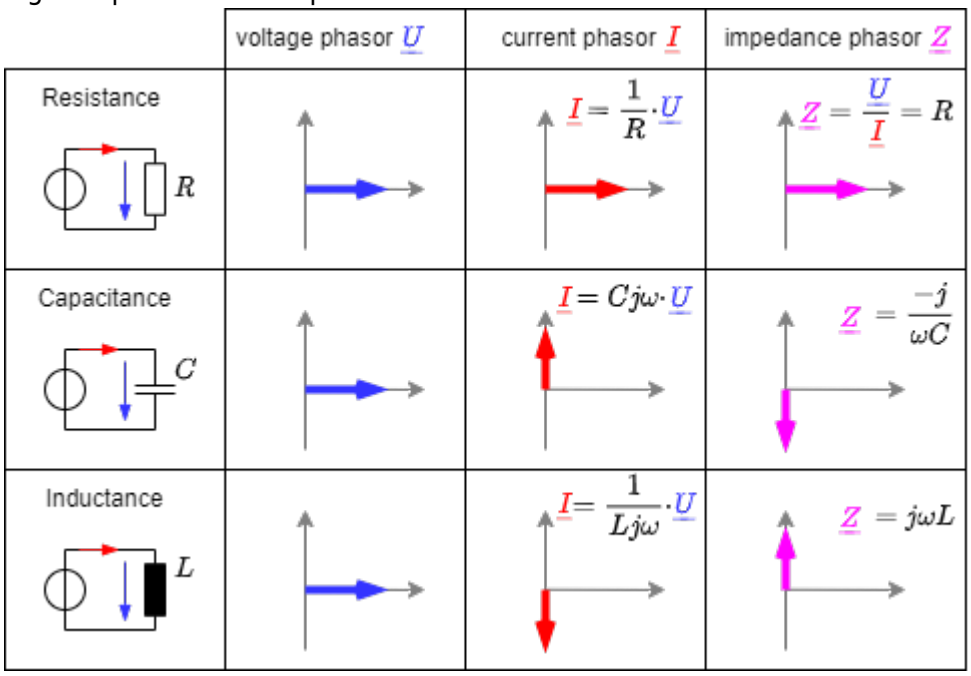
The relationship between j and integral calculus should be clear:

1. The derivative of a sinusidal value - and therefore a phasor - can simply be written as “$\cdot j$”, which also means a phase shift of $+90°$:
   ${{d}\over{dt}} e^{j(\omega t + \varphi_x)} = j \cdot e^{j(\omega t + \varphi_x)}$
2. The integral of a sinusidal value - and therefore a phasor - can simply be written as “$\cdot (-j)$”, which also means a phase shift of $-90°$.[1)]
   $\int e^{j(\omega t + \varphi_x)} = {{1}\over{j}} \cdot e^{j(\omega t + \varphi_x)} = - j \cdot e^{j(\omega t + \varphi_x)}$

Once a fixed input voltage is given, the voltage phasor $\underline{U}$, the current phasor

$\underline{I}$, and the impedance phasor $\underline{Z}$. In figure 13 these phasors are shown.

Fig. 13: phasors of the pure loads



6.5.3 Application on Impedance Networks

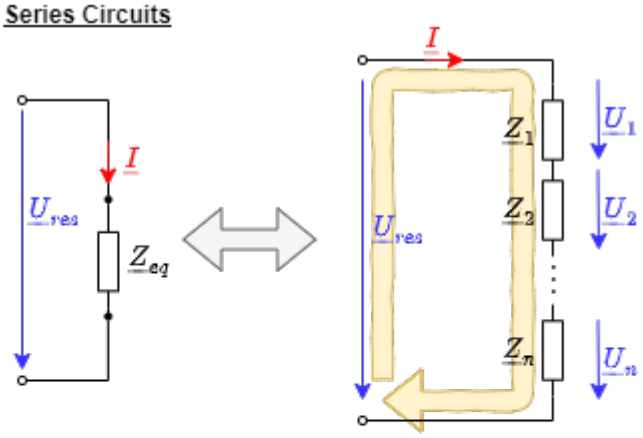
Simple Networks

In the cahpter Kirchhoff's Circuit Laws we already had a look onto simple networks like a series or parallel circuit of resistors. These formulas not only apply for ohmic resistors but also for impedances:

Series Circuit

Fig. 14: Simple Networks

Series Circuits

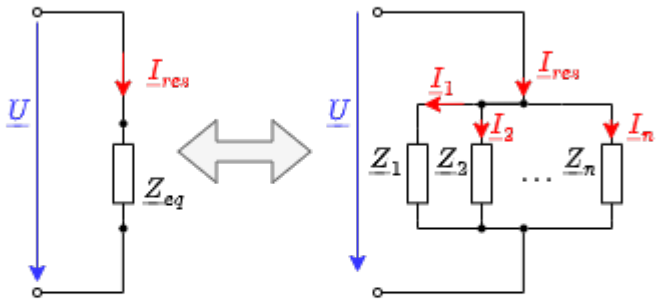


$$\underline{U}_{res} = \underline{U}_1 + \underline{U}_2 + \ldots + \underline{U}_n = \sum_n \underline{U}_i$$

$$\underline{Z}_{eq} \cdot \underline{I} = \underline{Z}_1 \cdot \underline{I} + \underline{Z}_2 \cdot \underline{I} + \ldots + \underline{Z}_n \cdot \underline{I} = \sum_n \underline{Z}_i \cdot \underline{I}$$

$$\boxed{\underline{Z}_{eq} = \underline{Z}_1 + \underline{Z}_2 + \ldots + \underline{Z}_n = \sum_n \underline{Z}_i}$$

Parallel Circuits



$$\underline{I}_{res} = \underline{I}_1 + \underline{I}_2 + \ldots + \underline{I}_n = \sum_n \underline{I}_i$$

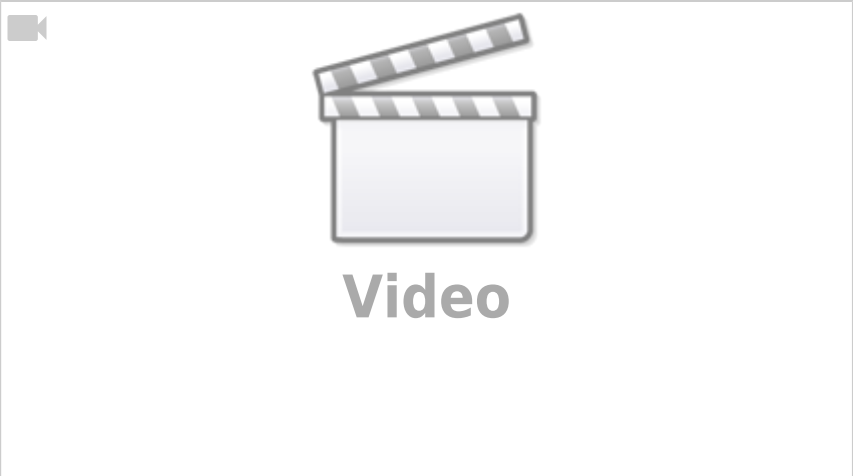
$$\frac{1}{\underline{Z}_{eq}} \cdot \underline{U} = \frac{1}{\underline{Z}_1} \cdot \underline{U} + \frac{1}{\underline{Z}_2} \cdot \underline{U} + \ldots + \frac{1}{\underline{Z}_n} \cdot \underline{U} = \sum_n \frac{1}{\underline{Z}_i} \cdot \underline{U}$$

$$\boxed{\frac{1}{\underline{Z}_{eq}} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \ldots + \frac{1}{\underline{Z}_n} = \sum_n \frac{1}{\underline{Z}_i}}$$

\begin{align*} \underline{U}_{eq}&=\sum_n \underline{U}_n \\ \underline{Z}_{eq}\cdot \underline{I} &=\sum_n \underline{Z}_{n}\cdot \underline{I} \\ \rightarrow {Z}_{eq} =\sum_n \underline{Z}_{n} \end{align*}

Video

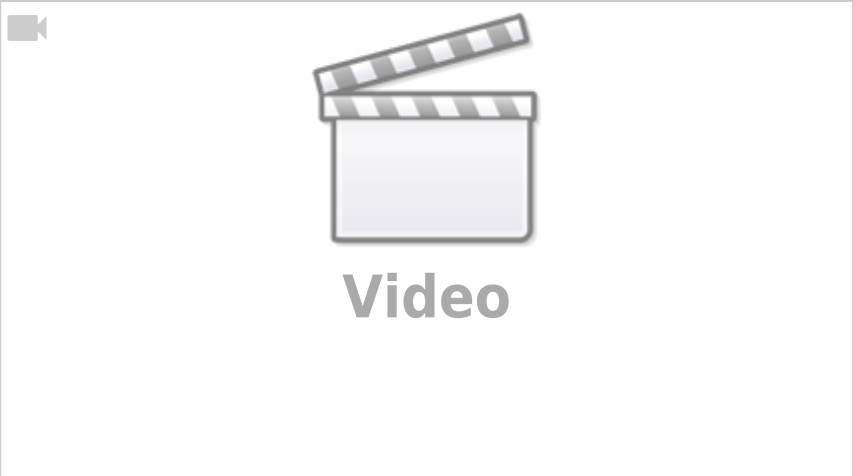
Pointer diagrams; complex alternating current calculus.



Complex alternating current calculation - basic terms: Impedance, Reactance, Resistance

Capacitor and inductance as complex resistors; pointer diagram

detailed explanation of impedances



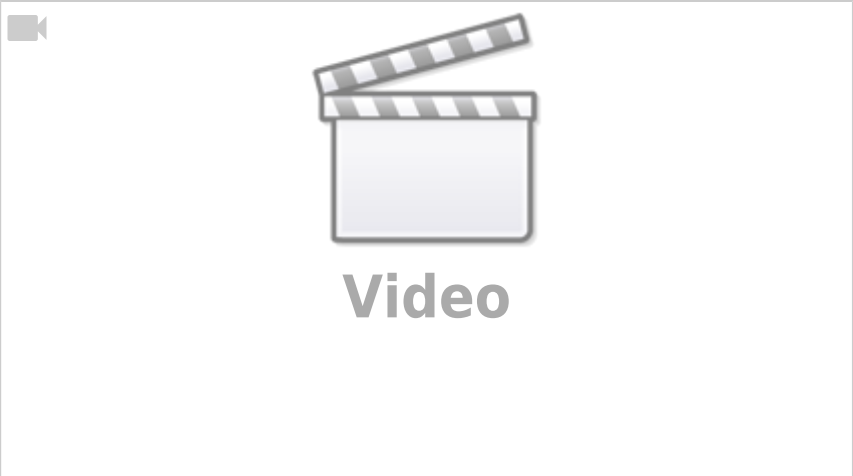
Exercises

Video

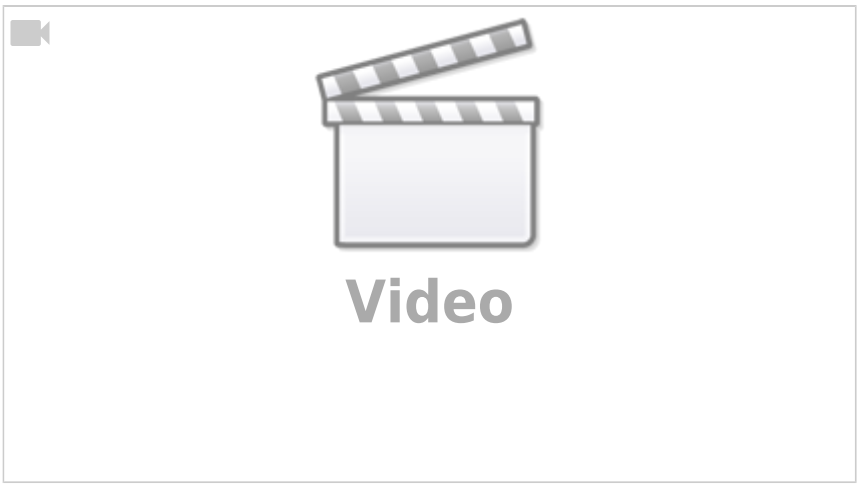
Parallel connection of complex resistors / impedances
Why can a circuit with impedances only act purely ohmic?

more complex exam task: complex circuit I

more complex exam task: complex circuit II



Examination task: complex circuit III



1)

in general, here the integration constant must be considered. This is however often be neclectable, since only AC values (without a DC value) are considered.

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