

Exam Summer Semester 2023

Student Group

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Exercise E1 Resistivity and temperature dependent Resistance (written test, approx. 7 % of a 60-minute written test, SS2023)

The resistivity of a dielectric material is described by the Arrhenius law: $\rho = \rho_0 \exp\left(\frac{E_a}{kT}\right)$ with $\rho_0 = 10^{17} \Omega \cdot \text{m}$ and $E_a = 0.8 \text{ eV}$. The resistivity of the material is $\rho = 10^{17} \Omega \cdot \text{m}$ at $T = 20^\circ \text{C}$. Calculate the resistance of a dielectric material with a thickness of $d = 100 \text{ nm}$ and a width of $A = 1 \text{ cm}$ at $T = 20^\circ \text{C}$ and $T = 55^\circ \text{C}$.

Solution: The resistivity of the dielectric material is $\rho_{\text{PP}}(20^\circ \text{C}) = 10^{17} \Omega \cdot \text{m}$.

For the given material the temperature coefficients in the range of 20°C and 55°C

are given as $\alpha = -0.048 \text{ 1/K}$ and $\beta = +0.00057 \text{ 1/K}^2$.

$$\begin{aligned} R(55^\circ \text{C}) &= R(20^\circ \text{C}) \cdot (1 + \alpha \cdot \Delta T + \beta \cdot T^2 + \dots) \\ &= 80 \text{ G}\Omega \cdot (1 - 0.048 \text{ 1/K} \cdot (35^\circ \text{C}) + 0.00057 \text{ 1/K}^2 \cdot \Delta T^2) \end{aligned}$$

Calculate the resistance for the dielectric material for 20°C .

Solution

$$R(20^\circ \text{C}) = \rho \cdot \frac{d}{A} = 10^{17} \Omega \cdot \text{m}$$

$$\rho_m \cdot \frac{\{0.8 \cdot 10^{-6} \cdot \rho_m\}}{1 - \rho_m^2} \quad \text{\texttt{\textbackslash end\{align*\}}}$$

Exercise E2 Resistivity and temperature dependent Resistance (written test, approx. 7 % of a 60-minute written test, SS2023)

The resistivity of a dielectric material is given by the formula $\rho_m(T) = \rho_0 \cdot \exp\left(\frac{E_a}{k_B T}\right)$ where ρ_0 is a material constant, E_a is the activation energy, and k_B is the Boltzmann constant. The resistivity of a dielectric material is given by the formula $\rho_m(T) = \rho_0 \cdot \exp\left(\frac{E_a}{k_B T}\right)$ where ρ_0 is a material constant, E_a is the activation energy, and k_B is the Boltzmann constant. The resistivity of a dielectric material is given by the formula $\rho_m(T) = \rho_0 \cdot \exp\left(\frac{E_a}{k_B T}\right)$ where ρ_0 is a material constant, E_a is the activation energy, and k_B is the Boltzmann constant.

The resistivity of the dielectric material is $\rho_m(T) = 10^{17} \cdot \exp\left(\frac{E_a}{k_B T}\right) \Omega \cdot \text{m}$.

For the given material the temperature coefficients in the range of $20 \sim 55 \text{ }^\circ\text{C}$ and $55 \sim 80 \text{ }^\circ\text{C}$ are given as $\alpha = -0.048 \text{ }^\circ\text{C}^{-1}$ and $\beta = +0.00057 \text{ }^\circ\text{C}^{-2}$.

$$\begin{aligned} R(55 \text{ }^\circ\text{C}) &= R(20 \text{ }^\circ\text{C}) \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2 + \dots) \\ &= 80 \text{ }^\circ\text{C} \cdot (1 - 0.048 \text{ }^\circ\text{C}^{-1} \cdot (35 \text{ }^\circ\text{C}) + 0.00057 \text{ }^\circ\text{C}^{-2} \cdot (35 \text{ }^\circ\text{C})^2) \end{aligned} \quad \text{\texttt{\textbackslash end\{align*\}}}$$

Calculate the resistance for the dielectric material for $20 \text{ }^\circ\text{C}$.

Solution

$$\begin{aligned} R(20 \sim \text{°C}) &= \rho \cdot \frac{d}{A} \\ &= 10^{17} \sim \Omega \cdot \text{m} \cdot \frac{0.8 \cdot 10^{-6} \sim \text{m}}{1 \sim \text{m}^2} \end{aligned}$$

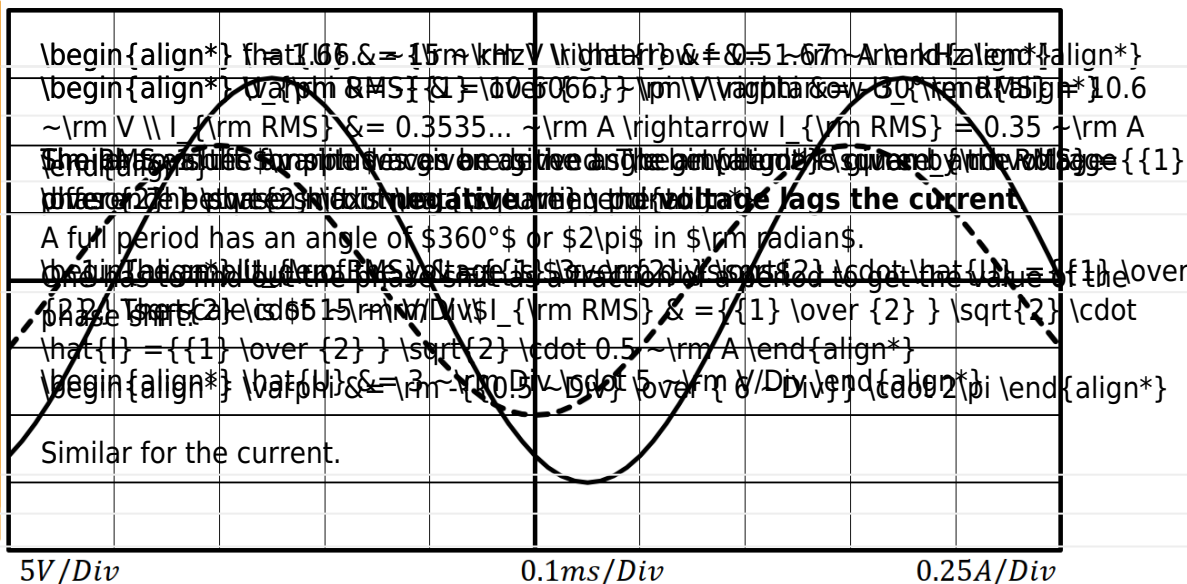
Exercise E3 Analyzing a Scope Plot (written test, approx. 12 % of a 60-minute written test, SS2023)

Q. What is the RMS value of the voltage (in radian and degree)?

Result
The measured current curve shall be visible as a dashed line.

The continuous line shows the voltage.

Solution



Use the correct symbols and units in your answers!

1. Calculate the frequency f of the periodic signals.

Solution

Frequency f is given by the period T . The period can be measured in the image of the scope.

1. The sine waves repeat after $6 \sim \text{divisions}$ (e.g. from falling turning point to falling turning point of one curve)
2. The scale is $0.1 \sim \text{ms/Div}$

$$f = \frac{1}{T} \quad T = 6 \sim \text{Div} \cdot 0.1 \sim \text{ms/Div}$$

$$\rightarrow f = \frac{1}{6 \cdot \text{Div} \cdot 0.1 \cdot \text{ms/Div}} \quad \text{\textbackslash end\{align*\}}$$

Exercise E4 Complex voltage dividers

(written test, approx. 16 % of a 60-minute written test, SS2023)

1. Calculate the two impedances Z_L and Z_C in the circuit below resulting phase shift between the voltage $\underline{U}_I$ and $\underline{U}_O$. Choose an appropriate scaling factor and write it down.

- $R = 1.1 \text{ k}\Omega$

Solution $L = 3.5 \text{ mH}$

Result $\underline{U}_I = 5 \text{ V}$

$$\underline{Z} = 50 \text{ m}\Omega \quad \text{\textbackslash end\{align*\}}$$

$$\underline{U}_O = 0.5 \text{ V} - j \cdot 1.5 \text{ V} \quad \text{\textbackslash end\{align*\}}$$

At the cut-off frequency the absolute values of impedances $\underline{Z}_L$ is equal to $\underline{Z}_C$. This leads to $\frac{1}{\omega C} = \omega L$ $\Rightarrow \omega = \frac{1}{\sqrt{LC}}$ $\Rightarrow f = \frac{1}{2\pi \sqrt{LC}}$ $\Rightarrow f = \frac{1}{2\pi \sqrt{1.1 \cdot 10^{-3} \cdot 3.5 \cdot 10^{-6}}} = 10.3 \text{ kHz}$



.. Calculate the impedance $\underline{Z}_L$.

Solution

$$\underline{Z}_L = j \cdot \omega \cdot L = j \cdot 2\pi \cdot 150 \text{ kHz} \cdot 3.5 \text{ mH} \quad \text{\textbackslash end\{align*\}}$$

Exercise E5 Pure Resistor Network Simplification

(written test, approx. 12 % of a 60-minute written test, SS2023)

1. Calculate the voltage U_K , when switch S is closed.

Result

The values in the circuit are

Solution

- $R_1 = 60 \Omega$

When switch S is closed, the circuit is simplified to a single resistor R_1 in series with a voltage source U_K . $\text{\textbackslash begin\{align*\}}$

$$R_1 = 60 \Omega \quad \text{\textbackslash end\{align*\}}$$

- $R_4 = 100 \text{ } \Omega$

The voltage divider for node K has the same proportionality as the voltage divider for node K' . Therefore, the potential of K is the same as for K' . There will be no current flow through R_3 . The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node K , when switch S is open.
It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch S is opened, the upper part is a parallel circuit. Therefore, R_{eq} is given as:

$$R_{\text{eq}} = (R_1 + R_2) \parallel (R_1 + R_2) + R_4 \\ = \frac{1}{2} \cdot (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (60 \, \Omega + 40 \, \Omega) + 100 \, \Omega$$

Exercise E6 Pure Resistor Network Simplification I (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below shall be given as $U_{\text{AB}} = 60 \, \text{V}$. What is the value for I_{AB} in the circuit?

Solution

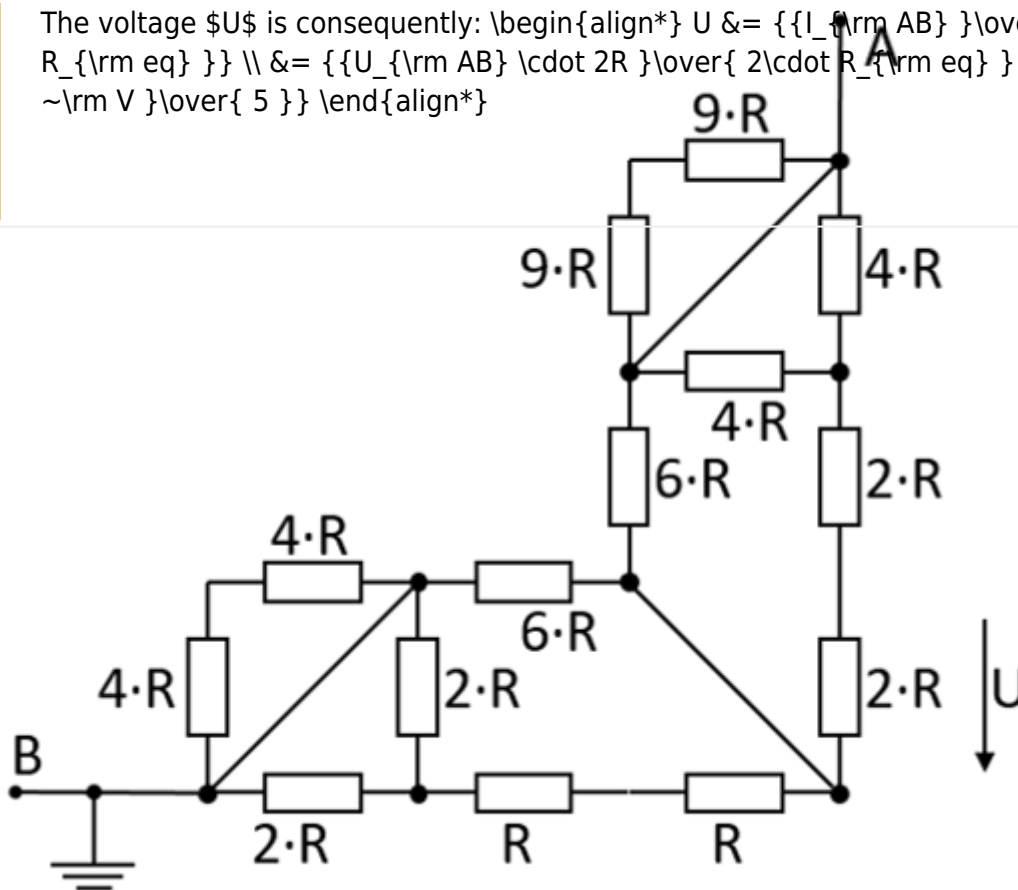
$$I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}} = \frac{60 \, \text{V}}{60 \, \Omega} = 1 \, \text{A}$$

The current through the circuit is given as $I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}}$.

This current has to flow in summary through parallel branches. The voltage U in question in the upper right branch given by $(4R \parallel 4R) + 2R + 2R$. Its resistance is just the same as the upper left branch $6R$.

Therefore, half of the current flows to the left half to the right side.

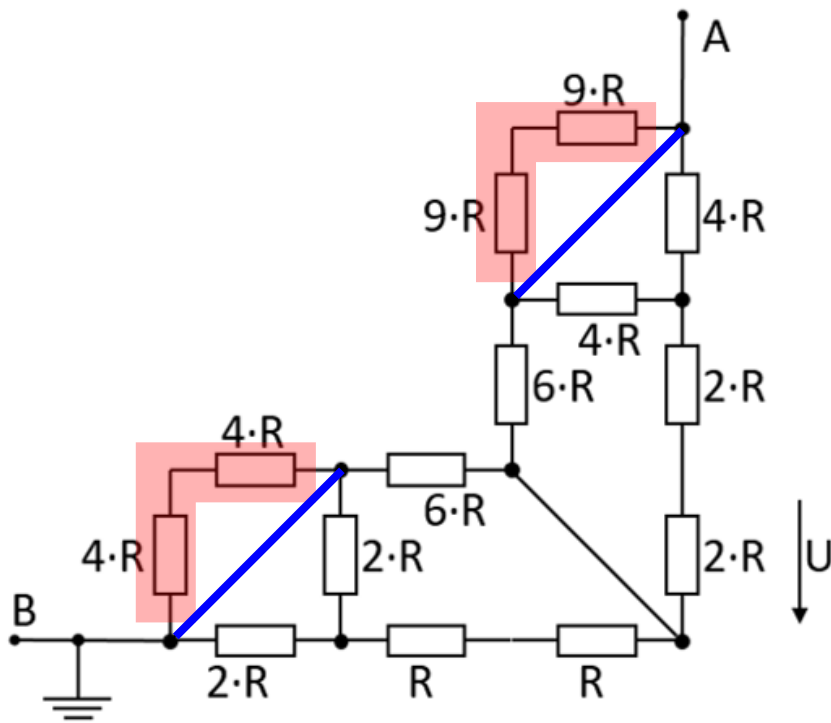
The voltage U is consequently:
$$U = \frac{I_{\text{arm AB}}}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{U_{\text{AB}} \cdot 2R}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{60 \text{ V}}{5}$$



1. What is the equivalent resistance R_{eq} ?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

Therefore, R_{eq} is given as:
$$R_{\text{eq}} = (2R || 2R + R +$$

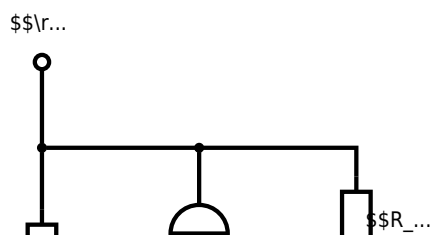
$$R) \parallel 6R \text{ \&\&+ } 6R \parallel (2R + 2R + 4R) \parallel 4R \text{ \&\&= } (R + R + R) \parallel 6R \text{ \&\&+ } 6R \parallel (2R + 2R + 2R) \text{ \&\&= } 3R \parallel 6R \text{ \&\&+ } 6R \parallel 6R \text{ \&\&= } \left\{ \frac{3R \cdot 6R}{3R + 6R} \right\} \text{ \&\&+ } 3R \text{ \&\& \end{align*}}$$

Exercise E7 Equivalent Linear Source (written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification.

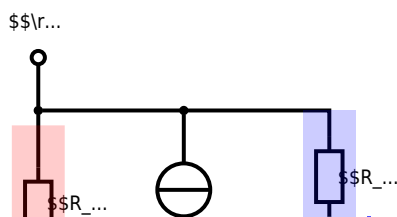
Calculate the internal resistance R_{i} and the source voltage U_{s} of an equivalent linear voltage source.

- $R_1 = 5 \text{ } \Omega$
 - $U_1 = 10 \text{ V}$
 - $R_2 = 5 \text{ } \Omega$
 - $I_3 = 0.5 \text{ A}$
 - $R_4 = 10 \text{ } \Omega$
 - $U_5 = 4 \text{ V}$
- $$U_{\text{AB}} = 1.11 \text{ V} \quad R_{\text{i}} = 5.55 \text{ } \Omega$$



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = \frac{U_x}{R_x}$

$$\begin{aligned} I_0 &= \frac{U_0}{R_1} = \frac{10 \text{ V}}{5 \text{ } \Omega} = 2 \text{ A} \\ I_5 &= \frac{U_5}{R_4} = \frac{4 \text{ V}}{10 \text{ } \Omega} = 0.4 \text{ A} \end{aligned}$$

I_3 and I_0 can be combined to $I_{03} = I_0 - I_3$ facing upwards:

$$I_{03} = 1.5 \text{ A}$$

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03} = R_1 \cdot I_{03}$ facing down.

$$U_{03} = 7.5 \text{ V}$$

Then, the resistors R_1 and R_2 can be combined to $R_{12} = R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12} . The current will be $I_{0123} = \frac{U_{03}}{R_{12}}$, facing up again.

$$I_{0123} = 0.6 \text{ A}$$

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235} = I_{0123} - I_5$ and the resistors as $R_{124} = R_{12} || R_4$.

$$I_{01235} = 0.2 \text{ A} \quad R_{124} = 5.55 \text{ } \Omega$$

The final step is the back-transformation to a linear voltage source, with $U_{\text{AB}} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.

Here, there is also a full final formula given:

$$\begin{aligned} U_{\text{AB}} &= U_{\text{AB}} = I_{01235} \cdot R_{124} \quad \&= (I_{0123} - I_5) \cdot (R_{12} \parallel R_4) \quad \&= \left(\frac{U_{03}}{R_{12}} - I_5 \right) \cdot (R_{12} \parallel R_4) \\ &= \left(\frac{R_1 \cdot I_{03}}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \quad \&= \left(\frac{R_1 \cdot \left(\frac{U_0}{R_1} - I_3 \right)}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \end{aligned}$$

Exercise E8 (Dis)Charging Capacities

(written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit shown in the drawing consists of a DC voltage source $U = 10 \text{ V}$, a capacitor $C = 200 \text{ nF}$, and three resistors $R_1 = 8 \text{ k}\Omega$, $R_2 = 17 \text{ k}\Omega$, and $R_3 = 25 \text{ k}\Omega$. The switch S_1 is initially open and the capacitor is fully discharged. At $t = 0$, the switch S_1 is closed. What is the new time constant?

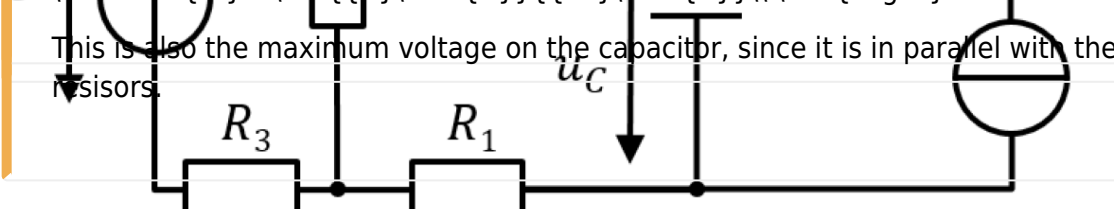
- $C = 200 \text{ nF}$

Solution: $R_1 = 8 \text{ k}\Omega$

Solution: $\tau = 8 \text{ k}\Omega \cdot 200 \text{ nF} = 1.6 \text{ ms}$

$$\begin{aligned} U_C(t) &= U \cdot \left(1 - e^{-t/\tau} \right) \\ U_C(1.6 \text{ ms}) &= 10 \text{ V} \cdot \left(1 - e^{-1.6 \text{ ms} / 1.6 \text{ ms}} \right) = 10 \text{ V} \cdot (1 - e^{-1}) \approx 6.32 \text{ V} \end{aligned}$$

Again, the time constant is given by the equivalent circuit consisting of C in parallel with $R_1 \parallel R_2$. Without the parallel resistors, the current source would charge the capacitor "to infinity" ($C \rightarrow \infty$). This is here limited by the parallel resistors R_1 and R_2 . When S_2 is closed, then, the source U drives the current through the series circuit given by U , C , R_1 and R_2 . Therefore, the voltage on the branch with the resistors ($R_1 \parallel R_2$) is $U_C = (8 \text{ k}\Omega + 17 \text{ k}\Omega) \cdot 0.2 \text{ mA} = 25 \text{ k}\Omega \cdot 0.2 \text{ mA} = 5 \text{ V}$. This is also the maximum voltage on the capacitor, since it is in parallel with the resistors.



Before t_0 all switches are switched as shown and the capacitor is fully discharged. At $t_0 = 0 \text{ s}$ the switch S_1 shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

Solution

The time constant is generally given as: $\tau = R \cdot C$

Once S_1 is closed and S_2 is open at t_0 , the source U drives the current through the series circuit given by S_1 , C , R_1 and R_3 .

Therefore, $R = R_1 + R_3$ $\tau_1 = (R_1 + R_3) \cdot C \quad \&= (8 \cdot 10^3 + 7 \cdot 10^3) \cdot 0.2 \cdot 10^{-6} \text{ s} = 15 \cdot 10^{-3} \cdot 0.2 \cdot 10^{-6} \text{ s} = 3 \cdot 10^{-6} \text{ s}$

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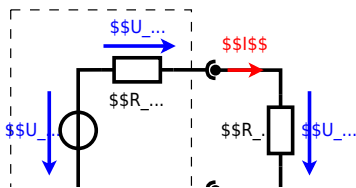
Solution

Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ($1 - e^{-x}$).

Exercise E9 Impedances at Frequencies

(written test, approx. 14 % of a 60-minute written test, SS2023)

At a high frequency with $C = 20 \text{ nF}$ and $R = 50 \text{ }\Omega$, the impedance Z of the parallel combination of C and R is approximately $Z \approx R$.



Exercise E11 Analyzing a Scope Plot (written test, approx. 12 % of a 60-minute written test, SS2023)

What does the RMS value of the voltage and current (in radian and degree)?

The measured current curve shall be visible as a dashed line.

The continuous line shows the voltage.

Solution

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\begin{align*} f &= 15 \text{ kHz} \\ U_{\text{eff}} &= 1.67 \text{ V} \\ I_{\text{eff}} &= 0.35 \text{ A} \end{align*}
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The RMS value of the voltage is given by the RMS value of the voltage. The RMS value of the voltage is given by the RMS value of the voltage.

The RMS value of the current is given by the RMS value of the current. The RMS value of the current is given by the RMS value of the current.

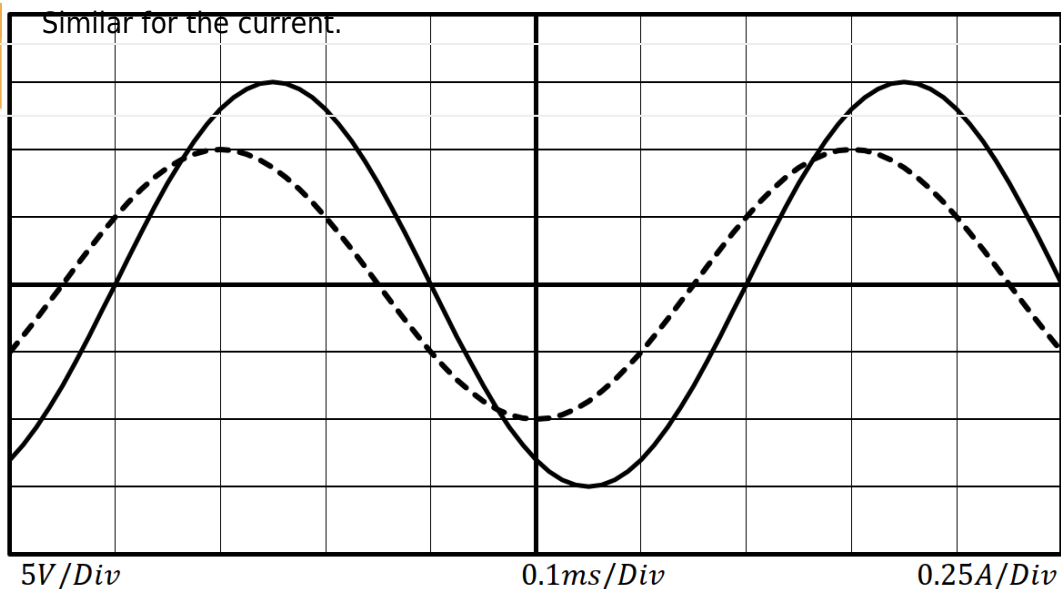
The phase shift between the voltage and the current is given by the phase shift between the voltage and the current.

A full period has an angle of 360° or 2π in radians.

The phase shift between the voltage and the current is given by the phase shift between the voltage and the current.

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The phase shift between the voltage and the current is given by the phase shift between the voltage and the current.



Use the correct symbols and units in your answers!

1. Calculate the frequency f of the periodic signals.

Solution

Frequency f is given by the period T . The period can be measured in the image of the scope.

1. The sine waves repeat after $6 \sim \text{rm divisions}$ (e.g. from falling turning point to falling turning point of one curve)
2. The scale is $0.1 \sim \text{rm ms/Div}$

$$\begin{aligned} f &= \frac{1}{T} \quad T = 6 \sim \text{rm Div} \cdot 0.1 \sim \text{ms/Div} \\ \rightarrow f &= \frac{1}{6 \sim \text{rm Div} \cdot 0.1 \sim \text{ms/Div}} \end{aligned}$$

Exercise E12 Complex voltage dividers

(written test, approx. 16 % of a 60-minute written test, SS2023)

1. Calculate the two simplified impedances Z_1 and Z_2 by resulting phase for the total impedance Z_{total} . Choose an appropriate scaling factor and write it down.

- $R = 1.1 \sim \text{rm k}\Omega$

Solution

Result: $\underline{U}_I = 5 \sim \text{rm V}$

$$\underline{Z}_1 = 1.1 \sim \text{rm k}\Omega \quad \underline{Z}_2 = 50 \sim \text{rm m}\Omega \quad \underline{Z}_{\text{total}} = 1.1 \sim \text{rm k}\Omega$$

$$\underline{U}_O = 0.5 \sim \text{rm V} - j \cdot 1.5 \sim \text{rm V}$$

At cut off frequency the absolute values of impedances $\underline{Z}_L$ is equal to $\frac{R}{\sqrt{1 - \frac{R^2}{Z_L^2}}}$. This leads to the complex voltage divider for cutoff $\frac{R}{R + j\omega L}$.
 The voltage divider is $\frac{U_I}{U_O} = \frac{R}{R + j\omega L}$.
 The voltage divider is $\frac{U_I}{U_O} = \frac{R}{R + j\omega L}$.
 The voltage divider is $\frac{U_I}{U_O} = \frac{R}{R + j\omega L}$.



.. Calculate the impedance $\underline{Z}_L$.

Solution

$$\underline{Z}_L = j\omega L = j2\pi \cdot 150 \text{ kHz} \cdot 3.5 \text{ mH}$$

Exercise E13 Pure Resistor Network Simplification (written test, approx. 12 % of a 60-minute written test, SS2023)

Calculate the voltage across R_3 , when switch S is closed.

Result

The values in the circuit are

Solution

- $R_1 = 60 \text{ } \Omega$
- $R_2 = 40 \text{ } \Omega$
- $R_3 = 40 \text{ } \Omega$
- $R_4 = 10 \text{ } \Omega$
- The voltage divider for R_3 has the same proportionality as the voltage divider for R_1 . Therefore, the potential of R_3 is the same as for R_1 . There will be no current flow through R_3 . The resistance does not create a voltage drop and therefore does not interfere with the circuit.

1. Calculate the voltage at node K , when switch S is open.
It might be beneficial to redraw the circuit first.

Solution

Rearranging the circuit one can get:

Once the switch S is opened, the upper part is a parallel circuit. Therefore, R_{eq} is given as:

$$R_{\text{eq}} = (R_1 + R_2) \parallel (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (R_1 + R_2) + R_4 = \frac{1}{2} \cdot (60 \, \Omega + 40 \, \Omega) + 100 \, \Omega$$

Exercise E14 Pure Resistor Network Simplification I (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit below should be given as $U_{\text{AB}} = 60 \, \text{V}$. What is the value for I_{AB} in the circuit?

Solution

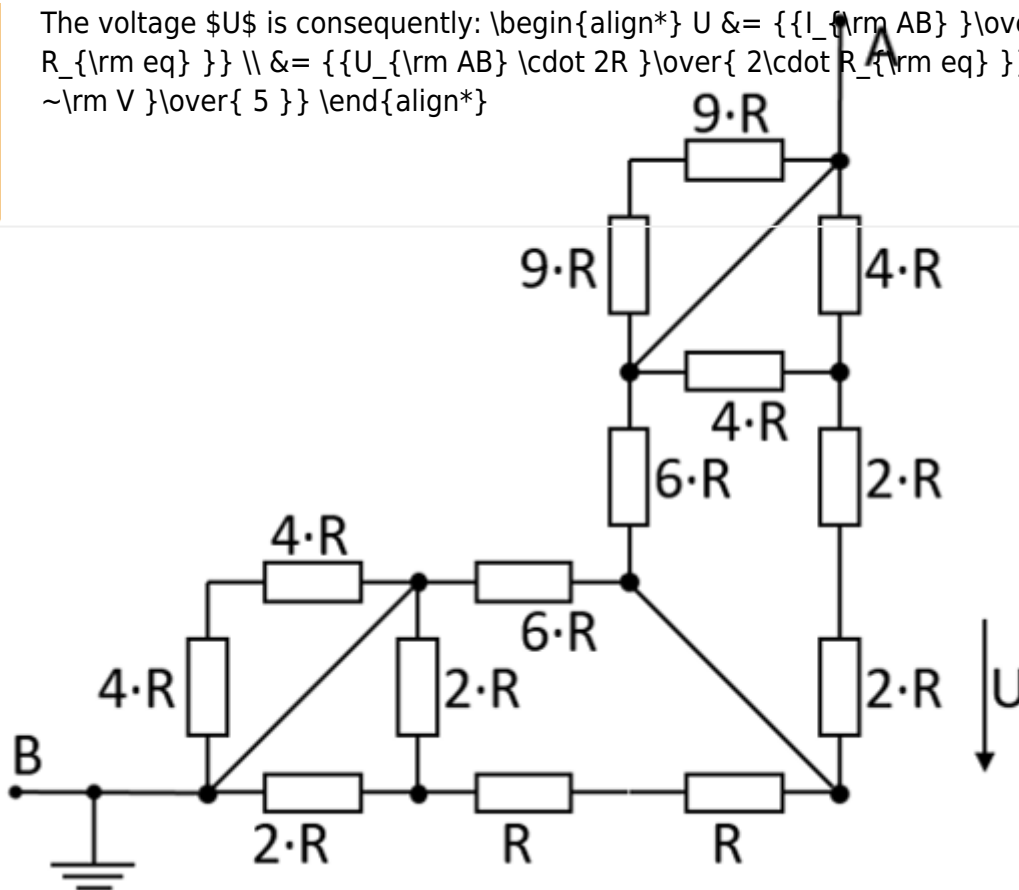
$$I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}} = \frac{60 \, \text{V}}{60 \, \Omega} = 1 \, \text{A}$$

The current through the circuit is given as $I_{\text{AB}} = \frac{U_{\text{AB}}}{R_{\text{eq}}}$.

This current has to flow in summary through parallel branches. The voltage U in question in the upper right branch given by $(4R \parallel 4R) + 2R + 2R$. Its resistance is just the same as the upper left branch $6R$.

Therefore, half of the current flows to the left half to the right side.

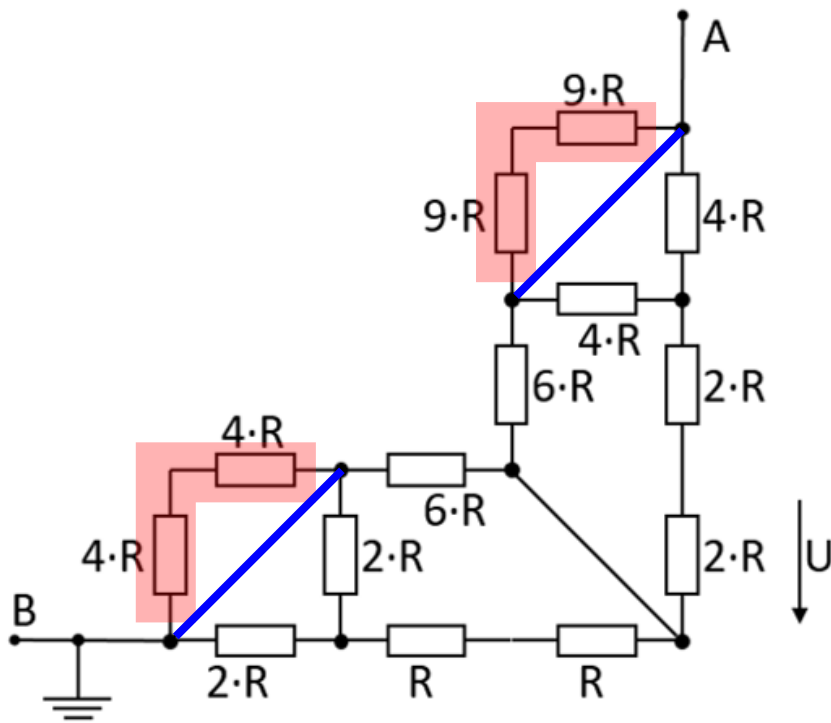
The voltage U is consequently:
$$U = \frac{I_{\text{arm AB}}}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{U_{\text{AB}} \cdot 2R}{2 \cdot R_{\text{eq}}} \quad \&= \quad \frac{60 \text{ V}}{5}$$



1. What is the equivalent resistance R_{eq} ?

Solution

Part of the circuit is shorted. Here the resistors (marked in red) are shorted by the connections marked in blue:



The circuit can then be rearranged for better interpretation:

Therefore, R_{eq} is given as:
$$R_{\text{eq}} = (2R \parallel 2R + R +$$

$$R) \parallel 6R \text{ \&\&+ } 6R \parallel (2R + 2R + 4R) \parallel 4R \text{ \&\&= } (R + R + R) \parallel 6R \text{ \&\&+ } 6R \parallel (2R + 2R + 2R) \text{ \&\&= } 3R \parallel 6R \text{ \&\&+ } 6R \parallel 6R \text{ \&\&= } \left\{ \frac{3R \cdot 6R}{3R + 6R} \right\} \text{ \&\&+ } 3R \text{ \&\& \end{align*}}$$

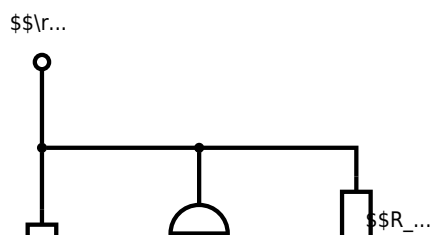
Exercise E15 Equivalent Linear Source

(written test, approx. 10 % of a 60-minute written test, SS2023)

The circuit below has to be simplified. Use equivalent linear sources for simplification.

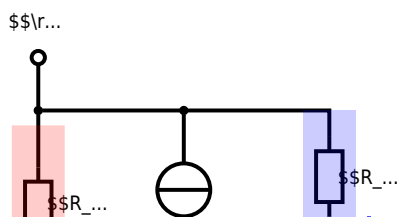
Calculate the internal resistance R_{i} and the source voltage U_{s} of an equivalent linear voltage source.

- $R_1 = 5 \text{ } \Omega$
- $U_1 = 10 \text{ V}$
- $R_2 = 5 \text{ } \Omega$
- $I_3 = 0.5 \text{ A}$
- $R_4 = 10 \text{ } \Omega$
- $U_5 = 4 \text{ V}$



Solution

The principle idea here is to find parts of the circuit which are already a linear (voltage or current) source. Then this can be transformed into the equivalent other source, as shown in the next picture.



In order to get the currents one has to calculate it by $I_x = \frac{U_x}{R_x}$

$$\begin{aligned} I_0 &= \frac{U_0}{R_1} = \frac{10 \text{ V}}{5 \text{ } \Omega} = 2 \text{ A} \\ I_5 &= \frac{U_5}{R_4} = \frac{4 \text{ V}}{10 \text{ } \Omega} = 0.4 \text{ A} \end{aligned}$$

I_3 and I_0 can be combined to $I_{03} = I_0 - I_3$ facing upwards:

$$I_{03} = 1.5 \text{ A}$$

Then, the linear current source I_{03} with R_1 gets transformed into a linear voltage source with $U_{03} = R_1 \cdot I_{03}$ facing down.

$$U_{03} = 7.5 \text{ V}$$

Then, the resistors R_1 and R_2 can be combined to $R_{12} = R_1 + R_2$.

After this, the next step is to make a linear current source out of U_{03} and R_{12} . The current will be $I_{0123} = \frac{U_{03}}{R_{12}}$, facing up again.

$$I_{0123} = 0.6 \text{ A}$$

The second-last step is the sum up of the current sources I_{0123} and I_5 as $I_{01235} = I_{0123} - I_5$ and the resistors as $R_{124} = R_{12} || R_4$.

$$I_{01235} = 0.2 \text{ A} \quad R_{124} = 5.55 \text{ } \Omega$$

The final step is the back-transformation to a linear voltage source, with $U_{\text{AB}} = R_{124} \cdot I_{01235}$.

The simplest and fastest (= for exams) is to work with interim results in the calculation.

Here, there is also a full final formula given:

$$\begin{aligned} U_{\text{AB}} &= U_{\text{AB}} = I_{01235} \cdot R_{124} \quad \&= (I_{0123} - I_5) \cdot (R_{12} \parallel R_4) \quad \&= \left(\frac{U_{03}}{R_{12}} - I_5 \right) \cdot (R_{12} \parallel R_4) \\ &= \left(\frac{R_1 \cdot I_{03}}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \quad \&= \left(\frac{R_1 \cdot \left(\frac{U_0}{R_1} - I_3 \right)}{R_1 + R_2} - I_5 \right) \cdot (R_{12} \parallel R_4) \end{aligned}$$

Exercise E16 (Dis)Charging Capacities (written test, approx. 14 % of a 60-minute written test, SS2023)

The circuit shown in the drawing consists of a DC voltage source $U = 10 \text{ V}$, a capacitor $C = 200 \text{ nF}$, and three resistors $R_1 = 8 \text{ k}\Omega$, $R_2 = 17 \text{ k}\Omega$, and $R_3 = 1 \text{ k}\Omega$. The switch S_1 is initially open and the capacitor is fully discharged. At $t = 0$, the switch S_1 is closed. What is the new time constant?

- $C = 200 \text{ nF}$

Solution: $R_1 = 8 \text{ k}\Omega$

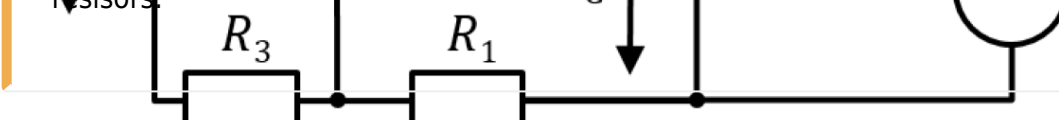
Solution: $\tau = 8 \text{ k}\Omega \cdot 200 \text{ nF} = 1.6 \text{ ms}$

$$\begin{aligned} U_C(t) &= U \cdot \left(1 - e^{-t/\tau} \right) \\ U_C(1.6 \text{ ms}) &= 10 \text{ V} \cdot \left(1 - e^{-1.6 \text{ ms} / 1.6 \text{ ms}} \right) = 10 \text{ V} \cdot (1 - e^{-1}) \approx 6.32 \text{ V} \end{aligned}$$

Again, the time constant is given by the equivalent circuit consisting of C in parallel with R_1 and R_2 . Without the parallel resistors, the current source would charge the capacitor "to infinity" ($C \rightarrow \infty$). This is here limited by the parallel resistors R_1 and R_2 . When S_2 is closed, then, the source U drives the current through the series circuit given by R_1 , C , R_1 and R_2 .

Therefore, the voltage on the capacitor (with the resistors R_1 and R_2) is $U_C = (8 \text{ k}\Omega + 17 \text{ k}\Omega) \cdot 0.2 \text{ }\mu\text{F} = 25 \cdot 10^3 \cdot 0.2 \cdot 10^{-6} \text{ V} = 5 \text{ V}$.

This is also the maximum voltage on the capacitor, since it is in parallel with the resistors.



Before t_0 all switches are switched as shown and the capacitor is fully discharged. At $t_0 = 0 \text{ s}$ the switch S_1 shall switch to the voltage source.

1. Calculate the time constant for charging the capacitor.

Solution

The time constant is generally given as: $\tau = R \cdot C$

Once S_1 is closed and S_2 is open at t_0 , the source U drives the current through the series circuit given by S_1 , C , R_1 and R_3 .

Therefore, $R = R_1 + R_3$ $\tau_1 = (R_1 + R_3) \cdot C \quad \&= (8 \cdot 10^3 + 7 \cdot 10^3) \cdot 0.2 \cdot 10^{-6} \text{ s} = 15 \cdot 10^{-3} \cdot 0.2 \cdot 10^{-6} \text{ s} = 3 \cdot 10^{-6} \text{ s}$

⚡...

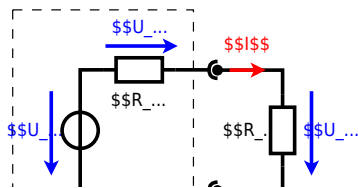
Solution

Both courses of the voltage for charging and discharging are described with an exponential function. However, the curve for charging increases first steep and flattens out for longer time scales ($1 - e^{-x}$).

Exercise E17 Impedances at Frequencies

(written test, approx. 14 % of a 60-minute written test, SS2023)

At a high frequency with S_2 open and S_1 closed, the voltage across R_3 is 6.8 V. The



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