

Exam Winter Semester 2022

Student Group

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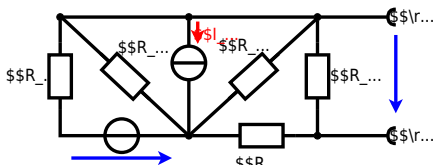
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**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

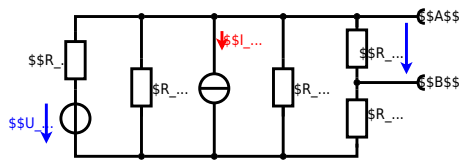
$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



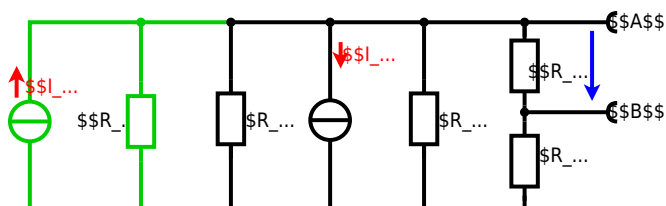
Calculate the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B .
$$R_1=5.0 \text{ } \Omega, \quad U_2=6.0 \text{ V}, \quad R_3=10 \text{ } \Omega, \quad I_4=4.2 \text{ A}, \quad R_5=10 \text{ } \Omega, \quad R_6=7.5 \text{ } \Omega, \quad R_7=15 \text{ } \Omega$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_4 + R_5)$$

$$U_{24} = U_{23} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5}$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) = \left(\frac{U_{23}}{R_1} - I_4 \right) \cdot \left(\frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

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Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The graph shows the temperature dependence of the resistance of a resistor. The resistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Calculate the resistance of the resistor at -40°C .

Its temperature coefficients are: $\alpha = 0.01 \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \frac{1}{\text{K}^2}$.

The temperature inside the refrigeration system can reach down to -40°C .

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistance of the resistor R depends on the temperature T and the heat Q . Therefore, a solution is to use a heat pump to heat up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

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Exercise E9 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{u} (in V) at $t = 30$ ms for the circuit shown in the figure. The components R and X_L shall be given.

After analysis, the following phasor voltage \underline{u} is extracted: $\underline{u} = 48.2 \angle -19.8^\circ$ V. In the time domain, the voltage is $u(t) = 48.2 \sqrt{2} \cos(2\pi \cdot 15 \cdot t - 19.8^\circ)$ V.

Solution
.. Calculate the physical values of the components.
Solution $R = 10 \Omega$, $X_L = 20 \Omega$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{10 + j20} = 2.5 \angle -63.4^\circ$$

The current $i(t) = 2.5 \sqrt{2} \cos(2\pi \cdot 15 \cdot t - 63.4^\circ)$ A.
The voltage across the capacitor is $\underline{u}_C = jX_C \underline{I} = j10 \cdot 2.5 \angle -63.4^\circ = 25 \angle 26.6^\circ$ V.
The voltage across the inductor is $\underline{u}_L = jX_L \underline{I} = j20 \cdot 2.5 \angle -63.4^\circ = 50 \angle 26.6^\circ$ V.
The total voltage is $\underline{u} = \underline{u}_C + \underline{u}_L = 25 \angle 26.6^\circ + 50 \angle 26.6^\circ = 75 \angle 26.6^\circ$ V.
In the time domain, $u(t) = 75 \sqrt{2} \cos(2\pi \cdot 15 \cdot t + 26.6^\circ)$ V.

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Exercise E13 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage \underline{u} (in V) at $t = 30$ ms for the circuit shown in the figure. The components R and X_L shall be given. The voltage source is $u(t) = 3.0 \cos(2\pi \cdot 15 \cdot t)$ V. The circuit consists of a resistor $R = 10 \Omega$, an inductor $X_L = 20 \Omega$, and a capacitor $X_C = 10 \Omega$.

Solution
The linear source is connected with an inductor of $330 \mu\text{H}$ and a capacitor of $30.22 \mu\text{F}$, all in series.
Result

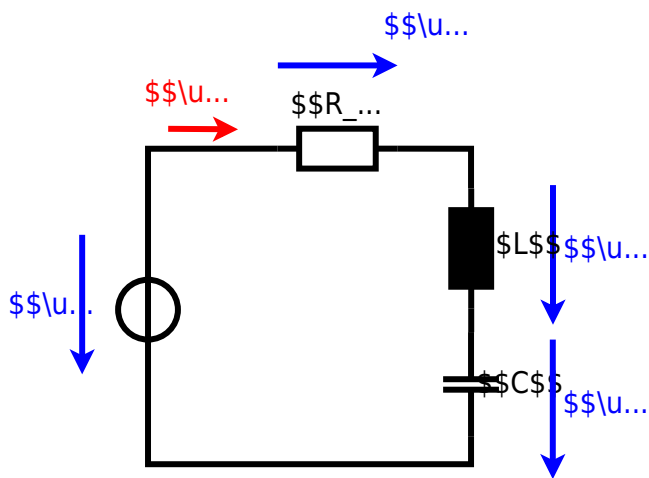
.. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.
Solution $Z = 10 - j10 + j20 = 10 + j10 \Omega$, $\underline{I} = \frac{3 \angle 0^\circ}{10 + j10} = 0.212 \angle -45^\circ$ A, $\underline{u} = \underline{I} Z = 2.12 \angle -45^\circ$ V.

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\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi \cdot f \cdot C} \quad \omega = 2\pi \cdot 15 \\
\text{Winkel } \varphi &= \arctan\left(\frac{X_L}{X_C}\right) = \arctan\left(\frac{2\pi \cdot f \cdot L}{\frac{1}{2\pi \cdot f \cdot C}}\right) \\
\end{align*}
\begin{align*} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\
\end{align*}
\begin{align*} Z_C &= \frac{1}{2\pi \cdot f \cdot C} \quad \omega = 2\pi \cdot 15 \\
&\sim \text{kHz} \cdot 330 \sim \mu\text{H} \\
\end{align*}
\begin{align*} \underline{Z} &= R + j \underline{Z}_L - j \underline{Z}_C \\
&= R + j(\underline{Z}_L - \underline{Z}_C) \quad |\underline{Z}| = \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2} \\
\end{align*}

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Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad R &= \rho \cdot \frac{l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

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Exercise E7 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance R_1 of the battery. The capacitor C is initially uncharged. The switch S_1 is closed at $t_0 = 0 \text{ s}$ and the voltage across the capacitor is again U_C at the moment $t_2 = 1 \text{ ms}$ after closing the switch. Calculate the voltage $u_C(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
The ideal voltage source U is in series with the internal resistance R_1 and the external resistance R_2 . The voltage across the capacitor is $u_C(t) = U \cdot (1 - e^{-t/\tau})$, where $\tau = (R_1 + R_2) \cdot C$.
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

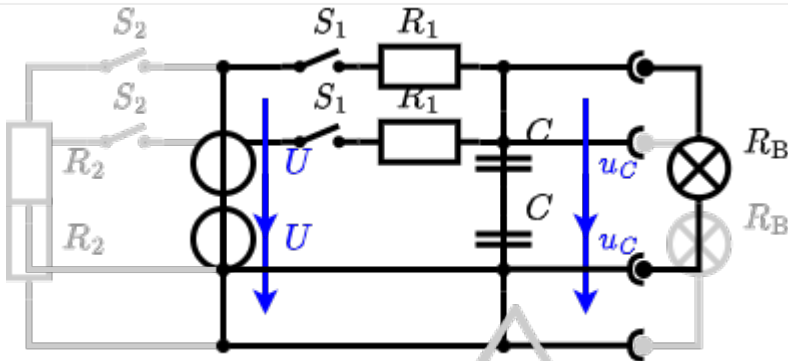


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

- .. First do not consider the light bulb - it is not connected to the RC circuit.
- Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \Rightarrow e^{-t/\tau} = 0.5 \Rightarrow -t/\tau = \ln(0.5) \Rightarrow t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \Omega$, short-circuit).
 $R_i = R_1 \parallel R_B = 10 \Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms} / (10 \Omega \cdot 100 \mu\text{F})})$$

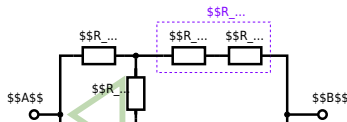
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Exercise E3 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0°C. On the left, $R_1 = R_2 = 1.5 \text{ k}\Omega$ and the voltage source $U = 10 \text{ V}$. The internal resistance of the source is $R_s = 100 \Omega$. The current I through the resistor R_B is to be determined.

Solution

$R_2 = R_3 = 100 \Omega$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

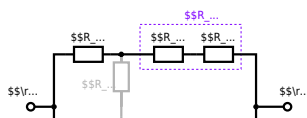
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

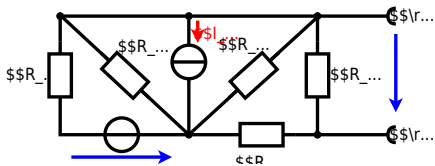
$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

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**Exercise E6 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



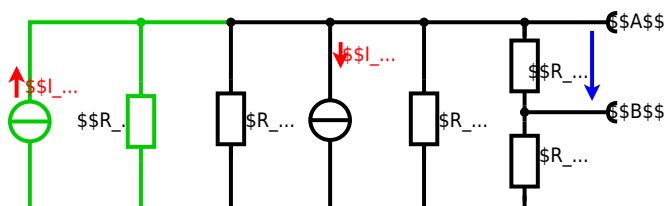
Calculate the internal resistance R_{in} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_6 + R_7)$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(\frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

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Exercise E2 Temperature-dependent Resistance
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. The graph shows the temperature dependence of the resistance of a resistor. The resistor has a resistance of $10 \text{ k}\Omega$ at 25°C . Calculate the resistance of the thermistor at -40°C .

Its temperature coefficients are: $\alpha = 0.01 \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \frac{1}{\text{K}^2}$.

The temperature inside the refrigeration system can reach down to -40°C .

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

Resistor transfer resistor R and U are given. The power P is given by $P = \frac{U^2}{R}$. Therefore, a solution is to increase the resistance R up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

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Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{U} and the phasor current \underline{I} in the circuit shown in the figure. The components R and X_L shall be given.

After analysis, the following phasors can be determined: $\underline{U} = 48.2 \angle 19.8^\circ \text{ V}$ and $\underline{I} = 19.8 \angle -48.2^\circ \text{ A}$.
Result
Solution

1. Calculation of the physical values of the components.
Solution $\underline{U} = 48.2 \angle 19.8^\circ \text{ V}$ and $\underline{I} = 19.8 \angle -48.2^\circ \text{ A}$

Solution
$$\underline{U} = \underline{I} \cdot \underline{Z} = \{50 \angle 0^\circ\} \cdot \{4.68 \angle -90^\circ + j3.0\} = 48.2 \angle 19.8^\circ \text{ V}$$

The current and voltage are in phase since the circuit is purely resistive.
Resulting current $I = 0.24 \text{ A}$ and voltage $U = 4.68 \cdot 0.24 = 1.12 \text{ V}$.
The phase shift is $\varphi = \arctan\left(\frac{-4.68}{3.0}\right) = -56.9^\circ$.
With the complex part comes the complex value $\underline{Z} = 4.68 \angle -90^\circ + j3.0$.
The phase φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(\underline{Z})}{\text{Re}(\underline{Z})}\right) = \arctan\left(\frac{-4.68}{3.0}\right) = -56.9^\circ$.

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Exercise E14 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the resulting phasor voltage \underline{U} and the phasor current \underline{I} in the circuit shown in the figure. The components R and X_L shall be given.
Result
Solution

1. Draw the circuit diagram of the given circuit.
Label all components, voltages, and currents.
Result
 $\underline{U} = 48.2 \angle 19.8^\circ \text{ V}$ and $\underline{I} = 19.8 \angle -48.2^\circ \text{ A}$

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\begin{align*} Z &= \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \\
&\end{align*} \\
\begin{align*} Z_C &= \frac{1}{2\pi f C} \quad \omega = 2\pi f = 2\pi \cdot 15 \\
&= \frac{1}{2\pi \cdot 15 \cdot 330 \cdot 10^{-6}} = 265.258 \text{ } \Omega \\
&\end{align*} \\
\begin{align*} Z_L &= 2\pi f L = 2\pi \cdot 15 \cdot 10^{-3} = 0.942 \text{ } \Omega \\
&\end{align*} \\
\begin{align*} Z &= R + jZ_L - jZ_C \\
&= 10 + j0.942 - j265.258 = 10 - j264.316 \text{ } \Omega \\
&= 264.316 \angle -89.28^\circ \text{ } \Omega \\
&\end{align*} \\
\underline{Z} &= R + \underline{Z}_L - \underline{Z}_C \\
&= 10 + j0.942 - j265.258 = 10 - j264.316 \text{ } \Omega \\
&= \sqrt{10^2 + (-264.316)^2} \angle \arctan\left(\frac{-264.316}{10}\right) \\
&= 264.316 \angle -89.28^\circ \text{ } \Omega \\
&\end{align*}

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Exercise E12 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022)

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{R1R2} = R_1 + R_2$
 Parallel circuit means that the voltage is the same on R_3 and C_1
 The equivalent impedance for R_3 and C_1 combined is given by $Z_{R3C1} = \frac{R_3 \cdot X_{C1}}{R_3 + jX_{C1}}$
 Since X_{C1} is perpendicular to R_3 , this can be simplified to $Z_{R3C1} = \frac{R_3 \cdot X_{C1}}{\sqrt{R_3^2 + X_{C1}^2}}$
 Therefore the resulting current of the parallel circuit is given as:
 $I_{R3C1} = \frac{U}{Z_{R3C1}} = \frac{U \cdot \sqrt{R_3^2 + X_{C1}^2}}{R_3 \cdot X_{C1}}$
 Back to the first formula: $I_{R1R2} \cdot Z_{R1R2} = I_{R3C1} \cdot Z_{R3C1}$
 $I_{R1R2} \cdot (R_1 + R_2) = \frac{U \cdot \sqrt{R_3^2 + X_{C1}^2}}{R_3 \cdot X_{C1}} \cdot \frac{R_3 \cdot X_{C1}}{\sqrt{R_3^2 + X_{C1}^2}}$
 $I_{R1R2} = \frac{U}{R_1 + R_2}$

Solution

$R_1 = 1.00 \cdot \Omega$

$R_2 = 10.0 \cdot \Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{R1R2} = R_1 + R_2$
 Parallel circuit means that the voltage is the same on R_3 and C_1
 The equivalent impedance for R_3 and C_1 combined is given by $Z_{R3C1} = \frac{R_3 \cdot X_{C1}}{R_3 + jX_{C1}}$
 Since X_{C1} is perpendicular to R_3 , this can be simplified to $Z_{R3C1} = \frac{R_3 \cdot X_{C1}}{\sqrt{R_3^2 + X_{C1}^2}}$
 Therefore the resulting current of the parallel circuit is given as:
 $I_{R3C1} = \frac{U}{Z_{R3C1}} = \frac{U \cdot \sqrt{R_3^2 + X_{C1}^2}}{R_3 \cdot X_{C1}}$
 Back to the first formula: $I_{R1R2} \cdot Z_{R1R2} = I_{R3C1} \cdot Z_{R3C1}$
 $I_{R1R2} \cdot (R_1 + R_2) = \frac{U \cdot \sqrt{R_3^2 + X_{C1}^2}}{R_3 \cdot X_{C1}} \cdot \frac{R_3 \cdot X_{C1}}{\sqrt{R_3^2 + X_{C1}^2}}$
 $I_{R1R2} = \frac{U}{R_1 + R_2}$

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

The heating element is used to heat the wire with a temperature of 1800 K .
 Result: power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.
 Calculate the current I needed to operate it.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m}$.

The heating element is 3 m long and has a diameter of 3.57 mm .
 Solution: $R = 10.3 \cdot \Omega$

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi \\ \frac{1}{4} d^2 \cdot \pi \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

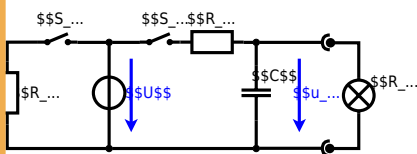
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Exercise E8 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the real battery) also takes into account the internal resistance R_1 of the battery. The capacitor C is initially uncharged. The switch S_1 is closed at $t_0 = 0 \text{ s}$ and the voltage across the capacitor is again U_C at the moment $t_2 = 1 \text{ ms}$ after closing the switch. Calculate the voltage $u_C(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

Solution
The ideal voltage source U is in series with the internal resistance R_1 and the external resistor R_2 . The voltage across the capacitor is $u_C(t) = U \cdot (1 - e^{-t/\tau})$, where $\tau = (R_1 + R_2) \cdot C$.
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

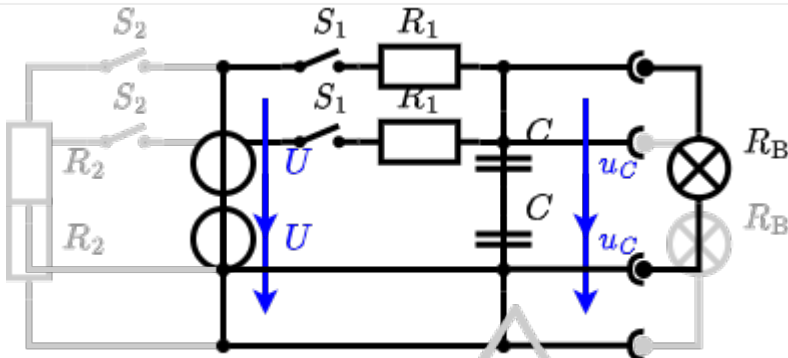


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ } \Omega$ and a capacitor of $C = 100 \text{ } \mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0 \text{ V}$.

- 1. First do not consider the light bulb - it is not connected to the RC circuit.
- 2. Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$:

$$u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$

$$e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$$



An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($R_i = 0 \Omega$, short-circuit).
 $R_i = R_1 \parallel R_B = 10 \Omega$

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ms} / (10 \Omega \cdot 100 \mu\text{F})})$$

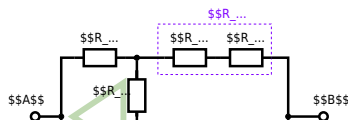
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Exercise E4 Pure Resistor Network Simplification
 (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 10:00h. $R_1 = R_2 = R_3 = 10 \Omega$ and the voltage source $U = 10 \text{V}$.
 Result given: R_B .

Solution

$R_2 = R_3 = 100 \Omega$



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

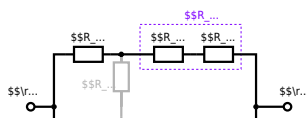
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2) \parallel R_5 = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega \cdot 200 \sim \Omega\} \over {500 \sim \Omega + 200 \sim \Omega}$$

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Last update: **2023/03/11 04:21**

