

# Exam Winter Semester 2022

## Student Group

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## Table of Contents

Exercise E5 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) .....	3
Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) .....	6
Exercise E9 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) .....	7
Exercise E13 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) .....	7
Exercise E11 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) .....	10
Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) .....	10
Exercise E7 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) .....	11
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) .....	12
Exercise E6 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) .....	14
Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) .....	18
Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) .....	19
Exercise E14 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) .....	19
Exercise E12 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) .....	22
Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) .....	22
Exercise E8 Charging Capacitors (written test, approx. 16 % of a 60-minute written test,	

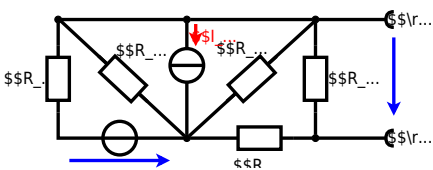
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WS2022) .....	23
Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) .....	24

**Exercise E5 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$\begin{aligned} U_{\text{S}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \Omega \end{aligned}$$



Calculated the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{S}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ .  
 $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$   
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( R_1 || R_3 || R_5 \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration system. The refrigerator has a resistance of  $10 \Omega$  at  $25^\circ\text{C}$  and  $25 \Omega$  at  $-40^\circ\text{C}$ .

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

**Result** The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

$$R = 10 \cdot (1 + 0.01 \cdot \Delta T + 71 \cdot 10^{-6} \cdot \Delta T^2)$$

The power of the resistor is  $P = U^2 / R$  and  $Q = P \cdot t$ . Therefore, a solution is to use a heat pump to heat up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left( 1 + 0.01 \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

**Exercise E9 Analyzing complex Impedances**  
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power  $\underline{S}$  and the real power  $P$  in the circuit shown through the components.  $\underline{U}$  and  $\underline{X}_L$  shall be given.

After analysis, the full complex power  $\underline{S}$  and the real power  $P$  shall be extracted and given in phasor notation.  $\underline{S} = P + jQ$  and  $P = \text{Re}\{\underline{S}\}$ .

Solution  
 .. Calculate the physical values of the two components.  
 Solution  $R = 0.2 \Omega$  and  $X_L = j4.68 \Omega$

Solution  

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{0.2 + j4.68} = 10.5 \angle -87.9^\circ \text{ A}$$
 The voltage across the resistor is  $\underline{U}_R = \underline{I} R = 2.1 \angle -87.9^\circ \text{ V}$   
 The voltage across the inductor is  $\underline{U}_L = \underline{I} X_L = 49.1 \angle -1.1^\circ \text{ V}$   
 The complex power is  $\underline{S} = \underline{U} \underline{I}^* = 50 \angle 0^\circ \cdot 10.5 \angle 87.9^\circ = 525 \angle 87.9^\circ \text{ VA}$   
 The real power is  $P = \text{Re}\{\underline{S}\} = 525 \cos(87.9^\circ) = 52.5 \text{ W}$   
 The reactive power is  $Q = \text{Im}\{\underline{S}\} = 525 \sin(87.9^\circ) = 519.5 \text{ var}$

**Exercise E13 Complex Impedance Circuit**  
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the circuit impedance  $\underline{Z}$  for a sinusoidal voltage  $\underline{u}(t) = 3.0 \sin(2\pi \cdot 15 \cdot t) \text{ V}$  and a current  $\underline{i}(t) = 0.22 \sin(2\pi \cdot 15 \cdot t + \varphi) \text{ A}$  in a circuit.

The circuit consists of an inductor of  $330 \mu\text{H}$  and a capacitor of  $0.22 \mu\text{F}$ , all in series.

Solution  
 Result  
 .. Draw the circuit diagram of the given circuit.  

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0 \angle 0^\circ}{0.22 \angle \varphi} = 13.6 \angle -\varphi \Omega$$

$$\underline{Z} = j\omega L - j\omega C = j2\pi \cdot 15 \cdot 330 \cdot 10^{-6} - j2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6} = j19.8 - j0.2 \Omega = j19.6 \Omega$$

$$\varphi = \arctan\left(\frac{\text{Im}\{\underline{Z}\}}{\text{Re}\{\underline{Z}\}}\right) = \arctan\left(\frac{19.6}{0}\right) = 90^\circ$$

$$\underline{Z} = 13.6 \angle -90^\circ = -j13.6 \Omega$$





**Exercise E11 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ , and an inductor with an inductance of  $L_1 = 4.7 \text{ }\mu\text{H}$ . The circuit is connected to an AC voltage source with a voltage of  $U = 10 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance  $|Z|$  of the circuit.

Solution

$$R_1 = 1.00 \text{ k}\Omega$$

$$R_2 = 10.0 \text{ }\Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for  $R_1$  and  $R_2$  combined is given by 
$$R_{\text{total}} = R_1 + R_2$$

Parallel circuit means that the voltage is the same on  $R_3$  and  $C_3$  
$$\frac{1}{X_{\text{parallel}}} = \frac{1}{R_3} + \frac{1}{X_{C_3}}$$

Since  $X_{C_3}$  is perpendicular to  $R_3$ , this can be simplified to 
$$X_{\text{parallel}} = \frac{R_3 \cdot X_{C_3}}{\sqrt{R_3^2 + X_{C_3}^2}}$$

$R_3$  is perpendicular to  $X_{L_3}$  (It has to, since  $R_3$  is perpendicular to  $X_{C_3}$  and  $X_{L_3}$  is perpendicular to  $X_{C_3}$ ) 
$$Z_{\text{total}} = \sqrt{(R_{\text{total}})^2 + (X_{\text{parallel}})^2}$$

Therefore, the resulting current of the parallel circuit is given as: 
$$I_{\text{total}} = \frac{U}{Z_{\text{total}}}$$

This can be rearranged to determine  $R_3$  
$$R_3 = \frac{U \cdot I_{\text{total}}}{\sqrt{I_{\text{total}}^2 - (2\pi \cdot f \cdot 450 \text{ kHz} \cdot 4.7 \text{ }\mu\text{H})^2}}$$

Back to the first formula: 
$$R_3 \cdot I_{\text{total}} = X_{C_3} \cdot \frac{I_{\text{total}}}{\sqrt{I_{\text{total}}^2 - (2\pi \cdot f \cdot C_3)^2 - I_{\text{total}}^2}}$$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. Heating elements are used to heat the oven with a temperature of  $180 \text{ }^\circ\text{C}$ . The electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary. Calculate the current  $I$  needed to operate the heating elements.

The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .

The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .

Solution

Calculate the resistance  $R$  of the heating element.

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

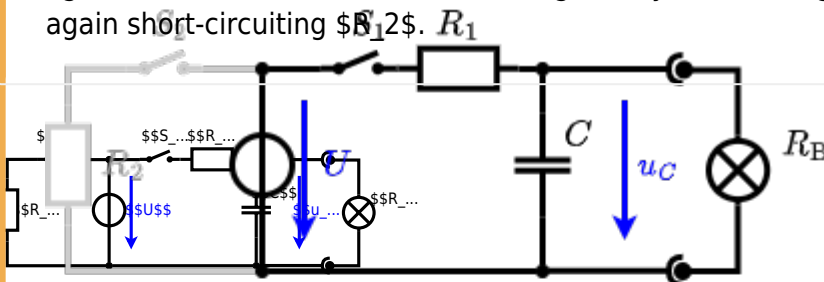
**Exercise E7 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (with the light bulb) is a series of  $R_1$  and  $R_2$  and a capacitor  $C$  and a switch  $S_1$  and a switch  $S_2$ . The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U$  is in series with  $R_1$  and  $R_2$ . The voltage  $u_c$  is independent of this series.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_2$ .

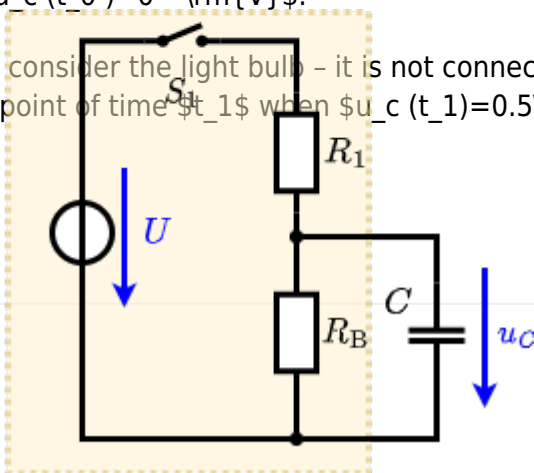


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \Omega$  and a capacitor of  $C = 100 \mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

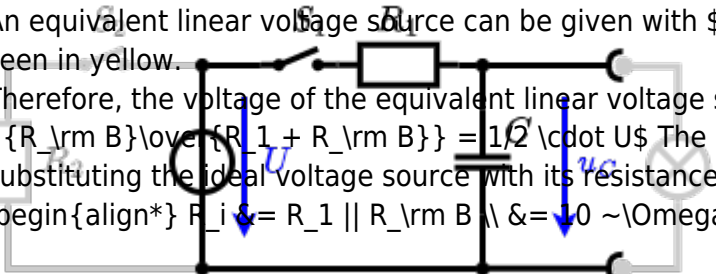
$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $t$ :  

$$(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) \implies t = R_1 \cdot C \cdot \ln(0.5)$$



**Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be simplified to a one-port network  $R_{eq}$  between terminals A and B.  $R_1 = 20 \Omega$ ,  $R_2 = R_3 = 10 \Omega$ ,  $R_4 = 15 \Omega$  and the switch is closed.

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



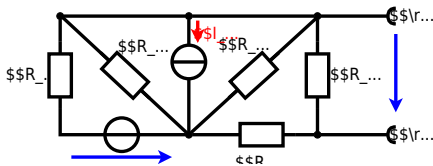
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E6 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



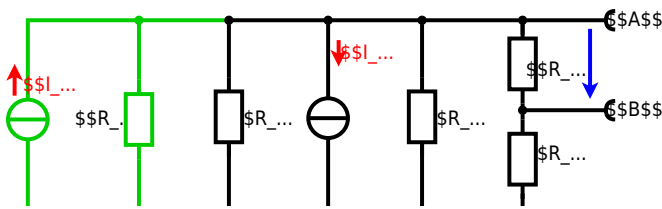
Calculate the internal resistance  $R_{int}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( R_1 || R_3 || R_5 \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \Omega \cdot \left( \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration systems. The refrigerator has a resistance of  $10 \Omega$  at  $25^\circ\text{C}$  and  $25 \Omega$  at  $-40^\circ\text{C}$ .

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

**Result** The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

$$R_{-40} = 10 \cdot (1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2) = 25 \Omega$$

The power transfer is  $P = U \cdot I = \frac{U^2}{R}$ . Therefore, a solution is to use a heat pump.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left( 1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2 \right)$$

### Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

1. Calculate the complex impedance  $Z$  of the circuit shown in the figure through the components.  $R$  and  $X_L$  shall be given.

After analysis, the full bridge circuit can be simplified to a series circuit in phasor domain. The voltage  $U$  and current  $I$  are given by  $U = 50 \sin(\omega t)$  and  $I = I_m \sin(\omega t + \varphi)$ .

Solution  
.. Calculate the physical values of the two components.  
Solution  $R = 10 \Omega$  and  $X_L = 20 \Omega$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \text{with } \underline{U} = 50 \angle 0^\circ \text{ V and } \underline{Z} = R + jX_L = 10 + j20 \Omega$$
  
The voltage  $U$  and current  $I$  are given by  $U = 50 \sin(\omega t)$  and  $I = I_m \sin(\omega t + \varphi)$ .  
resulting impedance  $Z = R + jX_L = 10 + j20 \Omega$ .  
Therefore, the component  $X_L$  is a pure inductor with the same angular frequency  $\omega$ .  
Impedance  $Z = R + jX_L = 10 + j20 \Omega$ .  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{10 + j20} = \frac{50}{\sqrt{10^2 + 20^2}} \angle -\arctan\left(\frac{20}{10}\right) = 2 \angle -63.4^\circ \text{ A}$$
  
The phase  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{-X_L}{R}\right) = \arctan\left(\frac{-20}{10}\right) = -63.4^\circ$ .  
With the complex part  $Z = R + jX_L = 10 + j20 \Omega$  and  $\underline{U} = 50 \angle 0^\circ \text{ V}$ ,  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{10 + j20} = 2 \angle -63.4^\circ \text{ A}$$
  
The phase  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{-X_L}{R}\right) = \arctan\left(\frac{-20}{10}\right) = -63.4^\circ$ .

### Exercise E14 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the circuit impedance  $Z$  for the circuit shown in the figure. The voltage source  $u(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$  V is connected to a series circuit of an inductor of  $L = 330 \mu\text{H}$  and a capacitor of  $C = 0.22 \mu\text{F}$ .

Solution  
Result  
.. Draw the equivalent circuit of the given circuit.  
Label all components, voltages, and currents.

$$Z = \frac{U}{I} = \frac{3.0 \text{ V}}{0.22 \text{ A}} = 13.6 \Omega$$
  
$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j19.2 \text{ k}\Omega$$
  
$$Z_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} = j3.1 \text{ }\Omega$$
  
Result  $Z = 13.6 \text{ }\Omega - j19.2 \text{ k}\Omega + j3.1 \text{ }\Omega = 13.6 \text{ }\Omega - j19.2 \text{ k}\Omega$





**Exercise E12 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

**2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$  and a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ . The voltage across the resistor is  $U_{R_1} = 100 \text{ V}$  at a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance of the capacitor  $Z_C$  and the total impedance  $Z_{\text{total}}$  of the circuit.**

**Solution**

$$Z_C = \frac{1}{j\omega C_1} = \frac{1}{j \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = -j 0.995 \text{ k}\Omega$$

$$Z_{\text{total}} = R_1 + Z_C = 1.00 \text{ k}\Omega - j 0.995 \text{ k}\Omega$$

$$|Z_{\text{total}}| = \sqrt{1.00^2 + 0.995^2} \text{ k}\Omega = 1.41 \text{ k}\Omega$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R_1$  and  $Z_C$  combined is given by  $Z_{\text{total}} = R_1 + Z_C$ .  
 Parallel circuit means that the voltage is the same on  $R_1$  and  $Z_C$ .  

$$U_{R_1} = I \cdot R_1 \Rightarrow I = \frac{U_{R_1}}{R_1} = \frac{100 \text{ V}}{1.00 \text{ k}\Omega} = 0.1 \text{ A}$$
  

$$U_{Z_C} = I \cdot Z_C \Rightarrow Z_C = \frac{U_{Z_C}}{I} = \frac{100 \text{ V}}{0.1 \text{ A}} = 1.00 \text{ k}\Omega$$
  
 Therefore, the resulting current of the parallel circuit is given as:  

$$I_{\text{total}} = I_{R_1} + I_{Z_C} = 0.1 \text{ A} + 0.1 \text{ A} = 0.2 \text{ A}$$
  
 This current is the same as the current through  $R_1$ .  

$$U_{\text{total}} = I_{\text{total}} \cdot Z_{\text{total}} = 0.2 \text{ A} \cdot 1.41 \text{ k}\Omega = 282 \text{ V}$$
  
 Back to the first formula:  $Z_{\text{total}} = R_1 + Z_C$   

$$|Z_{\text{total}}| = \sqrt{R_1^2 + |Z_C|^2} \Rightarrow |Z_C| = \sqrt{|Z_{\text{total}}|^2 - R_1^2} = \sqrt{1.41^2 - 1.00^2} \text{ k}\Omega = 1.00 \text{ k}\Omega$$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

**2. For a heating element, it is required to heat the water in a tank with a volume of  $V = 180 \text{ l}$  by electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary. Calculate the current  $I$  and the operating voltage  $U$  for heating elements. The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .**

The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

**Solution**

$$P = U \cdot I = R \cdot I^2 \Rightarrow I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

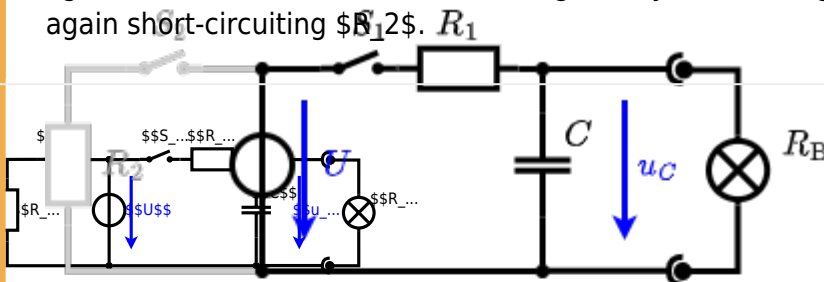
**Exercise E8 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (with the light bulb) is a series of  $R_1$  and  $R_2$  and a capacitor  $C$  as indicated in Figure 8.5. Initially the switch  $S_1$  is open. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U$  is in series with  $R_1$  and  $R_2$ . The voltage  $u_c$  is independent of this series combination.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_2$ .

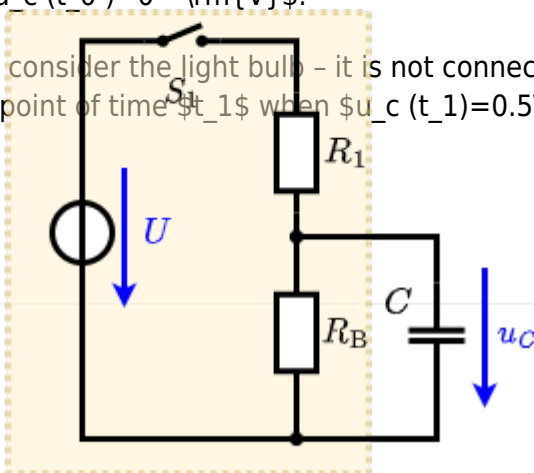


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$   
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5)$



**Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at 0 degree,  $R_1 = R_2 = R_3 = 10 \Omega$  and the voltage  $U = 10V$  is given.  $R_B$ .

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

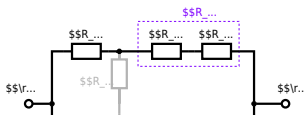
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega}$$

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