

Exam Winter Semester 2022

Student Group

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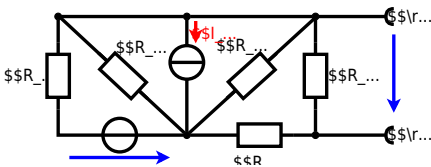
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**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$\begin{aligned} U_{\text{S}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \Omega \end{aligned}$$



Calculated the internal resistance R_{i} and the source voltage U_{S} of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{135} + I_{24} \cdot R_6 + I_{24} \cdot R_7$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot R_1 || R_3 || R_5$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot 15 \Omega \cdot 2.5 \Omega \cdot \frac{1}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration systems. The refrigerator has a resistance of 10Ω at 25°C and 25 W of power.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result
The temperature inside the refrigeration system can reach down to -40°C .

$$R = 6.5 \text{ k}\Omega$$

The power of the resistor is $P = U \cdot I = 2 \text{ W}$ and $Q = P \cdot t = 2 \text{ W} \cdot 2 \text{ h} = 4 \text{ Wh}$. Therefore, a solution is to use a heat pump to heat up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

Exercise E9 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power \underline{S} and the real power P in the circuit shown through the components. \underline{U} and \underline{X}_L shall be given.

After analysis, the full bridge network can be simplified, and the voltage \underline{U} and the current \underline{I} in the branch can be determined. $\underline{S} = \underline{U} \cdot \underline{I}^*$ and $P = \text{Re}\{\underline{S}\}$.

Solution
 .. Calculate the physical values of the two components.
 Solution $R = 10 \Omega$ and $X_L = 20 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{0.24 - j4.68 + j9.36} = \frac{50 \angle 0^\circ}{0.24 + j4.68} = 10.4 \angle -87.06^\circ \text{ A}$$
 The voltage \underline{U} is the voltage across the 0.24Ω resistor. The resulting impedance is $\underline{Z} = 0.24 - j4.68 \Omega$.
 Therefore, the component 0.24Ω is in series with the same current \underline{I} .

$$\underline{U} = \underline{I} \cdot \underline{Z} = 10.4 \angle -87.06^\circ \cdot (0.24 - j4.68) = 2.5 \angle -87.06^\circ - j48.5 \angle -87.06^\circ$$

$$\underline{U} = 2.5 \cos(\omega t - 87.06^\circ) - 48.5 \sin(\omega t - 87.06^\circ) \text{ V}$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{-48.5}{2.5}\right) = -87.06^\circ$.
 With the complex part $\cos(\omega t - 87.06^\circ)$ and $\sin(\omega t - 87.06^\circ)$ $\underline{U} = 2.5 \cos(\omega t - 87.06^\circ) - 48.5 \sin(\omega t - 87.06^\circ)$ V

$$\underline{S} = \underline{U} \cdot \underline{I}^* = (2.5 - j48.5) \cdot (10.4 \angle 87.06^\circ) = 26.0 - j500.0 \text{ VA}$$
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{-500}{26}\right) = -87.06^\circ$.

Exercise E13 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the circuit impedance \underline{Z} for a source $\underline{u}(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V and $\underline{Z}_C = 0.22 \mu\text{F}$ in a voltage source $\underline{u}(t) = 3.0 \sin(2\pi \cdot 15 \cdot t)$ V. The circuit consists of a series combination of a resistor of 10Ω and a capacitor of $0.22 \mu\text{F}$.

Solution
 Result $\underline{Z} = 10 - j19.8 \Omega$

.. Draw the circuit diagram of the given network and label all components, voltages, and currents.

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0 \angle 0^\circ}{0.15 \angle 0^\circ} = 20 \Omega$$

$$\underline{Z}_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j19.8 \Omega$$

$$\underline{Z} = 10 - j19.8 \Omega$$



Exercise E11 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$, a capacitor with a capacitance of $C_1 = 40 \text{ nF}$, and an inductor with an inductance of $L_1 = 4.7 \text{ }\mu\text{H}$. The circuit is connected to an AC voltage source with a voltage of $U = 10 \text{ V}$ and a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance $|Z|$ of the circuit.

Solution

$$R_1 = 1.00 \text{ k}\Omega$$

$$R_2 = 10.0 \text{ }\Omega$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and R_2 combined is given by $R_{1+2} = R_1 + R_2$
 Parallel circuit means that the voltage is the same on R_3 and C_1 $\frac{1}{Z} = \frac{1}{R_3} + \frac{1}{Z_{C_1}}$

$$\frac{1}{Z} = \frac{1}{R_3} + \frac{1}{\frac{1}{j\omega C_1}}$$

$$\frac{1}{Z} = \frac{1}{R_3} + j\omega C_1$$

$$Z = \frac{R_3}{1 + j\omega C_1 R_3}$$

$$|Z| = \frac{R_3}{\sqrt{1 + (\omega C_1 R_3)^2}}$$

$$|Z| = \frac{10 \text{ V}}{\sqrt{1 + (2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9})^2}}$$

$$|Z| = \frac{10 \text{ V}}{\sqrt{1 + (100.53)^2}}$$

$$|Z| = \frac{10 \text{ V}}{100.53}$$

$$|Z| = 0.0994 \text{ V}$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. Heating elements are used to heat the oven with a temperature of $180 \text{ }^\circ\text{C}$. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.
 Calculate the current I needed to operate the heating elements.
 The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.
 The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.
 Calculate the resistance R of the heating element.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

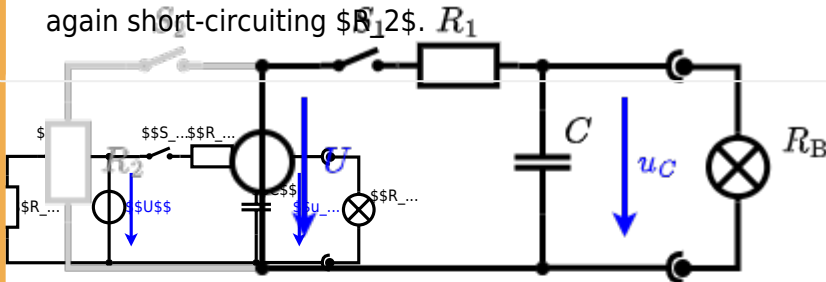
Exercise E7 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) is a series of R_1 and R_2 and a capacitor C and a light bulb R_B . The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

The ideal voltage source U is in series with R_1 and R_2 . The voltage u_c is independent of this series combination.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .



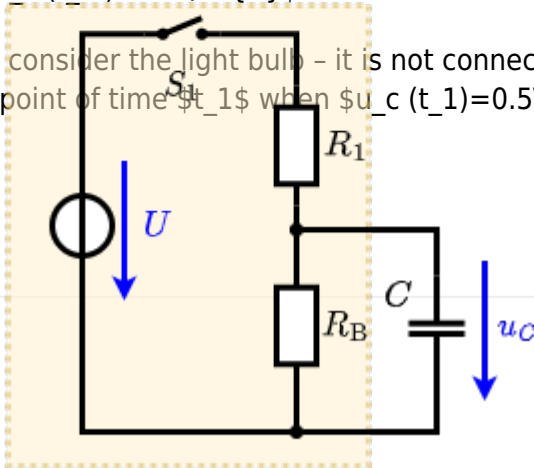
The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be simplified to a single resistor R_{eq} and the voltage U_{AB} across it. $R_1 = 20 \Omega$, $R_2 = R_3 = 10 \Omega$, $R_4 = 15 \Omega$ and the voltage $U = 10 \text{ V}$ is given.

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



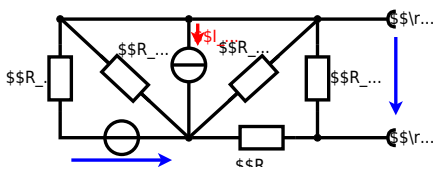
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel R_{\text{eq}}$$

**Exercise E6 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

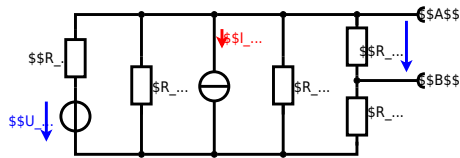
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



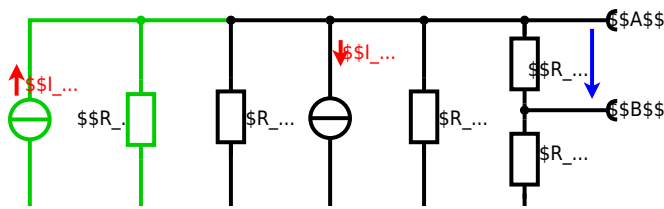
Calculate the internal resistance R_{int} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{24} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \text{ A} \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. The diagram explains the effect of resistance on the refrigeration system. The circuit has a resistance of 10Ω at 25°C and 25Ω at 0°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

Result The temperature inside the refrigeration system can reach down to -40°C .

$$R_{25} = 10 \Omega$$

The power transfer is $P = U \cdot I$ and $P = \frac{U^2}{R}$. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$



Exercise E12 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Question: A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$ and a capacitor with a capacitance of $C_1 = 40 \text{ nF}$. The circuit is connected to an AC voltage source with a peak voltage of $U_0 = 10 \text{ V}$ and a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance $|Z|$ of the circuit.

Solution:

The impedance of a resistor is $Z_R = R$ and the impedance of a capacitor is $Z_C = -j / (\omega C)$. The total impedance of a series circuit is the sum of the individual impedances:

$$Z = R_1 + Z_{C1} = R_1 - j / (\omega C_1)$$

The absolute value of the impedance is given by:

$$|Z| = \sqrt{R_1^2 + \left(\frac{1}{\omega C_1} \right)^2}$$

Substituting the given values:

$$|Z| = \sqrt{(1000)^2 + \left(\frac{1}{2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} \right)^2}$$

$$|Z| \approx \sqrt{1000000 + 1562500} \approx \sqrt{2562500} \approx 1600 \text{ }\Omega$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

Question: A heating element made of nichrome wire is used to heat water with a temperature rise of $\Delta T = 180 \text{ K}$. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the current I needed to operate the heating element. The nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

Solution:

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.

Calculate the resistance R of the heating element.

$$R = \rho \cdot \frac{l}{A} = \rho \cdot \frac{l}{\pi \cdot \left(\frac{d}{2} \right)^2}$$

$$R = 1.10 \cdot 10^{-6} \cdot \frac{3}{\pi \cdot \left(\frac{3.57 \cdot 10^{-3}}{2} \right)^2} \approx 0.001 \text{ }\Omega$$

Using the power formula $P = I^2 R$:

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40}{0.001}} = \sqrt{40000} = 200 \text{ A}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} \cdot d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

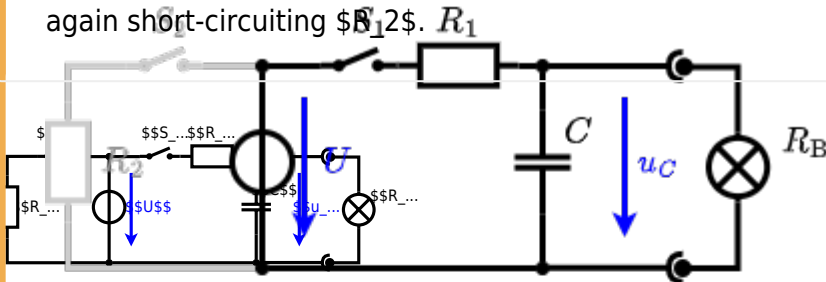
Exercise E8 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) is a series of R_1 and R_2 and a capacitor C as indicated in Figure 8.5. Initially the voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

The ideal voltage source U is in series with R_1 and R_2 . The voltage u_c is $U \cdot \frac{R_B}{R_1 + R_2 + R_B}$.

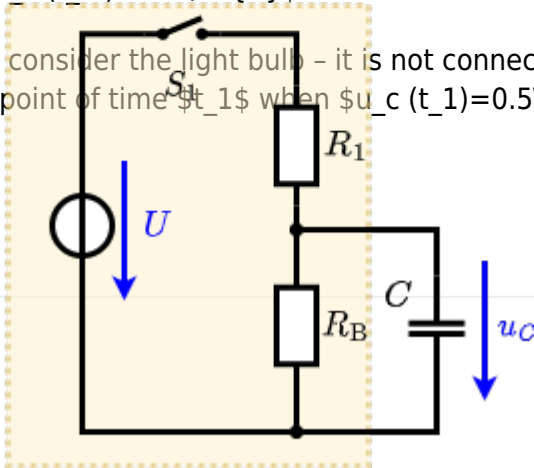
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be simplified with $R_1 = 20 \Omega$, $R_2 = R_3 = 10 \Omega$, $R_4 = 5 \Omega$ and the voltage $U = 10 \text{ V}$. Result: R_{eq} .

Solution

$$R_{eq} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

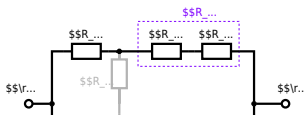


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \text{ } \Omega)^2}{3 \cdot 100 \text{ } \Omega} = \frac{1}{3} \cdot 100 \text{ } \Omega = 33.33 \text{ } \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \text{ } \Omega + (33.33 \text{ } \Omega + 400 \text{ } \Omega) \parallel (33.33 \text{ } \Omega + 100 \text{ } \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega \} \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega \cdot 200 \sim \Omega \} \over {500 \sim \Omega + 200 \sim \Omega} \parallel$$

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