

Exam Winter Semester 2022

Student Group

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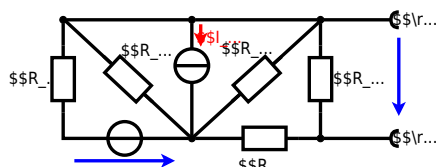
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**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{rs}} = U_{\text{AB}} = 4.5 \text{ V}$$

$$R_{\text{i}} = R_{\text{AB}} = 6 \text{ } \Omega$$



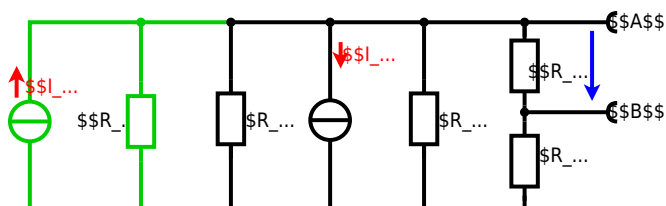
Calculated the internal resistance R_{i} and the source voltage U_{rs} of an equivalent linear voltage source on the connectors A and B .
$$R_1=5.0 \text{ } \Omega, \quad U_2=6.0 \text{ V}, \quad R_3= 10 \text{ } \Omega, \quad I_4=4.2 \text{ A}, \quad R_5=10 \text{ } \Omega, \quad R_6=7.5 \text{ } \Omega, \quad R_7=15 \text{ } \Omega$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1, R_3, R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(R_1 || R_3 || R_5 \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \text{ A} \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration system. The refrigerator has a resistance of 10Ω at 25°C and 25Ω at -40°C . Your answer.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

Result
The temperature inside the refrigeration system can reach down to -40°C .

$$R_{-40} = 10 \cdot (1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2) = 25 \Omega$$

The power transfer is $P = U \cdot I = \frac{U^2}{R}$. Therefore, a solution is to heat up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left(1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2 \right)$$

Exercise E9 Analyzing complex Impedances
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power \underline{S} (W and var) of the load \underline{Z}_L through the components. \underline{S}_R and \underline{S}_{X_1} shall be given.

After analysis, the full load impedance \underline{Z}_L can be extracted and brought in phasor notation $\underline{Z}_L = |Z_L| \cdot e^{j\varphi}$ with $\varphi = \varphi_L - \varphi_U$.

Solution
 .. Calculate the physical values of the load components.
 Solution $R = 10 \Omega$, $X_1 = 20 \Omega$, $X_2 = 20 \Omega$, $X_3 = 20 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle 0^\circ}{\frac{50}{\sqrt{2}} \angle 45^\circ} = \sqrt{2} \angle -45^\circ$$
 The voltage across the load is $\underline{U}_L = \underline{I} \cdot \underline{Z}_L = \sqrt{2} \angle -45^\circ \cdot (10 + j20) = 20\sqrt{2} \angle 0^\circ$
 resulting in $P = \text{Re}\{\underline{S}\} = \text{Re}\{\underline{U}_L \cdot \underline{I}^*\} = 20\sqrt{2} \cdot \sqrt{2} \cdot \cos(0^\circ - (-45^\circ)) = 40\sqrt{2} \cdot \cos(45^\circ) = 40$ W
 The reactive power is $Q = \text{Im}\{\underline{S}\} = \text{Im}\{\underline{U}_L \cdot \underline{I}^*\} = 20\sqrt{2} \cdot \sqrt{2} \cdot \sin(0^\circ - (-45^\circ)) = 40\sqrt{2} \cdot \sin(45^\circ) = 40$ var
 With the complex power $\underline{S} = P + jQ = 40 + j40$ var, the physical values are $P = 40$ W and $Q = 40$ var.
 The phase φ can be calculated as $\varphi = \arctan\left(\frac{Q}{P}\right) = \arctan\left(\frac{40}{40}\right) = 45^\circ$

Exercise E13 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the circuit impedance \underline{Z} for $f = 15$ kHz. The load \underline{Z}_L is a parallel combination of \underline{Z}_1 and \underline{Z}_2 . The voltage source $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V is connected to a series combination of an inductor of $330 \mu\text{H}$ and a capacitor of $0.22 \mu\text{F}$.

Solution
 Result
 .. Draw the circuit diagram of the given circuit.

Solution

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0 \angle 0^\circ}{\sqrt{2} \angle -45^\circ} = \frac{3.0}{\sqrt{2}} \angle 45^\circ = 2.12 \angle 45^\circ \Omega$$

$$\underline{Z}_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 10^3 \cdot 0.22 \cdot 10^{-6}} = -j1.51 \Omega$$

$$\underline{Z}_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 10^3 \cdot 330 \cdot 10^{-6} = j31.6 \Omega$$

$$\underline{Z} = \frac{\underline{Z}_C \cdot \underline{Z}_L}{\underline{Z}_C + \underline{Z}_L} = \frac{-j1.51 \cdot j31.6}{-j1.51 + j31.6} = \frac{47.7}{j30.1} = 1.58 \angle -90^\circ \Omega$$



Exercise E11 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$ and a capacitor with a capacitance of $C_1 = 40 \text{ nF}$. The voltage across the resistor is $U_{R_1} = 100 \text{ V}$ at a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance of the capacitor Z_C and the total impedance Z_{total} of the circuit.

Solution

$$Z_C = \frac{1}{j\omega C_1} = \frac{1}{j \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = -j0.995 \text{ k}\Omega$$

$$Z_{\text{total}} = R_1 + Z_C = 1.00 \text{ k}\Omega - j0.995 \text{ k}\Omega$$

$$|Z_{\text{total}}| = \sqrt{1.00^2 + 0.995^2} \approx 1.41 \text{ k}\Omega$$

A series circuit means that the current is constant on every component. The equivalent impedance for R_1 and Z_C combined is given by $Z_{\text{total}} = R_1 + Z_C$. Parallel circuit means that the voltage is the same on R_1 and Z_C . $U_{R_1} = U_{Z_C} = 100 \text{ V}$. Since Z_C is perpendicular to R_1 , the resulting current of the parallel circuit is given as: $I_{\text{total}} = I_{R_1} + I_{Z_C}$. $I_{R_1} = \frac{U_{R_1}}{R_1} = \frac{100 \text{ V}}{1.00 \text{ k}\Omega} = 0.1 \text{ A}$. $I_{Z_C} = \frac{U_{Z_C}}{|Z_C|} = \frac{100 \text{ V}}{0.995 \text{ k}\Omega} \approx 0.1005 \text{ A}$. $I_{\text{total}} = 0.1 \text{ A} + 0.1005 \text{ A} = 0.2005 \text{ A}$. $Z_{\text{total}} = \frac{U_{R_1}}{I_{\text{total}}} = \frac{100 \text{ V}}{0.2005 \text{ A}} \approx 499 \text{ }\Omega$.

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. Heating elements are used to heat the oven with a temperature of $180 \text{ }^\circ\text{C}$. The electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the current I and the resistance R of the heating element. The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$R = \frac{\rho \cdot l}{A} = \frac{1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m} \cdot 3 \text{ m}}{\pi \cdot (0.00357 \text{ m})^2} \approx 8.1 \text{ }\Omega$$

$$I = \sqrt{\frac{40 \text{ W}}{8.1 \text{ }\Omega}} \approx 2.2 \text{ A}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} \cdot d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

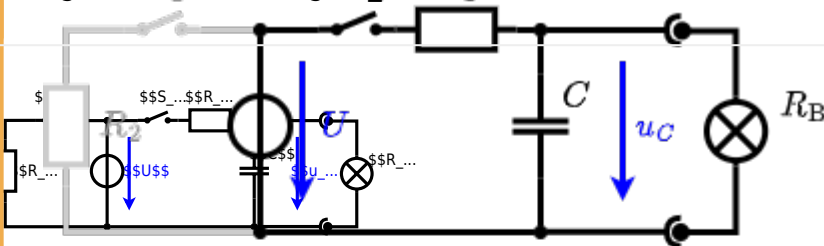
Exercise E7 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) also consists of a DC voltage source $U = 12 \text{ V}$, a resistor $R_1 = 20 \text{ }\Omega$, a capacitor $C = 100 \text{ }\mu\text{F}$, and a light bulb $R_B = 5 \text{ }\Omega$. The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution
 Hint: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

Solution
 The ideal voltage source U is in series with R_1 and R_B . The voltage u_c is independent of this series combination.

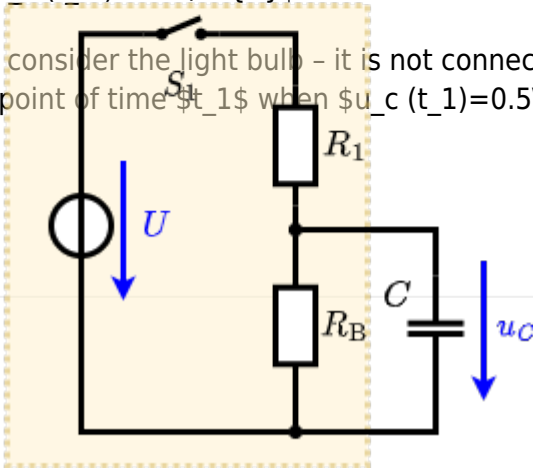
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_1 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

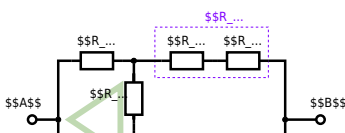
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at a rate of $R_1 = R_2 = R_3 = 10 \Omega$ and the voltage $U = 10V$ is given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

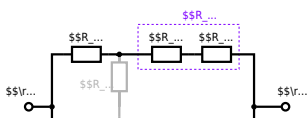
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \text{ } \Omega)^2}{3 \cdot 100 \text{ } \Omega} = \frac{1}{3} \cdot 100 \text{ } \Omega = 33.33 \text{ } \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \text{ } \Omega + (33.33 \text{ } \Omega + 400 \text{ } \Omega) \parallel (33.33 \text{ } \Omega + 100 \text{ } \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



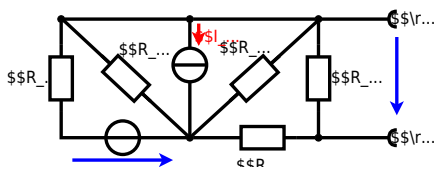
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E6 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



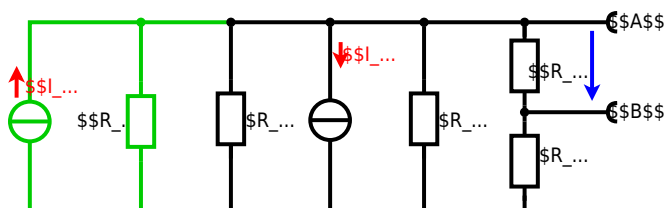
Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B .
 $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$,
 $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{6135}$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \text{ A} \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right)$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator is explained with the effect of resistance on refrigeration system. The refrigerator has a resistance of 10Ω at 25°C and 25 W of power.

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

Result
The temperature inside the refrigeration system can reach down to -40°C .

$$R = 10 \cdot (1 + 0.01 \cdot \Delta T + 71 \cdot 10^{-6} \cdot \Delta T^2)$$

The power of the resistor is $P = U \cdot I = \frac{U^2}{R}$ and $Q = P \cdot t$. Therefore, a solution is to use a heat pump to heat up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left(1 + 0.01 \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

1. Calculate the complex impedance Z of the circuit shown in the figure. The voltage $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.24 \cdot \cos(2\pi \cdot 15 \cdot t)$ A shall be given.

After analysis, the following dimensions must be indicated: Z in Ω , U in V, I in A, φ in degrees. The phase shift φ is defined as $\varphi = \varphi_U - \varphi_I$.

Solution
.. Calculate the physical values of the two components.
Solution $R = 10 \Omega$, $X_L = 2\pi \cdot 15 \cdot 0.2 = 1.88 \Omega$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \implies \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0 \cdot e^{j(2\pi \cdot 15 \cdot t)}}{0.24 \cdot e^{j(2\pi \cdot 15 \cdot t + \varphi)}} = 12.5 \cdot e^{-j\varphi}$$

The voltage $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V is a sine wave with an amplitude of 3.0 V and a phase shift of 0° . The current $i(t) = 0.24 \cdot \cos(2\pi \cdot 15 \cdot t)$ A is a cosine wave with an amplitude of 0.24 A and a phase shift of 90° . The resulting impedance Z is $12.5 \cdot e^{-j\varphi}$.

Therefore, the component R is a resistor with a value of $12.5 \cdot \cos(\varphi)$ Ω and the component X_L is an inductor with a value of $12.5 \cdot \sin(\varphi)$ Ω . The phase shift φ is $90^\circ - \varphi_I$.

With the complex part $Z = R + jX_L$ and the physical values $R = 10 \Omega$ and $X_L = 1.88 \Omega$, the phase shift φ can be calculated as $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{1.88}{10}\right) = 10.6^\circ$.

The phase shift φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{-4.68}{0.24}\right) = -10.6^\circ$.

Exercise E14 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the complex impedance Z of the circuit shown in the figure. The voltage $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V and the current $i(t) = 0.24 \cdot \cos(2\pi \cdot 15 \cdot t)$ A shall be given.

After analysis, the following dimensions must be indicated: Z in Ω , U in V, I in A, φ in degrees. The phase shift φ is defined as $\varphi = \varphi_U - \varphi_I$.

Solution
Result
.. Draw the circuit diagram of the given circuit.
Solution $Z = 10 \Omega$, $X_L = 2\pi \cdot 15 \cdot 0.2 = 1.88 \Omega$, $X_C = \frac{1}{2\pi \cdot 15 \cdot 0.22} = 1.52 \Omega$

With the complex part $Z = R + jX_L - jX_C$ and the physical values $R = 10 \Omega$, $X_L = 1.88 \Omega$, and $X_C = 1.52 \Omega$, the phase shift φ can be calculated as $\varphi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{1.88 - 1.52}{10}\right) = 1.8^\circ$.

The phase shift φ can be calculated as $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{-1.94}{19.28}\right) = -5.7^\circ$.



Exercise E12 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

2. A series circuit consists of a resistor with a resistance of $R_1 = 1.00 \text{ k}\Omega$ and a capacitor with a capacitance of $C_1 = 40 \text{ nF}$. The voltage across the resistor is $U_{R_1} = 100 \text{ V}$ at a frequency of $f = 4 \text{ MHz}$. Calculate the absolute value of the impedance of the capacitor Z_C and the total impedance Z_{total} of the circuit.

Solution

$$Z_C = \frac{1}{j\omega C_1} = \frac{1}{j \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = -j0.995 \text{ k}\Omega$$

$$Z_{\text{total}} = R_1 + Z_C = 1.00 \text{ k}\Omega - j0.995 \text{ k}\Omega$$

$$|Z_{\text{total}}| = \sqrt{1.00^2 + 0.995^2} \text{ k}\Omega \approx 1.41 \text{ k}\Omega$$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R_1 and Z_C combined is given by $Z_{\text{total}} = R_1 + Z_C$.
 Parallel circuit means that the voltage is the same on R_1 and Z_C .

$$U_{R_1} = I \cdot R_1 \Rightarrow I = \frac{U_{R_1}}{R_1} = \frac{100 \text{ V}}{1.00 \text{ k}\Omega} = 0.1 \text{ A}$$

$$U_{Z_C} = I \cdot Z_C \Rightarrow Z_C = \frac{U_{Z_C}}{I} = \frac{100 \text{ V}}{0.1 \text{ A}} = 1.00 \text{ k}\Omega$$

 Therefore, the resulting current of the parallel circuit is given as:

$$I_{\text{total}} = I_{R_1} + I_{Z_C} = 0.1 \text{ A} + 0.1 \text{ A} = 0.2 \text{ A}$$

 This current is the same as the current through R_1 .

$$U_{\text{total}} = I_{\text{total}} \cdot Z_{\text{total}} = 0.2 \text{ A} \cdot 1.41 \text{ k}\Omega = 282 \text{ V}$$

 Back to the first formula:

$$R_1 \cdot I_{\text{total}} = U_{R_1} \Rightarrow R_1 = \frac{U_{R_1}}{I_{\text{total}}} = \frac{100 \text{ V}}{0.2 \text{ A}} = 500 \text{ }\Omega$$

$$Z_C = \frac{U_{Z_C}}{I_{\text{total}}} = \frac{100 \text{ V}}{0.2 \text{ A}} = 500 \text{ }\Omega$$

Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

2. For heating elements used to heat the oven in a domestic use electric oven, a power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary. Calculate the current I and the resistance R of the heating element.
 The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$.
 The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.

Solution

$$P = U \cdot I \Rightarrow I = \frac{P}{U} = \frac{40 \text{ W}}{230 \text{ V}} \approx 0.174 \text{ A}$$

$$R = \frac{U}{I} = \frac{230 \text{ V}}{0.174 \text{ A}} \approx 1322 \text{ }\Omega$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \text{align*} \\ \frac{1}{4} d^2 \cdot \pi \cdot R &= \rho \cdot l \cdot \frac{1}{d^2 \cdot \pi} \quad \text{align*} \\ R &= \frac{4 \cdot l}{d^2 \cdot \pi} \quad \text{align*} \\ R &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \quad \text{align*} \end{aligned}$$

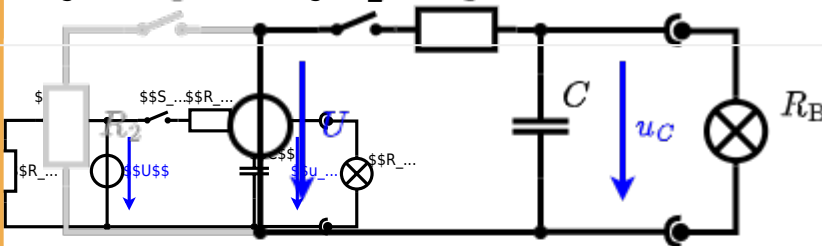
Exercise E8 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (with the light bulb) is a series RC circuit. The capacitor is initially uncharged. At $t_0 = 0 \text{ s}$ the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

The ideal voltage source U is in series with R_1 and R_2 . The voltage u_c is independent of the choice of R_1 and R_2 .

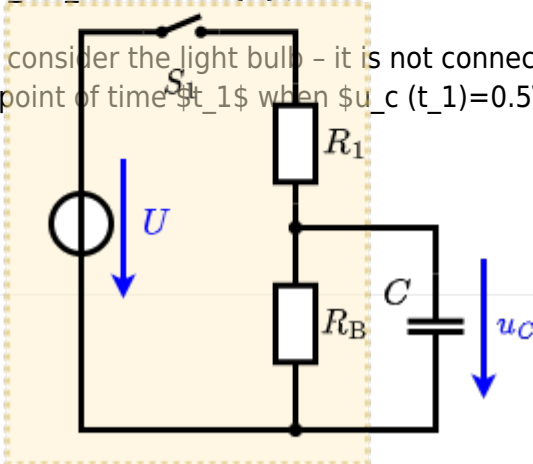
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting S_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first tasks. At the moment $t_0 = 0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0 \text{ V}$.

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.

Solution



An equivalent linear voltage source can be given with U_s , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($R_1 = 0 \Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



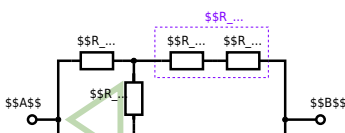
Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at 0.10 A, $R_1 = R_2 = R_3 = 10 \Omega$ and the voltage $U = 10 \text{ V}$ is given. R_B .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

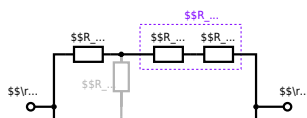
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \, \Omega)^2}{3 \cdot 100 \, \Omega} = \frac{1}{3} \cdot 100 \, \Omega = 33.33 \, \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \, \Omega + (33.33 \, \Omega + 400 \, \Omega) \parallel (33.33 \, \Omega + 100 \, \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = \{500 \sim \Omega\} \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

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