

# Exam Winter Semester 2022

## Student Group

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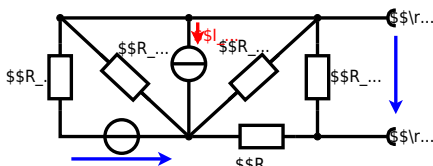
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**Exercise E5 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$\begin{aligned} U_{\text{rs}} &= U_{\text{AB}} = 4.5 \text{ V} \\ R_{\text{i}} &= R_{\text{AB}} = 6 \text{ } \Omega \end{aligned}$$



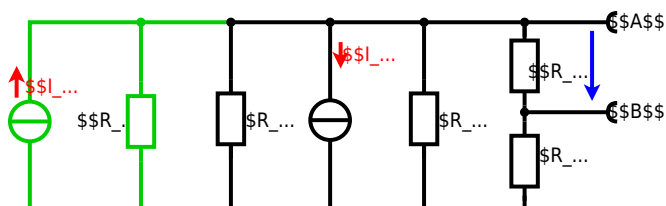
Calculated the internal resistance  $R_{\text{i}}$  and the source voltage  $U_{\text{rs}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ .  
 $R_1=5.0 \text{ } \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \text{ } \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \text{ } \Omega$ ,  $R_6=7.5 \text{ } \Omega$ ,  $R_7=15 \text{ } \Omega$   
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot (R_1 || R_3 || R_5)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} \right) - 4.2 \text{ A} \cdot 15 \Omega \cdot 2.5 \Omega \cdot \frac{1}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration system. The refrigerator has a resistance of  $10 \Omega$  at  $25^\circ \text{C}$  and  $25 \Omega$  at  $-40^\circ \text{C}$ .

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

**Result**  
The temperature inside the refrigeration system can reach down to  $-40^\circ \text{C}$ .

$$R = 10 \cdot (1 + 0.01 \cdot \Delta T + 71 \cdot 10^{-6} \cdot \Delta T^2)$$

The power transfer is  $P = U \cdot I = \frac{U^2}{R}$ . Therefore, a solution is to heat up the refrigeration system.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left( 1 + 0.01 \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2 \right)$$

**Exercise E9 Analyzing complex Impedances**  
 (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the complex power  $\underline{S}$  in a circuit with a voltage source  $\underline{u}(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V and a load  $\underline{Z} = 0.24 - j4.68 \Omega$ . The real and imaginary parts of  $\underline{S}$  shall be given.

After analysis, the real and imaginary parts of the complex power shall be extracted and given in physical units. The effective value of the voltage source shall be calculated as  $\underline{U} = 0.24 + j4.68 \Omega$ .

Solution  
 .. Calculation of the physical values of the load components.  
 Solution  $R = 0.24 \Omega$  and  $X_L = 4.68 \Omega$

Solution

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\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50}{0.24 - j4.68} = 107.31 \angle 87.96^\circ \text{ A}
The voltage of the source is 50 V (effective value). The resulting current is 107.31 A. The real power is 107.31 A * 50 V = 5365.5 W. The reactive power is 107.31 A * 4.68 V = 502.11 var. The complex power is 5365.5 W + j502.11 var. The phase angle is 87.96 degrees.

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**Exercise E13 Complex Impedance Circuit**  
 (written test, approx. 15 % of a 60-minute written test, WS2022)

2. Calculate the complex power  $\underline{S}$  in a circuit with a voltage source  $\underline{u}(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V and a load  $\underline{Z} = 0.24 - j4.68 \Omega$ . The real and imaginary parts of  $\underline{S}$  shall be given.

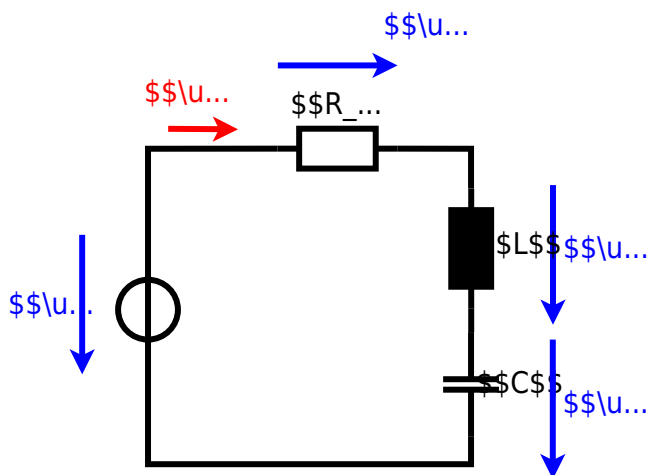
After analysis, the real and imaginary parts of the complex power shall be extracted and given in physical units. The effective value of the voltage source shall be calculated as  $\underline{U} = 0.24 + j4.68 \Omega$ .

Solution  
 Result  
 .. Draw the circuit diagram of the given circuit.  
 Solution  $Z = 0.24 - j4.68 \Omega$

Solution  $Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j19.24 \Omega$

Solution  $\underline{Z} = 0.24 - j4.68 - j19.24 = 0.24 - j23.92 \Omega$





**Exercise E11 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

**2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$ , a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ , and an inductor with an inductance of  $L_1 = 4.7 \text{ }\mu\text{H}$ . The circuit is connected to an AC voltage source with a voltage of  $U = 10 \text{ V}$  and a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance  $|Z|$  of the circuit.**

**Solution**

$$R_1 = 1.00 \text{ k}\Omega$$

$$R_2 = 10.0 \text{ }\Omega$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R_1$  and  $R_2$  combined is given by 
$$R_{\text{parallel}} = R_1 + R_2$$

Parallel circuit means that the voltage is the same on  $R_1$  and  $R_2$ .  

$$\frac{U}{R_1} = \frac{U}{R_2} = I$$
  
 Since  $U$  is perpendicular to  $R_1$  and  $R_2$ , the resulting current of the parallel circuit is given as:  

$$I_{\text{parallel}} = I_1 + I_2$$

This current  $I_{\text{parallel}}$  is perpendicular to  $R_3$ .  

$$I_{\text{parallel}}^2 = I_1^2 + I_2^2$$
  

$$\left(\frac{U}{R_{\text{parallel}}}\right)^2 = \left(\frac{U}{R_1}\right)^2 + \left(\frac{U}{R_2}\right)^2$$
  

$$\frac{1}{R_{\text{parallel}}^2} = \frac{1}{R_1^2} + \frac{1}{R_2^2}$$
  

$$R_{\text{parallel}} = \frac{R_1 R_2}{\sqrt{R_1^2 + R_2^2}}$$

Back to the first formula:  

$$R_3 \cdot I_{\text{parallel}} = X_{L1} \cdot I_{\text{parallel}}$$
  

$$R_3 = X_{L1} \cdot \frac{I_{\text{parallel}}}{I_{\text{parallel}}} = X_{L1}$$
  

$$R_3 = 2\pi \cdot f \cdot C_1 \cdot \frac{1}{\sqrt{1 + \left(\frac{R_1 R_2}{R_1^2 + R_2^2}\right)^2}}$$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

**2. Heating element made of nichrome wire with a temperature coefficient of  $\alpha = 1.80 \text{ } \text{K}^{-1}$ . The electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary.  
 Calculate the current  $I$  needed to operate the heating elements.  
 The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .**

The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 3.57 \text{ mm}$ .  
 Calculate the resistance  $R$  of the heating element.

**Solution**

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}}$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{\textbackslash align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{\textbackslash text{with } } A = r^2 \cdot \pi = \\ &= \frac{1}{4} \cdot d^2 \cdot \pi \quad \& \quad \& \quad \rho = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{\textbackslash m{m}} \cdot \frac{4 \cdot 3 \cdot \text{\textbackslash m{m}}}{(3.57 \cdot 10^{-3} \cdot \text{\textbackslash m{m}})^2 \cdot \pi} \quad \& \quad \& \quad \text{\textbackslash end{align*}} \end{aligned}$$

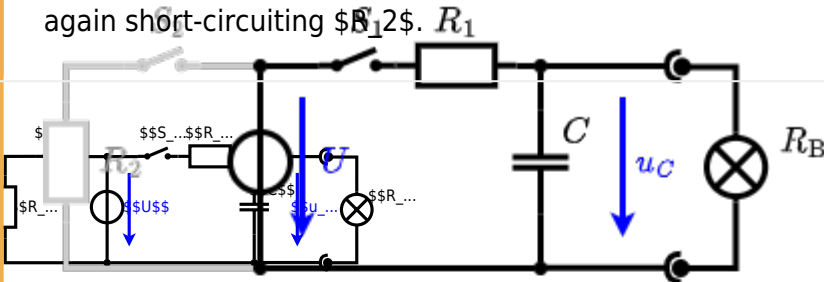
**Exercise E7 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (with the light bulb) is a series of  $R_1$  and  $R_2$  and a capacitor  $C$  and a switch  $S_1$  and a switch  $S_2$ . The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U$  is in series with  $R_1$  and  $R_2$ . The voltage  $u_c$  is independent of this series combination.

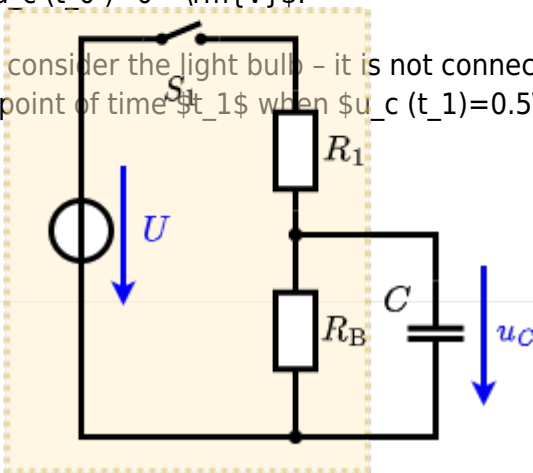
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $S_1$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \Omega$  and a capacitor of  $C = 100 \mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

.. First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



**Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at 0.10 A,  $R_1 = R_2 = R_3 = 1.5 \Omega$  and the voltage  $U = 10 \text{ V}$  is given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \text{ } \Omega)^2}{3 \cdot 100 \text{ } \Omega} = \frac{1}{3} \cdot 100 \text{ } \Omega = 33.33 \text{ } \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \text{ } \Omega + (33.33 \text{ } \Omega + 400 \text{ } \Omega) \parallel (33.33 \text{ } \Omega + 100 \text{ } \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



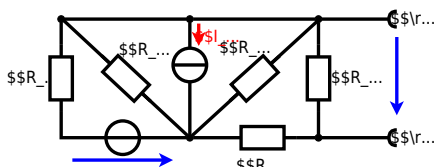
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel R_{\text{eq}}$$

**Exercise E6 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



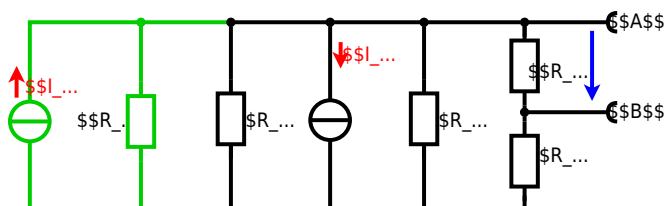
Calculate the internal resistance  $R_i$  and the source voltage  $U_s$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  
 $R_1=5.0 \text{ } \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \text{ } \Omega$ ,  $I_4=4.2 \text{ A}$ ,  
 $R_5=10 \text{ } \Omega$ ,  $R_6=7.5 \text{ } \Omega$ ,  $R_7=15 \text{ } \Omega$  Use equivalent sources in order to simplify the circuit!

### Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot (R_4 + R_6)$$

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left( \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left( R_1 || R_3 || R_5 \right)$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot 15 \Omega \cdot 2.5 \Omega \cdot \frac{1}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator is explained with the effect of resistance on refrigeration systems. The refrigerator has a resistance of  $10 \Omega$  at  $25^\circ \text{C}$  and  $25 \Omega$  at  $-40^\circ \text{C}$ .

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

**Result**  
The temperature inside the refrigeration system can reach down to  $-40^\circ \text{C}$ .

$$R_{-40} = 10 \Omega \cdot (1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2) = 25 \Omega$$

The power transfer is  $P = U \cdot I = \frac{U^2}{R}$ . Therefore, a solution is to use a heat pump.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$\Delta T = T_{\text{end}} - T_{\text{start}}$$

$$R = 10 \Omega \cdot \left( 1 + 0.01 \cdot (-40 - 25) + 71 \cdot 10^{-6} \cdot (-40 - 25)^2 \right)$$

### Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

1. Calculate the complex impedance  $Z$  of the circuit shown in the figure. The voltage  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V and the current  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$  A shall be given.

After analysis, the following dimensions must be provided:  $Z$  in  $\Omega$ ,  $\varphi$  in  $^\circ$ .  
Phase:  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$  A

Solution  
.. Calculate the physical values of the two components.  
Solution  $R = 10 \Omega$ ,  $X_L = 2\pi \cdot 15 \cdot 0.2 = 1.88 \Omega$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \Rightarrow \quad \underline{Z} = \frac{\underline{U}}{\underline{I}}$$
  
The voltage  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V is the reference voltage. The current  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$  A is the reference current. The resulting impedance  $Z$  is  $Z = \frac{U}{I} = \frac{3.0}{0.24} = 12.5 \Omega$ .  
The real part of the impedance is  $R = 10 \Omega$ . The imaginary part is  $X_L = 1.88 \Omega$ .  
The impedance  $Z$  is  $Z = R + jX_L = 10 + j1.88 \Omega$ .  
The phase  $\varphi$  is  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{1.88}{10}\right) = 10.7^\circ$ .  
With the complex part  $Z = 10 + j1.88 \Omega$ , the phase  $\varphi$  is  $\varphi = \arctan\left(\frac{1.88}{10}\right) = 10.7^\circ$ .  
The phase  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{1.88}{10}\right) = 10.7^\circ$ .

### Exercise E14 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the complex impedance  $Z$  of the circuit shown in the figure. The voltage  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V and the current  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$  A shall be given.

After analysis, the following dimensions must be provided:  $Z$  in  $\Omega$ ,  $\varphi$  in  $^\circ$ .  
Phase:  $i(t) = 0.24 \cdot \sin(2\pi \cdot 15 \cdot t - \varphi)$  A

Solution  
Result  
.. Draw the circuit diagram of the given circuit.  
Solution  $Z = 10 + j1.88 \Omega$ ,  $\varphi = 10.7^\circ$

Solution  
$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{3.0}{0.24} = 12.5 \Omega$$
  
The real part of the impedance is  $R = 10 \Omega$ . The imaginary part is  $X_L = 1.88 \Omega$ .  
The impedance  $Z$  is  $Z = R + jX_L = 10 + j1.88 \Omega$ .  
The phase  $\varphi$  is  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{1.88}{10}\right) = 10.7^\circ$ .  
With the complex part  $Z = 10 + j1.88 \Omega$ , the phase  $\varphi$  is  $\varphi = \arctan\left(\frac{1.88}{10}\right) = 10.7^\circ$ .  
The phase  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}(Z)}{\text{Re}(Z)}\right) = \arctan\left(\frac{1.88}{10}\right) = 10.7^\circ$ .





**Exercise E12 Impedances at different Frequencies**  
**(written test, approx. 18 % of a 60-minute written test, WS2022)**

**2. A series circuit consists of a resistor with a resistance of  $R_1 = 1.00 \text{ k}\Omega$  and a capacitor with a capacitance of  $C_1 = 40 \text{ nF}$ . The voltage across the resistor is  $U_{R_1} = 100 \text{ V}$  at a frequency of  $f = 4 \text{ MHz}$ . Calculate the absolute value of the impedance of the capacitor  $Z_C$  and the total impedance  $Z_{\text{total}}$  of the circuit.**

**Solution**

$$Z_C = \frac{1}{j\omega C_1} = \frac{1}{j \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9}} = -j 0.995 \text{ k}\Omega$$

$$Z_{\text{total}} = R_1 + Z_C = 1.00 \text{ k}\Omega - j 0.995 \text{ k}\Omega$$

$$|Z_{\text{total}}| = \sqrt{1.00^2 + 0.995^2} \text{ k}\Omega = 1.41 \text{ k}\Omega$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R_1$  and  $Z_C$  combined is given by  $Z_{\text{total}} = R_1 + Z_C$ .  
 Parallel circuit means that the voltage is the same on  $R_1$  and  $Z_C$ .  

$$U_{R_1} = I \cdot R_1 \Rightarrow I = \frac{U_{R_1}}{R_1} = \frac{100 \text{ V}}{1.00 \text{ k}\Omega} = 0.1 \text{ A}$$
  

$$U_{Z_C} = I \cdot Z_C \Rightarrow Z_C = \frac{U_{Z_C}}{I} = \frac{100 \text{ V}}{0.1 \text{ A}} = 1.00 \text{ k}\Omega$$
  
 Therefore, the resulting current of the parallel circuit is given as:  

$$I_{\text{total}} = I_{R_1} + I_{Z_C} = 0.1 \text{ A} + 0.1 \text{ A} = 0.2 \text{ A}$$
  
 This current is the same as the current through  $R_1$ .  

$$U_{\text{total}} = I_{\text{total}} \cdot Z_{\text{total}} = 0.2 \text{ A} \cdot 1.41 \text{ k}\Omega = 282 \text{ V}$$
  
 Back to the first formula:  $Z_{\text{total}} = R_1 + Z_C$   

$$|Z_{\text{total}}| = \sqrt{R_1^2 + |Z_C|^2} \Rightarrow |Z_C| = \sqrt{|Z_{\text{total}}|^2 - R_1^2} = \sqrt{1.41^2 - 1.00^2} \text{ k}\Omega = 1.00 \text{ k}\Omega$$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

**2. For a heating element, it is required to heat the water in a tank with a volume of  $V = 180 \text{ l}$  by electric power dissipation (= heat flow) of  $P = 40 \text{ W}$ . Calculate the current  $I$  needed to operate the heating element. The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ }\Omega \cdot \text{m}$ .**

**Solution**

$$P = U \cdot I \Rightarrow I = \frac{P}{U} = \frac{40 \text{ W}}{230 \text{ V}} = 0.174 \text{ A}$$

**Solution**

$$R = \frac{U}{I} = \frac{230 \text{ V}}{0.174 \text{ A}} = 1322 \text{ }\Omega$$

$$\sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \cdot \Omega}} \quad \text{align*}$$

$$\begin{aligned} R &= \rho \cdot l \cdot \frac{1}{A} \quad \& \quad | \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \& \quad R = \rho \cdot l \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \& \quad R = \\ &= 1.10 \cdot 10^{-6} \cdot \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \cdot \text{m}}{(3.57 \cdot 10^{-3} \cdot \text{m})^2 \cdot \pi} \quad \& \quad \end{aligned}$$

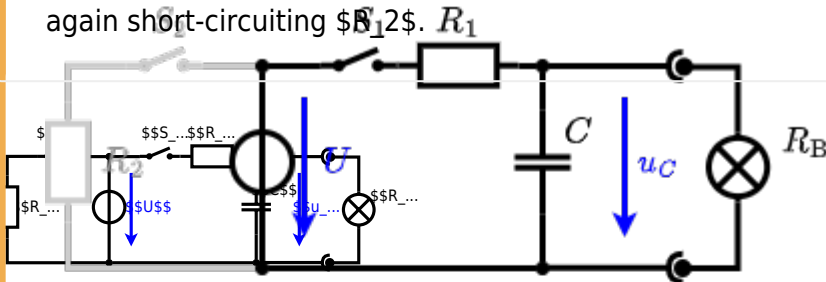
**Exercise E8 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (with the light bulb) also consists of a DC voltage source  $U = 12 \text{ V}$ , a resistor  $R_1 = 20 \text{ }\Omega$ , a capacitor  $C = 100 \text{ }\mu\text{F}$ , and a light bulb  $R_B = 5 \text{ }\Omega$ . The switch  $S_1$  is open. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution**  
 Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ .

**Solution**  
 Hint: To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ . The internal voltage source  $U_{int}$  is  $U_{int} = U \cdot \frac{R_B}{R_1 + R_B} = 12 \text{ V} \cdot \frac{5 \text{ }\Omega}{20 \text{ }\Omega + 5 \text{ }\Omega} = 2 \text{ V}$ . The internal resistance  $R_{int}$  is  $R_{int} = \frac{R_1 \cdot R_B}{R_1 + R_B} = \frac{20 \text{ }\Omega \cdot 5 \text{ }\Omega}{20 \text{ }\Omega + 5 \text{ }\Omega} = 3.85 \text{ }\Omega$ .

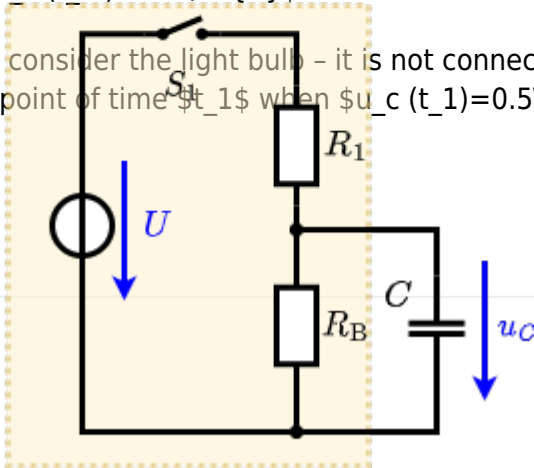
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_B$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first tasks. At the moment  $t_0 = 0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0 \text{ V}$ .

... First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .

**Solution**



An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \Omega$ , short-circuit).

$$R_i = R_1 \parallel R_B = 10 \Omega$$

$$u_c(t) = U_s \cdot (1 - e^{-t/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t/(10 \Omega \cdot 100 \mu F)})$$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$



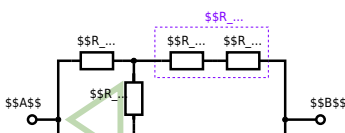
**Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at 0.10 A,  $R_1 = R_2 = R_3 = 1.5 \Omega$  and the voltage  $U = 10 \text{ V}$  is given.  $R_B$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

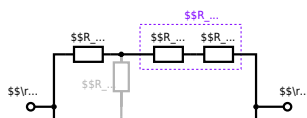


Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \text{ } \Omega)^2}{3 \cdot 100 \text{ } \Omega} = \frac{1}{3} \cdot 100 \text{ } \Omega = 33.33 \text{ } \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{\text{eq}} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{\text{eq}} = 33.33 \text{ } \Omega + (33.33 \text{ } \Omega + 400 \text{ } \Omega) \parallel (33.33 \text{ } \Omega + 100 \text{ } \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{\text{eq}}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel R_{\text{eq}} = (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel R_{\text{eq}} = \frac{500 \sim \Omega \cdot 200 \sim \Omega}{500 \sim \Omega + 200 \sim \Omega}$$

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