Exam Winter Semester 2022

Student Group

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Exercise E1 Resistance of a Wire by Resistivity
(written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of a Nichrome wire with a round cross-section is used in an electric oven.
Nichrome is a Nickel-Chromium alloy for heating elements.
The Nichrome wire has a resistivity of $1.10\cdot 10^{-6} ~\Omega \rm{m}$.
The heating element is $3 ~\rm{m}$ long and has a diameter of $3.57 ~\rm{mm}$.
1. Calculate the resistance R of the heating element.

2. The heating element is to heat the oven to a temperature of $180 ~\rm{°C}$. For this, a power dissipation (= heat flow) of $P=40 ~\rm{W}$ is necessary.
Calculate the current I needed to operate it.

Result

$$\begin{align*} R &= 0.33 ~\Omega \end{align*}$$

Solution

$$\begin{align*} P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 ~\rm{W}}{0.33 ~\Omega}} \end{align*}$$

$$\begin{align*} R &= \rho \cdot \frac{l}{A} && | \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} && \\ R &= 1.10\cdot 10^{-6} ~\Omega \rm{m} \cdot \frac{4 \cdot 3~\rm{m}}{(3.57\cdot 10^{-3}~\rm{m})^2 \cdot \pi} && \\ \end{align*}$$

Exercise E1 Resistance of a Wire by Resistivity
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Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

A thermistor is used as a temperature sensor in a refrigeration system. The thermistor has a resistance of $10 ~\rm{k}\Omega$ at $+25 ~°\rm{C}$.

Its temperature coefficients are: $\alpha=0.01 ~{{1}\over{\rm{K}}}$ and $\beta=71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}}$

The temperature inside the refrigeration system can reach down to $-40 ~°\rm{C}$.

1. Calculate the resistance of the thermistor at $-40 ~°\rm{C}$.

2. Additionally, explain whether a constant current or voltage would be more suitable for the … Give an explanation for your answer.

Solution

$$\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \text{with } \Delta T = T_{\rm end} - T_{\rm start}\\ R &= 10 ~\rm{k}\Omega \cdot \left(1 + 0.01 ~{{1}\over{\rm{K}}} \cdot (-40~°\rm{C} - 25~°\rm{C}) + 71 \cdot 10^{-6}~{{1}\over{\rm{K}^2}} \cdot (-40~°\rm{C} - 25~°\rm{C})^2 \right) \\ \end{align*}$$

Result

$$\begin{align*} R &= 6.5 ~\rm{k}\Omega \\ \end{align*}$$

The power of the electrical energy out of the circuit and generate heat. Therefore, a resistor transfers $P = U \cdot I = R \cdot I^2 = {U^2}\over{R}$ is equivalent to the heat flow. … might heat up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

Result

Exercise E2 Temperature-dependent Resistance
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Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following circuit with $R_1=200 ~\Omega$, $R_2=R_3=100 ~\Omega$ and the switch S is given.

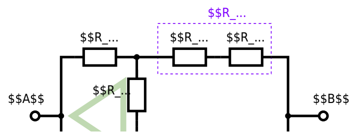
2. The switch shall now be closed. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Result

Solution

\begin{align*} R_{\rm eq} &= 1[illegible] ~\Omega \\ \end{align*}

Now a wye-delta transformation is necessary.

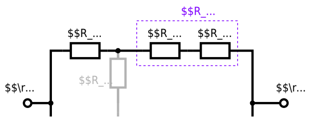


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega)&&||(100 ~\Omega + 100 ~\Omega) &&\\ R_{\rm eq} &= (500 ~\Omega)&&||(200 ~\Omega)&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}

Exercise E4 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following circuit with $R_1=200 ~\Omega$, $R_2=R_3=100 ~\Omega$ and the switch $\rm S$ is given.

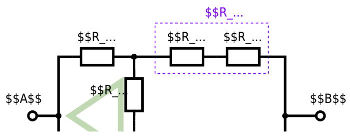
2. The switch shall now be closed. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Result

Solution

\begin{align*} R_{\rm eq} &= 142.8 ~\Omega \\ \end{align*}

Now a wye-delta transformation is necessary.

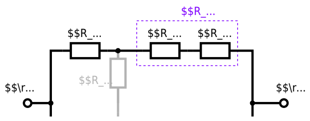


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as: \begin{align*} R_{Y} &= {{R_2 \cdot R_2}\over{R_2 + R_2 + R_2}} \\ &= {{(100 ~\Omega)^2}\over{3 \cdot 100 ~\Omega}} \\ &= {{1}\over{3}} \cdot 100 ~\Omega = 33.33 ~\Omega \end{align*}

The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= R_Y + (R_Y + R_1 + R_1)||(R_Y + R_2)\\ R_{\rm eq} &= 33.33 ~\Omega + (33.33 ~\Omega + 400 ~\Omega)||(33.33 ~\Omega + 100 ~\Omega)\\ \end{align*}

1. The switch shall now be open. Calculate the equivalent resistance $R_{\rm eq}$ between $\rm A$ and $\rm B$.

Solution

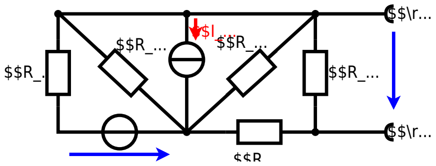


The equivalent resistor is given by a parallel configuration of resistors in series: \begin{align*} R_{\rm eq} &= (R_2 + R_1 + R_1)||(R_2 + R_2)\\ R_{\rm eq} &= (100 ~\Omega + 200 ~\Omega + 200 ~\Omega)&&||(100 ~\Omega + 100 ~\Omega) &&\\ R_{\rm eq} &= (500 ~\Omega)&&||(200 ~\Omega)&&\\ R_{\rm eq} &= {{500 ~\Omega \cdot 200 ~\Omega }\over{500 ~\Omega + 200 ~\Omega}}&&\\ \end{align*}

Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

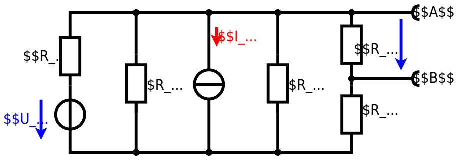
\begin{align*} U_{\rm s} &= U_{\rm AB} &&= 4.5 ~\rm{V}\\ R_{\rm i} &= R_{\rm AB} &&= 6 ~\Omega \end{align*}



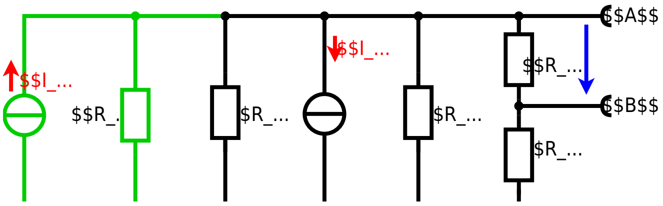
Calculated the internal resistance $R_\rm i$ and the source voltage $U_\rm s$ of an equivalent linear voltage source on the connectors $\rm A$ and $\rm B$. \begin{align*} R_1=5.0 ~\Omega, && U_2=6.0 ~\rm{V}, && R_3= 10 ~\Omega, \\ I_4=4.2 ~\rm{A}, && R_5=10 ~\Omega , && R_6=7.5 ~\Omega, \\ R_7=15 ~\Omega && && \end{align*} Use equivalent sources in order to simplify the circuit!

Solution

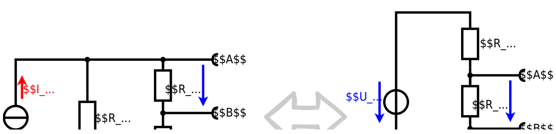
The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and R_1:



Now a lot of them can be combined. The resistors R_1, R_3, R_5 are in parallel, like also I_2 and I_4: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following: \begin{align*} U_{24}

&= R_{135} \cdot I_{24} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot R_1||R_3||R_5 \end{align*}

On the right side of the last circuit, there is a voltage divider given by R_{135}, R_6, and R_7.
Therefore the voltage between A and B is given as: \begin{align*} U_{\rm AB} &= U_{24} \cdot {{R_7}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ &= ({{U_2}\over{R_1}} - I_4) \cdot {{R_7 \cdot R_1||R_3||R_5}\over{R_6 + R_7 + R_1||R_3||R_5}} \\ \end{align*}

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || (R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| (7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

Exercise E6 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

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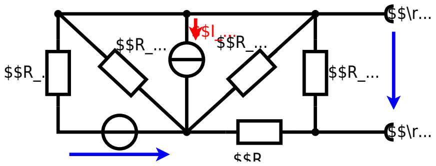

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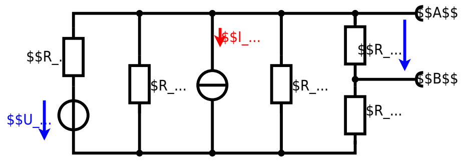
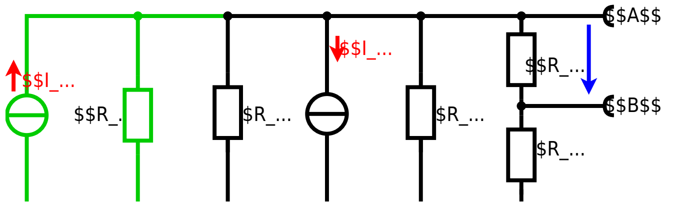
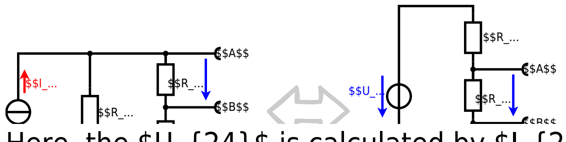


The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2={{U_2}\over{R_1}}$ and R_1:



Now a lot of them can be combined. The resistors R_1, R_3, R_5 are in parallel, like also I_2 and I_4: \begin{align*} R_{135} &= R_1||R_3||R_5\\ I_{24} &= I_2 - I_4 = {{U_2}\over{R_1}} - I_4 \end{align*} The resulting circuit can again be transformed:



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For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit): \begin{align*} R_{\rm AB} &= R_7 || (R_6 + R_1||R_3||R_5) \\ \end{align*}

with $R_1||R_3||R_5 = 5 ~\Omega || 10 ~\Omega || 10 ~\Omega = 5 ~\Omega || 5 ~\Omega = 2.5 ~\Omega$:

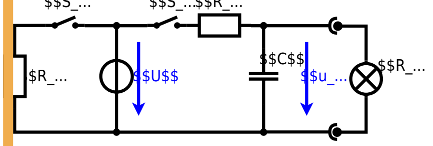
\begin{align*} U_{\rm AB} &= ({{6.0 ~\rm{V}}\over{5.0 ~\Omega}} - 4.2 ~\Omega) \cdot {{15 ~\Omega \cdot 2.5 ~\Omega}\over{7.5 ~\Omega + 15 ~\Omega + 2.5 ~\Omega}} \\ R_{\rm AB} &= 15 ~\Omega|| (7.5 ~\Omega + 2.5 ~\Omega) \\ \end{align*}

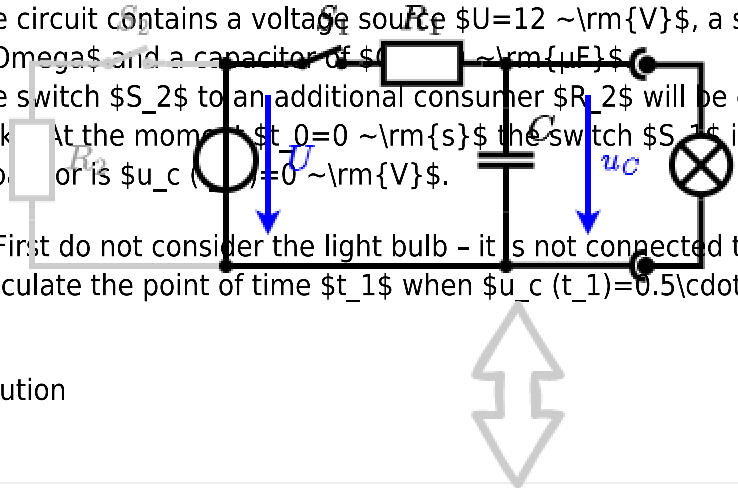
Exercise E7 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

[illegible]
[illegible] The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.

Result
Solution
Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and $R_\rm B$.
[illegible]

[illegible]
[illegible]
On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2.

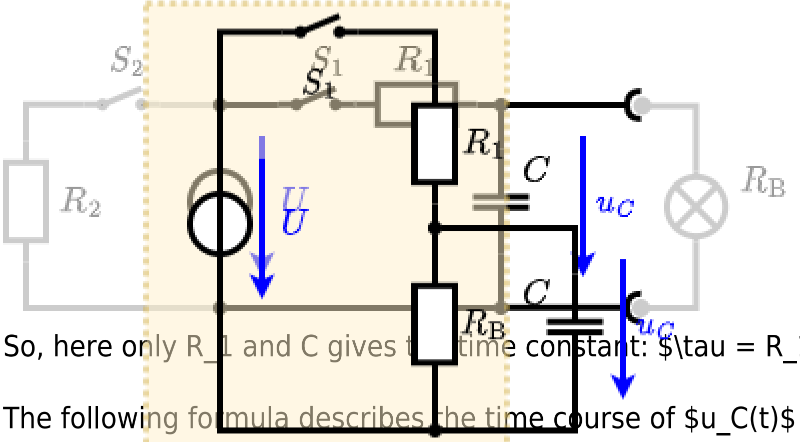




The circuit contains a voltage source $U=12 ~\rm{V}$, a switch S_1, a resistor of $R_1=20 ~\Omega$ and a capacitor of $C=$[illegible]$ ~\rm{µF}$.
The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0 ~\rm{s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c (t_0)=0 ~\rm{V}$.

1. First do not consider the light bulb – it is not connected to the RC circuit.
Calculate the point of time t_1 when $u_c (t_1)=0.5\cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}

An equivalent linear voltage source can be given with U, R_1, and $R_\rm B$ as seen in yellow.
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit). \begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}

\begin{align*} u_c (t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}

Exercise E8 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

2. Now, the light bulb is also connected, it has a resistance of $R_B=20 ~\Omega$. The switch S_2 is still open. $U=12 ~\rm{V}$. The voltage across the capacitor is again $0 ~\rm{V}$ at the moment $t_0=0 ~\rm{s}$ when the switch S_1 is closed. Calculate the voltage $u_c (t_2)$ across the capacitor at $t_2=1 ~\rm{ms}$ after closing the switch.
Hint: To solve this, first create an equivalent linear voltage source from U, R_1, and $R_\rm B$.

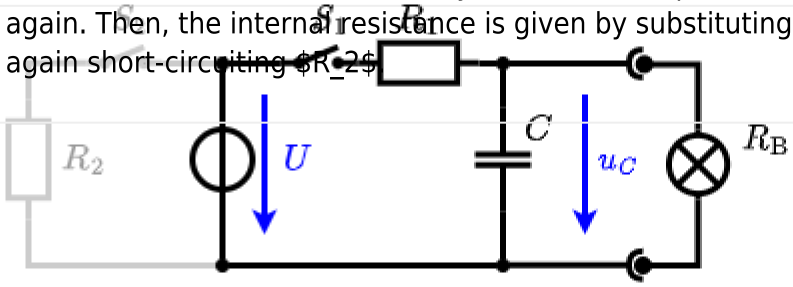
Result
Solution
\begin{align*} t_1 = ... \end{align*} \begin{align*} u_c(t_2) = 3.792 ~\rm{V} \end{align*}

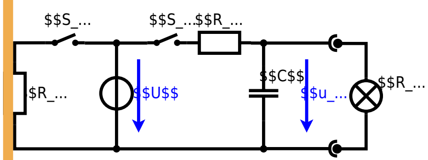
Solution

\begin{align*} \Delta W_R &= \Delta W_C \\ &= {{1}\over{2}} \cdot C \cdot U^2 \\ &= {{1}\over{2}} \cdot 100 ~\rm{µF} \cdot (12 ~\rm{V})^2 \end{align*}

It does not change anything: The ideal voltage source U provides the voltage of $U=12 ~\rm{V}$ independent of this resistor.

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2





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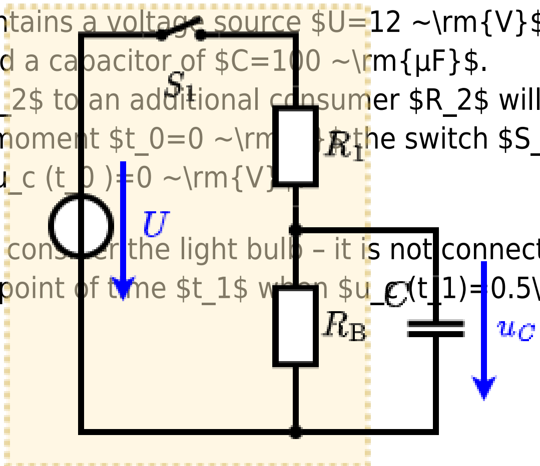

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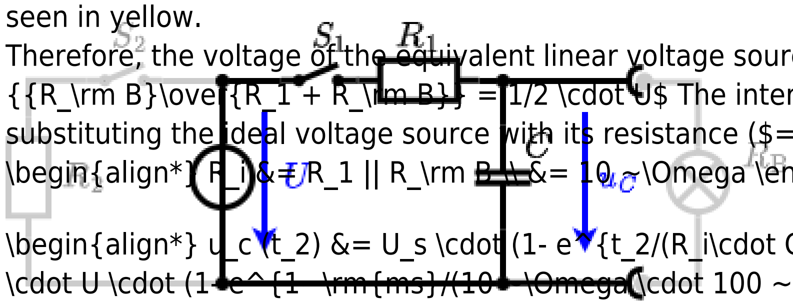
Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot {{R_\rm B}\over{R_1 + R_\rm B}} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 ~\Omega$, short-circuit). \begin{align*} R_i &= R_1 || R_\rm B \\ &= 10 ~\Omega \end{align*}



\begin{align*} u_c(t_2) &= U_s \cdot (1- e^{t_2/(R_i\cdot C)}) \\ &= {{1}\over{2}} \cdot U \cdot (1- e^{1 ~\rm{ms}/(10 ~\Omega \cdot 100 ~\rm{µF})}) \end{align*}
So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c (t_1)=0.5\cdot U$: \begin{align*} u_c (t) = U \cdot (1- e^{t/\tau}) = 0.5\cdot U \end{align*} It has to be rearranged to t \begin{align*} (1- e^{t/\tau}) &= 0.5 \\ e^{t/\tau} &= 0.5 \\ t/\tau &= ln(0.5) \\ t &= \tau \cdot ln(0.5) \\ t &= R_1 \cdot C \cdot ln(0.5) \end{align*}

Exercise E9 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

A complex impedance ... consists of two components. (R and $\underline{X}_1$) shall be given.

This component shall be dimensioned in such a way that ... \begin{align*} \underline{Z} = \left({{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega \end{align*}

Solution

1. Calculate the physical values of the two components.

\begin{align*} C = 103 ~\rm{µF} \end{align*}

Solution

\begin{align*} R &= 0.24 ~\Omega \\ L &= 2.26 ~\rm{mH} \end{align*}

Solution

\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }}

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.

Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$

\begin{align*} \underline{Z} &= \left({{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4j}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{9+16}} \cdot (3-4{\rm j}) + 5{\rm j} \right) ~\Omega \\ &= 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega \\ &= R + {\rm j} \cdot X_L \end{align*}

\begin{align*} X_C &= {{1}\over{\omega \cdot C}} = X_L \\ C &= {{1}\over{\omega \cdot X_L}} \\ &= {{1}\over{2\pi \cdot 300 ~\rm{Hz} \cdot 4.68 ~\Omega}} \\ \end{align*}

The absolute value $|\underline{I}|$ can be calculated as \begin{align*} |\underline{I}| &= {{50 ~\rm{V}}\over{| 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega |}} \\ &= {{50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}} \end{align*}

With the complex part comes the physical value: \begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \\ \end{align*}

The phaze φ_i can be calculated as \begin{align*} \varphi_i &= \arctan \left({{\Im()}\over{\Re()}} \right) \\ &= \arctan \left({{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

\begin{align*} |\underline{I}| &= 10.67 ~\rm{A} \\ \varphi_i &= -87.06° \end{align*}

Exercise E10 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

A complex impedance ... consists of two components. (R and $\underline{X}_1$) shall be given.

This component shall be dimensioned in such a way that ... \begin{align*} \underline{Z} = \left({{2}\over{3+4{\rm j}}}+5{\rm j} \right) \Omega \end{align*}

Solution

1. Calculate the physical values of the two components.

\begin{align*} C = 103 ~\rm{µF} \end{align*}

Solution

\begin{align*} R &= 0.24 ~\Omega \\ L &= 2.26 ~\rm{mH} \end{align*}

Solution

\begin{align*} \underline{I} &= {{\underline{U}}\over{\underline{Z}}} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \\ &= {{50 ~\rm{V}}\over{ 0.24 ~\Omega + {\rm j} \cdot 4.68 ~\Omega }} \cdot {{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }\over{ 0.24 ~\Omega - {\rm j} \cdot 4.68 ~\Omega }}

The current and voltage are in phase once there is only a pure ohmic (= pure real) resulting impedance $\underline{Z} + \underline{X}_2$.

Therefore, the component mus be a capacitor with the same absolute value of impedance: $|\underline{X}_C| = |\underline{X}_L|$

\begin{align*} \underline{Z} &= \left({{2}\over{3+4{\rm j}}} + 5{\rm j} \right) ~\Omega \\ &= \left({{2}\over{3+4{\rm j}}} \cdot {{3-4{\rm j}}\over{3-4j}} +

The absolute value of the current I can be calculated by: \begin{align*} [illegible] \\ &= 0.24 ~\Omega} + {\rm j} \cdot 4.68 ~\Omega} \\ &= R + {\rm j} X_L \\ \end{align*} [illegible] {50 ~\rm{V}}\over{\sqrt{ (0.24 ~\Omega)^2 + (4.68 ~\Omega)^2 }}} \end{align*}

With the complex part comes the physical value: \begin{align*} X_L &= \omega L \\ L &= {{X_L}\over{2\pi \cdot f}} \\ &= {{4.68 ~\Omega}\over{2\pi \cdot 300 ~\rm{Hz}}} \\ \end{align*}

The phase φ can be calculated as: \begin{align*} \varphi &= \arctan \left({{\rm Im}}\over{\rm Re}} \right) \\ &= \arctan \left({{-4.68 ~\Omega}\over{0.24 ~\Omega}} \right) \\ \end{align*}

Exercise E11 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

Calculate the resistor values.

1. A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

2. A RL series circuit with $L_2=4.7 ~\rm µH$ is used in an AC voltage source ($U_2=1.0 ~\rm V$, $f_2=450 ~\rm kHz$), which generates a current $I_2=60 ~\rm mA$.

3. A RC parallel circuit with $C_3=1 ~\rm nF$ ... generates a current $I_3=1.3 ~\rm A$ with $I_{3R}=1.0 ~\rm A$ through R_3.

Solution

Parallel circuit means that the voltage is the same on R_3 and C_3. \begin{align*} R_1 &= |\underline{X}_{C1}| \\ &= {{1}\over{2\pi \cdot f \cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~\rm nF}} \\ \end{align*}

Result

\begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}

Solution

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by \begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j} \cdot \omega L \end{align*} Since ${\rm j} \cdot \omega L$ is perpendicular to R_2 this can be simplified to: \begin{align*} \left| {{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left({{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}

This can be rearranged to get R_2: \begin{align*} R_2 &= \sqrt{ \left({{U}\over{I}} \right)^2 - {X_{L2}}^2 } \\ &= \sqrt{ \left({{1 ~\rm V}}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}

Result

\begin{align*} R_2 &= 10.0 ~\Omega \\ \end{align*}

Solution

Parallel circuit means that the voltage is the same on R_3 and C_3. [illegible] $\underline{I}_{3C}$ [illegible] gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$. (It has to, since R_3 is perpendicular to ${\rm j}\cdot{X}_{3C}$, too)

Therefore, the resulting current of the parallel circuit is given as: \begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ {I}_{3C} &= \sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2} \\ \end{align*}

Back to the first formula: \begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}

Exercise E12 Impedances at different Frequencies
(written test, approx. 18 % of a 60-minute written test, WS2022)

3. A R_3 parallel circuit with $C_3=4.7 ~\rm nF$ is under an AC voltage source of $U_3 = 1.0 ~\rm V$, $f_3=120 ~\rm kHz$, which generates a current of $I_{3R}=10 ~\rm A$ through R_3.

Calculate the resistor values R_2 ... $L_2=4.7 ~\rm µH$ ... $U_2=1.0 ~\rm V$, $f_2=450 ~\rm kHz$, which generates a current of $I_2=60 ~\rm mA$.

1. A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1=40 ~{\rm nF}$ at $f_1=4 ~{\rm MHz}$.

Solution

$$\begin{align*} R_1 &= 1.00 ~\Omega \\ \end{align*}$$

Solution

$$\begin{align*} R_2 &= 10.0 ~\Omega \\ \end{align*}$$

$$\begin{align*} R_3 &= 7.0 ~\Omega \\ \end{align*}$$

Solution

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by $$\begin{align*} {{\underline{U}}\over{\underline{I}}} &= R_2 + \underline{X}_{L2} \\ &= R_2 + {\rm j}\cdot \omega L \end{align*}$$ Since ${\rm j} \cdot \omega L$ is perpendicular to R_2 this can be simplified to: $$\begin{align*} \left|{{\underline{U}}\over{\underline{I}}} \right|^2 &= |R_2|^2 + |\underline{X}_{L2}|^2 \\ \left({{U}\over{I}} \right)^2 &= {R_2}^2 + {X_{L2}}^2 \\ \end{align*}$$

Parallel circuit means that the voltage is the same on R_3 and C_3. $$\begin{align*} R_1 &= |\underline{X}_{C1}| \\ &= {{1}\over{2\pi \cdot f_1\cdot C_1}} \\ &= {{1}\over{2\pi \cdot 4 ~{\rm MHz} \cdot 40 ~{\rm nF}}} \\ \end{align*}$$ gets clear, that $\underline{I}_{3R}$ is perpendicular to $\underline{I}_{3C}$ (It has to, since R_3 is perpendicular to ${\rm j}\cdot {X}_{3C}$, too).

Therefore, the resulting current of the parallel circuit is given as: $$\begin{align*} \underline{I}_{3} &= \underline{I}_{3R} + \underline{I}_{3C} \\ |\underline{I}_{3}|^2 &= |\underline{I}_{3R}|^2 + |\underline{I}_{3C}|^2 \\ \end{align*}$$

This can be rearranged to get R_2: $$\begin{align*} R_2 &= \sqrt{ \left({{U}\over{I}} \right)^2 - X_{L2}^2 } \\ &= \sqrt{ \left({{1 ~\rm V}}\over{60~\rm mA}} \right)^2 - (2\pi \cdot 450~{\rm kHz} \cdot 4.7 ~ {\rm µH})^2 } \\ \end{align*}$$

Back to the first formula: $$\begin{align*} R_3 \cdot {I}_{3R} &= {X}_{3C} \cdot {I}_{3C} \\ R_3 &= {X}_{3C} \cdot {{{I}_{3C}}\over{{I}_{3R}}} \\ &= {{1}\over{2\pi \cdot f \cdot C_3}} \cdot {{\sqrt{|{I}_{3}|^2 - |{I}_{3R}|^2}}\over{{I}_{3R}}} \\ \end{align*}$$

Exercise E13 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

A linear source is given by the voltage source $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ with an internal resistance of $10 ~\Omega$.

This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

1. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

2. Calculate the complex impedances $\underline{Z}_{C}$, $\underline{Z}_L$ and $\underline{Z}$. Choose an appropriate scaling factor and write it down.

3. Calculate the current in the circuit ... $|\underline{Z}|$ ... $\underline{Z}_R$ ... the overall ... $|\underline{Z}_L|$ such as ...

Result

Solution

Solution

Result

$$\begin{align*} Z_L &= 31 ~\Omega \\ Z_C &= 48.2 ~\Omega \\ Z &= 19.8 ~\Omega \end{align*}$$

$$\begin{align*} I &= 107 ~{\rm mA} \end{align*}$$

$$\begin{align*} Z &= {{\hat{U}}\over{\hat{I}}} \\ \hat{I} &= {{\hat{U}}\over{Z}} \\ \end{align*}$$

$$\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}\\ \end{align*}$$

Result

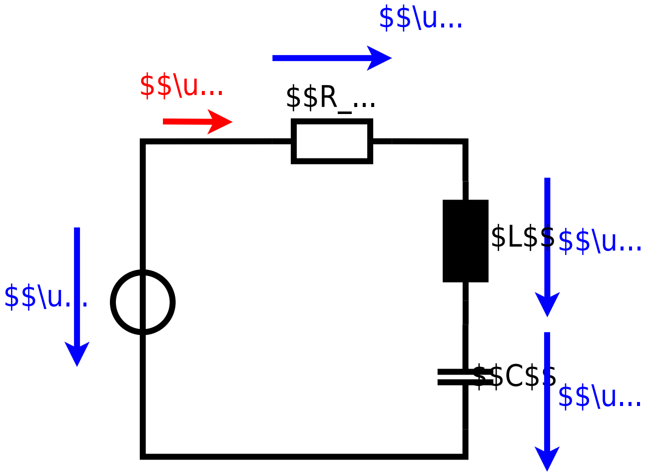
With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$: $$\begin{align*} I &= {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ &= {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}} \\ \end{align*}$$

$$\begin{align*} Z_L &= 2\pi \cdot f \cdot L\\ &= 2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}\\ \end{align*}$$

$$\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}}}\\ \end{align*}$$

$$\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}$$

`$$10...`


$$\u...
$$\u...
$$R_...
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Exercise E14 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

A voltage source generates a sinusoidal voltage $u(t) = 3.0 ~{\rm V} \cdot \sin(2\pi \cdot 15 ~{\rm kHz} \cdot t)$ with an internal resistance of $10 ~\Omega$.
This linear source is connected with an inductor of $330 ~ {\rm µH}$ and a capacitor of $0.22 ~{\rm µF}$, all in series.

1. Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

2. Calculate the complex impedances $\underline{Z}_C$, $\underline{Z}_L$ and $|\underline{Z}|$ of the overall circuit.

Solution

$$\begin{align*} Z_C &= {{1}\over{2\pi \cdot f \cdot C}}\\ &= {{1}\over{2\pi \cdot 15 ~{\rm kHz} \cdot 0.22 ~{\rm µF}}}\\ \end{align*}$$

$$\begin{align*} Z_L &= 2\pi \cdot f \cdot L\\ &= 2\pi \cdot 15 ~{\rm kHz} \cdot 330 ~{\rm µH}\\ \end{align*}$$

$$\begin{align*} \underline{Z} &= R + \underline{Z}_L + \underline{Z}_C \\ &= R + j \cdot {Z}_L - j \cdot {Z}_C \\ &= R + j \cdot ({Z}_L - {Z}_C) \\ |\underline{Z}| &= \sqrt{R^2 + (\underline{Z}_L - \underline{Z}_C)^2 }\\ \end{align*}$$

Result

$$\begin{align*} Z_L &= 31.1 ~\Omega \\ Z_C &= 48.2 ~\Omega \\ Z &= 19.8 ~\Omega \end{align*}$$

3. Calculate the current I through the circuit. Choose an appropriate scaling factor and write it down.

Solution

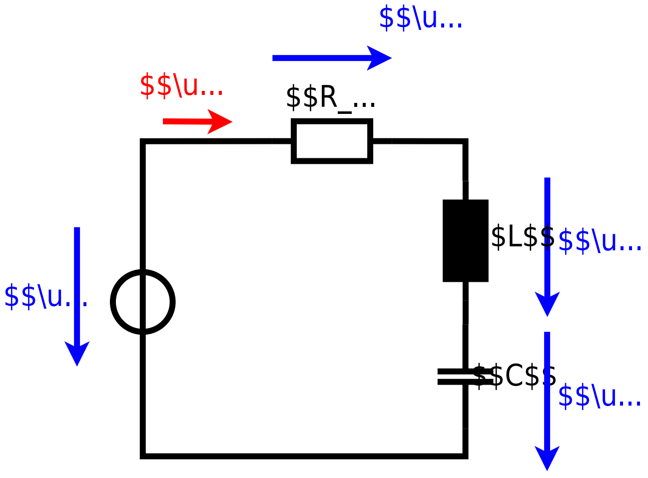
$$\begin{align*} Z &= {{\hat{U}}\over{\hat{I}}} \\ \hat{I} &= {{\hat{U}}\over{Z}} \\ \end{align*}$$

With $I = {{1}\over{\sqrt{2}}}\cdot \hat{I}$:

$$\begin{align*} I &= {{1}\over{\sqrt{2}}}\cdot {{\hat{U}}\over{Z}} \\ &= {{1}\over{\sqrt{2}}}\cdot {{3.0 ~{\rm V}}\over{19.28 ~\Omega}} \\ \end{align*}$$

Result

$$\begin{align*} I = 107.3 ~{\rm mA} \end{align*}$$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