

# Exam Winter Semester 2022

## Student Group

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**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

A heating element made of nichrome wire with a temperature coefficient of  $1.80 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$  is used. Electric power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary. Determine the current  $I$  needed to operate it. for heating elements. The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$ . The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .  
 Solution:  $R = 10^{-3} \text{ } \Omega$

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad || \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad || \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

**Exercise E1 Resistance of a Wire by Resistivity**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

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**Exercise E2 Temperature-dependent Resistance**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator, explains a thermodynamic process in a refrigeration system. The refrigerant has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$ . Your answer.

Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$

Result  
 The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermistor at  $-40^\circ\text{C}$ .

The power transfer is  $P = U \cdot I$  and  $I = \frac{U}{R}$ . Therefore, a solution is to increase the resistance of the thermistor.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot (1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2)$$

**Exercise E2 Temperature-dependent Resistance**  
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Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot (1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2)$$

**Exercise E3 Pure Resistor Network Simplification**  
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at once,  $R_2 = R_3 = 100 \Omega$  and the switch shall be closed. Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution

$$R_{eq} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel (R_Y + R_2) \parallel (R_Y + R_2) \parallel (R_Y + R_2)$$

.. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

**Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved with  $R_1 = 200 \Omega$ ,  $R_2 = R_3 = 100 \Omega$  and the source  $B = 15 \text{ V}$ .  
 Result given:  $R_{\text{eq}} = 132.8 \Omega$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E5 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance  $R_{in}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

### Solution

The best thing is to re-think the wiring like rubber bands and adjust them:





Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{67}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0\Omega$ , so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with  $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$ :

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

### Exercise E6 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



Calculate the internal resistance  $R_{in}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_{24} \cdot \frac{R_1}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4) \cdot R_1 || R_3 || R_5$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

**Exercise E7 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit below is a battery with an internal resistance of  $R_1 = 5 \Omega$  and a charging capacitor  $C = 2 \mu\text{F}$  connected in parallel with a switch  $S_1$ . The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Result:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

**Solution:** The ideal voltage source  $U$  is in series with  $R_1$  and  $R_2$ . The voltage  $U$  is  $12 \text{ V}$  and  $R_1 = 5 \Omega$  and  $R_2 = 10 \Omega$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U=12\text{ V}$ , a switch  $S_1$ , a resistor of  $R_1=20\text{ }\Omega$  and a capacitor of  $C=100\text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first task. At the moment  $t_0=0\text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0)=0\text{ V}$ .



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1)=0.5 \cdot U$ .

Solution



So, here  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_c(t_1)=0.5 \cdot U$ :  

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5$   
 An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow:  

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$$
 The internal resistance is given by substituting the ideal voltage source with its resistance ( $=0\text{ }\Omega$ , short-circuit).  

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $=0\text{ }\Omega$ , short-circuit).  

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$
  

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

**Exercise E8 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

The circuit (see the solution) consists of a  $12\text{ V}$  DC voltage source, a  $20\text{ }\Omega$  resistor, a  $100\text{ }\mu\text{F}$  capacitor, a  $10\text{ }\Omega$  resistor, and a light bulb. The voltage across the capacitor is again  $0\text{ V}$  at the moment  $t_0=0\text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2=1\text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ .  

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is  $U = 12 \text{ V}$ . The internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0) = 0$ .

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1) = 0.5 \cdot U$ .



Solution

An equivalent linear voltage source can be given with  $U$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \text{ }\Omega$ , short-circuit).  $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$

$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$   
So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1) = 0.5 \cdot U$ :  $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$   
It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

### Exercise E9 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage  $\underline{u}(t) = 50 \cos(300t - 10^\circ)$  V and the phasor current  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A are both through the components ( $R$  and  $X_L$ ) shall be given.

After analysis, the full and dimensioned complex impedance  $\underline{Z}$  shall be extracted and the magnitude  $|\underline{Z}|$  in phase (in  $^\circ$ ) shall be given.  $\underline{Z} = (2 + j4) \Omega$

Solution  
.. Calculation of physical values of the two components.  
Solution  $\varphi = \arctan\left(\frac{4}{2}\right) = 63.4^\circ$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle -10^\circ}{(2 + j4) \Omega} = 12.5 \angle -73.4^\circ$$
  
The current  $I$  lags the voltage  $U$  by  $63.4^\circ$ .  
The resulting impedance  $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{12.5 \angle -73.4^\circ} = 4 \angle 63.4^\circ = 1.55 + j3.69 \Omega$   
Therefore, the component  $R = 1.55 \Omega$  and  $X_L = 3.69 \Omega$ .  
Impedance  $\underline{Z} = R + jX_L = 1.55 + j3.69 \Omega$   
Magnitude  $|\underline{Z}| = \sqrt{1.55^2 + 3.69^2} = 4 \Omega$   
Phase  $\varphi = \arctan\left(\frac{3.69}{1.55}\right) = 67.5^\circ$   
The phase  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{3.69}{1.55}\right) = 67.5^\circ$

### Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage  $\underline{u}(t) = 50 \cos(300t - 10^\circ)$  V and the phasor current  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A are both through the components ( $R$  and  $X_L$ ) shall be given.

After analysis, the full and dimensioned complex impedance  $\underline{Z}$  shall be extracted and the magnitude  $|\underline{Z}|$  in phase (in  $^\circ$ ) shall be given.  $\underline{Z} = (2 + j4) \Omega$

Solution  
.. Calculation of physical values of the two components.  
Solution  $\varphi = \arctan\left(\frac{4}{2}\right) = 63.4^\circ$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{50 \angle -10^\circ}{(2 + j4) \Omega} = 12.5 \angle -73.4^\circ$$
  
The current  $I$  lags the voltage  $U$  by  $63.4^\circ$ .  
The resulting impedance  $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{12.5 \angle -73.4^\circ} = 4 \angle 63.4^\circ = 1.55 + j3.69 \Omega$   
Therefore, the component  $R = 1.55 \Omega$  and  $X_L = 3.69 \Omega$ .  
Impedance  $\underline{Z} = R + jX_L = 1.55 + j3.69 \Omega$   
Magnitude  $|\underline{Z}| = \sqrt{1.55^2 + 3.69^2} = 4 \Omega$   
Phase  $\varphi = \arctan\left(\frac{3.69}{1.55}\right) = 67.5^\circ$   
The phase  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{3.69}{1.55}\right) = 67.5^\circ$

The absolute value of the impedance is  $Z = \sqrt{R^2 + (X_L - X_C)^2}$  with  $R = 5 \Omega$ ,  $X_L = \omega L = 2\pi \cdot 4 \text{ MHz} \cdot 100 \text{ nH} = 2.51 \text{ m}\Omega$  and  $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 10 \text{ nF}} = 3.98 \text{ m}\Omega$ .  
 The phase  $\phi$  is given by  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{2.51 \text{ m}\Omega - 3.98 \text{ m}\Omega}{5 \Omega}\right) = -0.24 \text{ rad}$ .  
 With the complex part comes the physical value:  $I = \frac{U}{Z} = \frac{50 \text{ V}}{\sqrt{5^2 + (2.51 - 3.98)^2}} = 9.99 \text{ A}$ .  
 The phase  $\phi$  is  $\phi = -0.24 \text{ rad} = -13.7^\circ$ .

**Exercise E11 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with a resistor  $R_1 = 1 \text{ k}\Omega$ , a capacitor  $C_1 = 40 \text{ nF}$  and an inductor  $L_1 = 4.7 \text{ }\mu\text{H}$  in AC with a voltage  $U = 50 \text{ V}$  and a frequency  $f = 4 \text{ MHz}$ .  
 Result:  $Z = 1.00 \text{ }\Omega$ ,  $I = 50 \text{ A}$ ,  $\phi = 0^\circ$ .  
 A resistor  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ .

Solution  
 Solution  $R_1 = 1.00 \Omega$   
 Solution  $R_2 = 10.0 \Omega$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z = \sqrt{R^2 + X_L^2}$ .  
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C_2$ .  
 The equivalent impedance for  $R_2$  and  $C_2$  combined is given by  $Z_2 = \frac{R_2 \cdot X_{C2}}{\sqrt{R_2^2 + X_{C2}^2}}$ .  
 Since  $Z = Z_2$  and  $I = I_2$ , we have  $\frac{U}{\sqrt{R^2 + X_L^2}} = \frac{U}{\frac{R_2 \cdot X_{C2}}{\sqrt{R_2^2 + X_{C2}^2}}}$ .  
 This can be simplified to  $\sqrt{R^2 + X_L^2} = \frac{R_2 \cdot X_{C2}}{\sqrt{R_2^2 + X_{C2}^2}}$ .  
 Squaring both sides:  $R^2 + X_L^2 = \frac{R_2^2 \cdot X_{C2}^2}{R_2^2 + X_{C2}^2}$ .  
 This can be rearranged to  $R^2(R_2^2 + X_{C2}^2) + X_L^2(R_2^2 + X_{C2}^2) = R_2^2 \cdot X_{C2}^2$ .  
 Therefore, the resulting current of the parallel circuit is given as:  
 $I_2 = \frac{U}{Z_2} = \frac{U \cdot \sqrt{R_2^2 + X_{C2}^2}}{R_2 \cdot X_{C2}}$   
 This can be rearranged to  $I_2 = \frac{U}{R_2} \cdot \frac{\sqrt{R_2^2 + X_{C2}^2}}{X_{C2}}$ .  
 Back to the first formula:  $R_3 \cdot I_2 = X_{C3} \cdot I_2$   
 $R_3 = \frac{X_{C3}}{I_2} = \frac{X_{C3}}{\frac{U}{R_2} \cdot \frac{\sqrt{R_2^2 + X_{C2}^2}}{X_{C2}}} = \frac{R_2 \cdot X_{C2} \cdot X_{C3}}{U \cdot \sqrt{R_2^2 + X_{C2}^2}}$

**Exercise E12 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)



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**Exercise E14 Complex Impedance Circuit**  
**(written test, approx. 15 % of a 60-minute written test, WS2022)**

1. Calculate the current  $i(t)$  in the circuit shown in Fig. 1. The voltage source is  $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$  V. The circuit consists of a resistor of  $R = 10 \Omega$ , an inductor of  $L = 330 \mu\text{H}$ , and a capacitor of  $C = 0.22 \mu\text{F}$ , all in series.

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Result
.. \begin{align*} Z &= R + j\omega L - j\omega C = 10 + j(2\pi \cdot 15 \cdot 330 \cdot 10^{-6}) - j(2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}) \\ Z &= 10 + j0.031 - j0.020 \\ Z &= 10 + j0.011 \Omega \\ \end{align*}
\begin{align*} I &= \frac{U}{Z} = \frac{3.0}{10 + j0.011} \\ I &= 0.3 - j0.003 \text{ A} \\ i(t) &= 0.3 \cdot \sin(2\pi \cdot 15 \cdot t - 0.03) \text{ A} \\ \end{align*}

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