

# Exam Winter Semester 2022

## Student Group

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### Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of nichrome wire with a temperature coefficient of  $1.80 \times 10^{-5} \text{ } ^\circ\text{C}^{-1}$  is used. Electric power dissipation (= heat flow) of  $P=40 \text{ W}$  is necessary.

Calculate the current  $I$  and the operating voltage  $U$  for heating elements.

The Nichrome wire has a resistivity of  $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$ .

The heating element is  $3 \text{ m}$  long and has a diameter of  $3.57 \text{ mm}$ .

∴ Calculate the resistance  $R$  of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ I &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad | \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{l}{\frac{1}{4} d^2 \cdot \pi} \quad | \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

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**Exercise E2 Temperature-dependent Resistance**  
**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. A refrigerator, explained in the effect course, is used to refrigerate food. The refrigerator has a resistance of  $10 \text{ k}\Omega$  at  $25^\circ\text{C}$  and  $1 \text{ m}\Omega$  at  $-40^\circ\text{C}$ . Its temperature coefficients are:  $\alpha = 0.01 \text{ K}^{-1}$  and  $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$ .

Result  
 The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermostat at  $-40^\circ\text{C}$ .

$$R = 10 \text{ k}\Omega \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

The power transfer is  $P = U \cdot I$  and  $I = U/R$ . Therefore, a solution is to increase the resistance of the thermostat.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

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\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \\ \text{with } \Delta T &= T_{\text{end}} - T_{\text{start}} \quad | \quad R = 10 \text{ k}\Omega \\ &= 10 \text{ k}\Omega \cdot \left( 1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right) \end{align*}
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**Exercise E3 Pure Resistor Network Simplification**  
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at once,  $R_2 = R_3 = 100 \Omega$  and the switch shall be given.  $R_1 = 400 \Omega$ . Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution

$$R_{eq} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1) \parallel (R_Y + R_2)$$

$$= 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

**Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved with  $R_1 = 200 \Omega$ ,  $R_2 = R_3 = 100 \Omega$  and the source  $B = 15 \text{ V}$ .  
 Result given:  $R_{\text{eq}} = 132.8 \Omega$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E5 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance  $R_{int}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

### Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :  

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:  

$$U_{24} = I_{24} \cdot R_{6135}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - \left( \frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0\Omega$ , so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with  $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$ :

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

### Exercise E6 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



Calculated the internal resistance  $R_{in}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3= 10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$  Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_{24}}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24}$$

$$U_{AB} = R_{135} \cdot I_{24} = \left( \frac{U_2}{R_1} - I_4 \right) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left\{ \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \right\} = \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left\{ \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \right\}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with  $R_1 \parallel R_3 \parallel R_5 = 5 \Omega \parallel 10 \Omega \parallel 10 \Omega = 5 \Omega \parallel 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left\{ \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right\}$$

$$R_{AB} = 15 \Omega \parallel (7.5 \Omega + 2.5 \Omega)$$

**Exercise E7 Charging Capacitors**  
**(written test, approx. 16 % of a 60-minute written test, WS2022)**

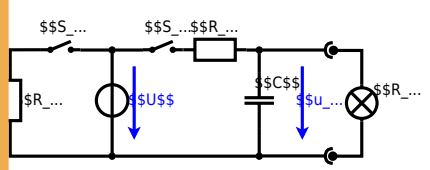
The circuit below is a battery with an internal resistance of  $R_1 = 5 \Omega$  and a charging capacitor  $C = 2 \mu\text{F}$  connected in parallel with a resistor  $R_2 = 10 \Omega$ . The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U_{eq}$  is given by:

$$U_{eq} = \frac{U}{1 + \frac{R_1}{R_2}} = \frac{12 \text{ V}}{1 + \frac{5 \Omega}{10 \Omega}} = 8 \text{ V}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U=12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1=20 \text{ }\Omega$  and a capacitor of  $C=100 \text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first task. At the moment  $t_0=0 \text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0)=0 \text{ V}$ .



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1)=0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_c(t)$  which has to be  $u_c(t_1)=0.5 \cdot U$ : 
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \Rightarrow e^{-t/\tau} = 0.5 \Rightarrow t/\tau = \ln(0.5) \Rightarrow t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$  An equivalent linear voltage source can be given with  $U_s$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $=0 \text{ }\Omega$ , short-circuit). 
$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms}/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

**Exercise E8 Charging Capacitors**  
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the solution) consists of a  $12 \text{ V}$  DC voltage source, a  $20 \text{ }\Omega$  resistor, a  $100 \text{ }\mu\text{F}$  capacitor, and a  $20 \text{ }\Omega$  resistor. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0=0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2=1 \text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ . 
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 6 \text{ V}$$
 
$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

Solution

The ideal voltage source is  $U = 12 \text{ V}$ . The internal resistance is  $R_1 = 20 \text{ }\Omega$ . The voltage across the capacitor is  $u_C(t)$ . The voltage across the light bulb is  $u_B(t)$ . The voltage across the resistor  $R_2$  is  $u_{R_2}(t)$ . The voltage across the capacitor is  $u_C(t)$ . The voltage across the light bulb is  $u_B(t)$ . The voltage across the resistor  $R_2$  is  $u_{R_2}(t)$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_C(t_0) = 0$ .

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_C(t_1) = 0.5 \cdot U$ .



Solution

An equivalent linear voltage source can be given with  $U$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \text{ }\Omega$ , short-circuit).  $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$ .

$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$   
So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_C(t_1) = 0.5 \cdot U$ :  $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$   
It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

### Exercise E9 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage  $\underline{u}(t) = 50 \cos(300t - 10^\circ)$  V and the phasor current  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A are both through the components ( $R$  and  $X_L$ ) shall be given.

After analysis, the full bandwidth of the circuit impedance  $Z$  can be extracted and the phase angle  $\varphi$  in phase with the voltage  $\underline{u}(t)$  is  $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) + 5^\circ$ .

Solution  
.. Calculation of physical values of the two components.  
Solution 
$$R = 20 \Omega, L = 2 \text{ mH} \quad \varphi = 87.06^\circ$$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \underline{U} = 50 \text{ V}$$
  
The voltage  $\underline{u}(t)$  is a cosine wave with an amplitude of 50 V and a phase angle of  $-10^\circ$ .  
The resulting current  $\underline{i}(t)$  is a cosine wave with an amplitude of 0.24 A and a phase angle of  $+90^\circ$ .  
Therefore, the component  $Z$  is a pure inductor with the same magnitude of 4.68  $\Omega$ .  
Impedance  $\underline{Z} = R + jX_L = 20 + j4.68 \Omega$ .  
$$\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) + 5^\circ = \arctan\left(\frac{4.68}{20}\right) + 5^\circ = 13.4^\circ + 5^\circ = 18.4^\circ$$
  
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) + 5^\circ = 18.4^\circ + 5^\circ = 23.4^\circ$ .  
With the complex part  $Z = 20 + j4.68 \Omega$ , the magnitude  $|Z| = \sqrt{20^2 + 4.68^2} = 20.11 \Omega$ .  
$$\varphi = \arctan\left(\frac{4.68}{20}\right) + 5^\circ = 13.4^\circ + 5^\circ = 18.4^\circ + 5^\circ = 23.4^\circ$$
  
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) + 5^\circ = 18.4^\circ + 5^\circ = 23.4^\circ$ .

### Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage  $\underline{u}(t) = 50 \cos(300t - 10^\circ)$  V and the phasor current  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A are both through the components ( $R$  and  $X_L$ ) shall be given.

After analysis, the full bandwidth of the circuit impedance  $Z$  can be extracted and the phase angle  $\varphi$  in phase with the voltage  $\underline{u}(t)$  is  $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) + 5^\circ$ .

Solution  
.. Calculation of physical values of the two components.  
Solution 
$$R = 20 \Omega, L = 2 \text{ mH} \quad \varphi = 87.06^\circ$$

Solution  
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \underline{U} = 50 \text{ V}$$
  
The voltage  $\underline{u}(t)$  is a cosine wave with an amplitude of 50 V and a phase angle of  $-10^\circ$ .  
The resulting current  $\underline{i}(t)$  is a cosine wave with an amplitude of 0.24 A and a phase angle of  $+90^\circ$ .  
Therefore, the component  $Z$  is a pure inductor with the same magnitude of 4.68  $\Omega$ .  
Impedance  $\underline{Z} = R + jX_L = 20 + j4.68 \Omega$ .  
$$\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) + 5^\circ = \arctan\left(\frac{4.68}{20}\right) + 5^\circ = 13.4^\circ + 5^\circ = 18.4^\circ$$
  
The phase angle  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{\text{Im}\{Z\}}{\text{Re}\{Z\}}\right) + 5^\circ = 18.4^\circ + 5^\circ = 23.4^\circ$ .

The absolute value of the impedance is  $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$  and the phase angle is  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$ .  
 With the complex part comes the physical value:  $I = \frac{U}{|Z|}$  and  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$ .

**Exercise E11 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with a resistor  $R = 1.00 \text{ k}\Omega$ , an inductor  $L = 4.7 \text{ }\mu\text{H}$  and a capacitor  $C = 40 \text{ nF}$  at  $f = 4 \text{ MHz}$ .  
 Result:  $Z = 1.00 \text{ k}\Omega$ ,  $\phi = 0^\circ$ .  
 A resistor  $R_1$  shall have the same absolute value of the impedance as a capacitor  $C_1 = 40 \text{ nF}$  at  $f_1 = 4 \text{ MHz}$ .

Solution  
 Solution:  $R_1 = 1.00 \text{ k}\Omega$   
 Solution:  $R_2 = 10.0 \text{ }\Omega$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R$  and  $L$  combined is given by  $Z_{RL} = R + j\omega L$ .  
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C_1$ .  
 The equivalent impedance for  $R_2$  and  $C_1$  combined is given by  $Z_{R_2C_1} = \frac{R_2 \cdot (-j/\omega C_1)}{R_2 - j/\omega C_1}$ .  
 Since  $Z_{RL}$  and  $Z_{R_2C_1}$  are perpendicular to each other, the resulting current of the parallel circuit is given as:  
 $I = \sqrt{I_{R_2}^2 + I_{C_1}^2}$   
 This can be simplified to  $I = \frac{U}{\sqrt{R_2^2 + \left(\frac{1}{\omega C_1}\right)^2}}$ .  
 Back to the first formula:  $R_2 \cdot \frac{1}{\omega C_1} = \sqrt{R_2^2 + \left(\frac{1}{\omega C_1}\right)^2} \cdot \frac{1}{\omega C_1}$   
 $R_2 = \sqrt{R_2^2 + \left(\frac{1}{\omega C_1}\right)^2}$

**Exercise E12 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)



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