

Exam Winter Semester 2022

Student Group

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of $d = 1.80 \text{ mm}$ is used for electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Determine the current I linked to the operating voltage for heating elements.

The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 3.57 \text{ mm}$.

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ &= \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \\ &= \frac{1}{4} d^2 \cdot \pi \quad \text{and } R = \rho \cdot \frac{l}{\frac{1}{4} d^2 \cdot \pi} \quad \text{and } R = \\ &= 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

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Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in a refrigeration system. The refrigerator has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal resistor at -40°C .

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

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\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{\textit{with } } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ \end{align*}

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\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{\textit{with } } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ \end{align*}

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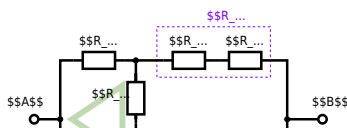
Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at once, $R_2 = R_3 = 100 \Omega$ and the switch shall be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution

$$R_{eq} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

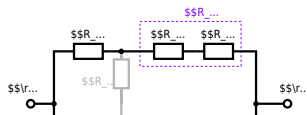
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \sim \Omega + 200 \sim \Omega + 200 \sim \Omega) \parallel (100 \sim \Omega + 100 \sim \Omega) \parallel (500 \sim \Omega) \parallel (200 \sim \Omega) \parallel (500 \sim \Omega \cdot 200 \sim \Omega) \over {500 \sim \Omega + 200 \sim \Omega}$$

Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 10 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel R_{\text{eq}}$$

**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance R_{in} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{34}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - I_4 \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

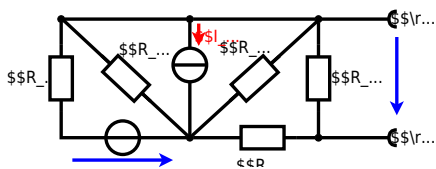
with $R_1 \parallel R_3 \parallel R_5 = 5\Omega \parallel 10\Omega \parallel 10\Omega = 5\Omega \parallel 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \parallel R_{AB} = 15\Omega \parallel (7.5\Omega + 2.5\Omega)$$

Exercise E6 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

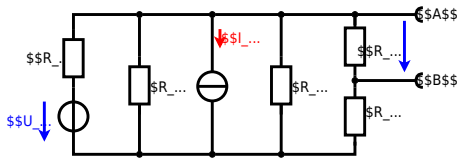
$$U_s = U_{AB} = 4.5\text{V} \parallel R_i = R_{AB} = 6\Omega$$



Calculate the internal resistance R_i and the source voltage U_s of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot R_{67}$$

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - (U_2 \cdot \frac{1}{R_1} - I_4) \cdot R_1 || R_3 || R_5$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \Omega \cdot \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega}$$

$$R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E7 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a battery with an internal resistance of R_1 and a switch S_1 and a capacitor C and a resistor R_2 in parallel. The voltage across the capacitor is again U_0 at the moment $t_0=0$ s when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1$ ms after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

$$U_{eq} = \frac{U \cdot R_2}{R_1 + R_2} \quad R_{eq} = R_1 || R_2$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$
 An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow:

$$\tau = R_i \cdot C \cdot \ln(0.5)$$

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E8 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the solution) consists of a 12 V DC voltage source, a capacitor of $100\text{ }\mu\text{F}$ and a resistor of $20\text{ }\Omega$. The voltage across the capacitor is again 0 V at the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

Solution
 To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 6\text{ V}$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source U is in series with the internal resistance R_1 . The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} . The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12 \text{ V}$, a switch S_1 , a resistor of $R_1=20 \text{ }\Omega$ and a capacitor of $C=100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0=0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0)=0$.

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_C(t_1)=0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R=0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$.

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1)=0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E9 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage $\underline{u}(t) = 50 \cos(300t - 10^\circ)$ V and the phasor current $\underline{i}(t) = 0.24 \cos(300t + \varphi)$ A are both through the components ($\$R\$ and $\$X_L\$) shall be given.$$

After analysis, the full bandwidth of the circuit impedance Z can be extracted and the phase shift φ in phase (in $^\circ$) can be determined. $Z = (R + j\omega L) \parallel (1/j\omega C) = (50 + j\omega L) \parallel (1/j\omega C)$

Solution
.. Calculation of the physical values of the two components.
Solution
$$R = 50 \Omega \quad L = 0.07 \text{ mH} \quad C = 2.2 \mu\text{F}$$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \varphi = \arg(\underline{U}) - \arg(\underline{Z}) = 50^\circ - \arg(Z)$$

The voltage $\underline{u}(t) = 50 \cos(300t - 10^\circ)$ V is the reference phasor ($\varphi_U = 0^\circ$ real) resulting in $\underline{U} = 50 \angle 0^\circ$ V. The impedance $\underline{Z} = R + j\omega L \parallel 1/j\omega C = 50 + j\omega L \parallel 1/j\omega C$.
Therefore, the component $1/j\omega C$ is in parallel with the same admittance $Y = 1/Z$.
Impedance $\underline{Z} = R + j\omega L \parallel 1/j\omega C = 50 + j\omega L \parallel 1/j\omega C = 50 + j\omega L \cdot \frac{1}{j\omega C} = 50 + \omega^2 L C$
$$\underline{Z} = 50 + j\omega L \parallel 1/j\omega C = 50 + \frac{j\omega L \cdot 1/j\omega C}{1 + \omega^2 L C} = 50 + \frac{L}{C} \frac{1}{1 + \omega^2 L C}$$

The admittance $\underline{Y} = 1/\underline{Z}$ can be calculated as $\varphi_i = \arg(\underline{I}) = \arg(\underline{U}) - \arg(\underline{Z}) = 50^\circ - \arg(Z)$
With the complex part $\varphi = \arg(Z) = \arctan(\frac{\omega L}{50 + \omega^2 L C})$
$$\varphi = \arctan\left(\frac{\omega L}{50 + \omega^2 L C}\right) = \arctan\left(\frac{300 \cdot 0.07 \cdot 10^{-3}}{50 + 300^2 \cdot 0.07 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-6}}\right) = \arctan\left(\frac{0.021}{50 + 0.1287}\right) = \arctan(0.0417) = 2.4^\circ$$

The phase φ can be calculated as $\varphi_i = \arg(\underline{I}) = \arg(\underline{U}) - \arg(\underline{Z}) = 50^\circ - 2.4^\circ = 47.6^\circ$

Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage $\underline{u}(t) = 50 \cos(300t - 10^\circ)$ V and the phasor current $\underline{i}(t) = 0.24 \cos(300t + \varphi)$ A are both through the components ($\$R\$ and $\$X_L\$) shall be given.$$

After analysis, the full bandwidth of the circuit impedance Z can be extracted and the phase shift φ in phase (in $^\circ$) can be determined. $Z = (R + j\omega L) \parallel (1/j\omega C) = (50 + j\omega L) \parallel (1/j\omega C)$

Solution
.. Calculation of the physical values of the two components.
Solution
$$R = 50 \Omega \quad L = 0.07 \text{ mH} \quad C = 2.2 \mu\text{F}$$

Solution
$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \quad \varphi = \arg(\underline{U}) - \arg(\underline{Z}) = 50^\circ - \arg(Z)$$

The voltage $\underline{u}(t) = 50 \cos(300t - 10^\circ)$ V is the reference phasor ($\varphi_U = 0^\circ$ real) resulting in $\underline{U} = 50 \angle 0^\circ$ V. The impedance $\underline{Z} = R + j\omega L \parallel 1/j\omega C = 50 + j\omega L \parallel 1/j\omega C$.
Therefore, the component $1/j\omega C$ is in parallel with the same admittance $Y = 1/Z$.
Impedance $\underline{Z} = R + j\omega L \parallel 1/j\omega C = 50 + j\omega L \parallel 1/j\omega C = 50 + \omega^2 L C$
$$\underline{Z} = 50 + j\omega L \parallel 1/j\omega C = 50 + \frac{j\omega L \cdot 1/j\omega C}{1 + \omega^2 L C} = 50 + \frac{L}{C} \frac{1}{1 + \omega^2 L C}$$

The admittance $\underline{Y} = 1/\underline{Z}$ can be calculated as $\varphi_i = \arg(\underline{I}) = \arg(\underline{U}) - \arg(\underline{Z}) = 50^\circ - \arg(Z)$
With the complex part $\varphi = \arg(Z) = \arctan(\frac{\omega L}{50 + \omega^2 L C})$
$$\varphi = \arctan\left(\frac{\omega L}{50 + \omega^2 L C}\right) = \arctan\left(\frac{300 \cdot 0.07 \cdot 10^{-3}}{50 + 300^2 \cdot 0.07 \cdot 10^{-3} \cdot 2.2 \cdot 10^{-6}}\right) = \arctan\left(\frac{0.021}{50 + 0.1287}\right) = \arctan(0.0417) = 2.4^\circ$$

The phase φ can be calculated as $\varphi_i = \arg(\underline{I}) = \arg(\underline{U}) - \arg(\underline{Z}) = 50^\circ - 2.4^\circ = 47.6^\circ$

The absolute value of the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and the phase angle is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.
 With the complex part comes the physical value: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.

Exercise E11 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with a resistor $R = 1.00 \text{ k}\Omega$, a capacitor $C = 40 \text{ nF}$ and an inductor $L = 4.7 \text{ }\mu\text{H}$ in AC with a voltage $U = 10 \text{ V}$ and a frequency $f = 450 \text{ kHz}$.
 Result: $Z = 1.00 \text{ k}\Omega$, $\phi = 0^\circ$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 Solution: $R_1 = 1.00 \text{ k}\Omega$
 Solution: $R_2 = 10.0 \text{ }\Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z_{RL} = \sqrt{R^2 + X_L^2}$.
 Parallel circuit means that the voltage is the same on R_2 and C_1 .
 The equivalent impedance for R_2 and C_1 combined is given by $Z_{R_2C_1} = \frac{R_2 X_C}{\sqrt{R_2^2 + X_C^2}}$.
 Since Z_{RL} and $Z_{R_2C_1}$ are perpendicular to each other, the resulting current of the parallel circuit is given as:
 $I = \sqrt{I_{R_2}^2 + I_{C_1}^2}$.
 This can be simplified to $I = \frac{U}{\sqrt{R_2^2 + X_C^2}}$.
 Back to the first formula: $R_2 \cdot I = X_C \cdot I$.
 $R_2 = X_C = \frac{1}{\omega C_1}$.

Exercise E12 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Resistor values $20 = 450 \text{ kHz}$ $4.7 \text{ } \mu\text{H}$ $30 \text{ } \mu\text{F}$ 3.0 V 15 kHz $330 \text{ } \mu\text{H}$ $0.22 \text{ } \mu\text{F}$

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\begin{align*} R_1 &= 1.00 \sim \Omega \\ R_2 &= 10.0 \sim \Omega \end{align*}

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A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by $Z = R + j\omega L$.
 Parallel circuit means that the voltage is the same on R_1 and R_2 .
 $Z = R_1 + R_2 + j\omega L$. Since ωL is perpendicular to R , this can be simplified to $Z = \sqrt{R^2 + (\omega L)^2}$. (It has to, since R is perpendicular to $j\omega L$.)

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (2\pi \cdot 15 \cdot 330 \cdot 10^{-6})^2}$$

Therefore, the resulting current of the parallel circuit is given as:

$$I = \frac{U}{Z} = \frac{3.0 \text{ V}}{\sqrt{10^2 + (\omega L)^2}}$$

Back to the first formula: $R_3 \cdot I = X_C \cdot I$

$$R_3 = X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 15 \cdot 30 \cdot 10^{-6}} = 353.68 \sim \Omega$$

Exercise E13 Complex Impedance Circuit
 (written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the resistor R in the circuit shown below.
 The voltage source is $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$.
 The circuit consists of a resistor $R = 10 \sim \Omega$, an inductor $L = 330 \sim \mu\text{H}$, and a capacitor $C = 0.22 \sim \mu\text{F}$, all in series.

Solution:
 Result: $i(t) = 197.3 \text{ mA} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t - 48.2^\circ)$

Draw the circuit diagram of the given circuit, label all components, voltages, and currents.

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$Z_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \sim \mu\text{F}}$$

$$\frac{1}{Z} = \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\frac{1}{Z} = \frac{1}{\sqrt{10^2 + (2\pi \cdot 15 \cdot 330 \cdot 10^{-6} - \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}})^2}}$$

$$\underline{Z} = R + j(\omega L - \frac{1}{\omega C}) = 10 + j(2\pi \cdot 15 \cdot 330 \cdot 10^{-6} - \frac{1}{2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}})$$

$$\underline{Z} = 10 - j19.8 \sim \Omega$$

$$|\underline{Z}| = \sqrt{10^2 + 19.8^2} = 22.1 \sim \Omega$$



Exercise E14 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the circuit impedance Z , the magnitude $|Z|$, and the phase ϕ and the real power P in the circuit. The voltage source $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V is connected in series with an inductor of $330 \mu\text{H}$ and a capacitor of $0.22 \mu\text{F}$.

Result: $Z = 19.8 - j48.2 \Omega$, $|Z| = 52.1 \Omega$, $\phi = -68.3^\circ$, $P = 0.12 \text{ W}$

Draw the circuit diagram of the given circuit.

Label all components, voltages, and currents.

Result: $Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}} = -j48.2 \Omega$

With $Z_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} = j31.1 \Omega$

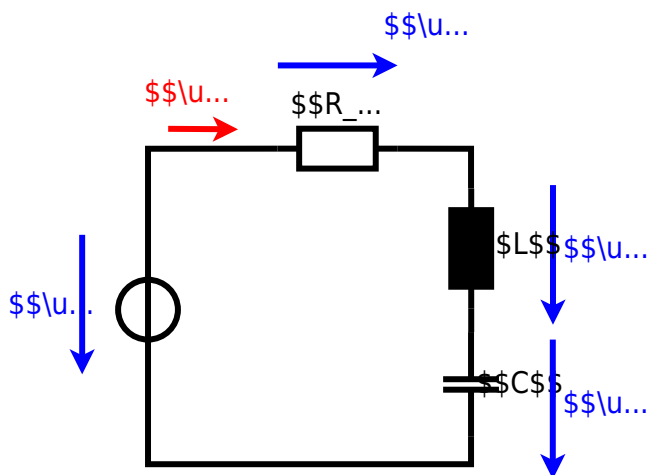
$Z = Z_L + Z_C = j31.1 - j48.2 = -j17.1 \Omega$

$|Z| = \sqrt{R^2 + X^2} = \sqrt{0^2 + (-17.1)^2} = 17.1 \Omega$

$\phi = \arctan\left(\frac{X}{R}\right) = \arctan\left(\frac{-17.1}{0}\right) = -90^\circ$

$P = I_{\text{eff}}^2 \cdot R = \left(\frac{U_{\text{eff}}}{|Z|}\right)^2 \cdot R = \left(\frac{1.5}{17.1}\right)^2 \cdot 0 = 0 \text{ W}$





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