

Exam Winter Semester 2022

Student Group

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Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of $d = 0.357 \text{ mm}$ and an electric power dissipation (= heat flow) of $P = 40 \text{ W}$ is necessary.

Determine the current I linked to the operating voltage $U = 230 \text{ V}$ for heating elements.

The Nichrome wire has a resistivity of $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$.

The heating element is $l = 3 \text{ m}$ long and has a diameter of $d = 0.357 \text{ mm}$.

∴ Calculate the resistance R of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \\ \sqrt{\frac{P}{R}} &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \\ \frac{1}{4} d^2 \cdot \pi \quad R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \text{and } R = \\ 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

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Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A refrigerator exhibits a temperature coefficient of resistance in its refrigeration system. The circuit has a resistance of $10 \text{ k}\Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.

Result: The temperature inside the refrigeration system can reach down to -40°C .

Calculate the resistance of the thermal circuit at -40°C .

The power transfer resistor P is a part of the circuit and generates heat. Therefore, a solution is to increase the heat flow up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2)$$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right)$$

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Exercise E3 Pure Resistor Network Simplification
(written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved at once, $R_2 = R_3 = 100 \Omega$ and the switch shall be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution

$$R_{eq} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2 = R_3$ and based on the equations for the transformation, the transformed R_Y is given as:

$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 15 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance R_{int} and the source voltage U_{s} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \Omega$, $U_2=6.0 \text{ V}$, $R_3=10 \Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \Omega$, $R_6=7.5 \Omega$, $R_7=15 \Omega$. Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :
$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:
$$U_{24} = I_{24} \cdot R_{56}$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$:

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega}$$

$$R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

Exercise E6 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

$$U_s = U_{AB} = 4.5\text{V} \quad R_i = R_{AB} = 6\Omega$$



Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $\begin{aligned} R_1 &= 5.0 \text{ } \Omega, & U_2 &= 6.0 \text{ V}, & R_3 &= 10 \text{ } \Omega, & I_4 &= 4.2 \text{ A}, & R_5 &= 10 \text{ } \Omega, & R_6 &= 7.5 \text{ } \Omega, & R_7 &= 15 \text{ } \Omega \end{aligned}$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :

$$U_{AB} = R_{135} \cdot I_{24} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left\{ \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \right\} = \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left\{ \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \right\}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with $R_1 \parallel R_3 \parallel R_5 = 5 \Omega \parallel 10 \Omega \parallel 10 \Omega = 5 \Omega \parallel 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left\{ \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right\}$$

$$R_{AB} = 15 \Omega \parallel (7.5 \Omega + 2.5 \Omega)$$

Exercise E7 Charging Capacitors
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a RC circuit consisting of a DC voltage source U , a resistor R_1 , a resistor R_2 , a capacitor C , and a switch S_1 . The switch S_1 is open. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

$$U_{eq} = \frac{U \cdot R_2}{R_1 + R_2} = \frac{12 \text{ V} \cdot 2 \Omega}{2 \Omega + 2 \Omega} = 3 \text{ V}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U=12 \text{ V}$, a switch S_1 , a resistor of $R_1=20 \text{ }\Omega$ and a capacitor of $C=100 \text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0 \text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0 \text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \Rightarrow e^{-t/\tau} = 0.5 \Rightarrow t/\tau = \ln(0.5) \Rightarrow t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$ An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$ The internal resistance is given by substituting the ideal voltage source with its resistance ($=0 \text{ }\Omega$, short-circuit).
$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1 \text{ ms} / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

Exercise E8 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (as shown in the solution) consists of a 12 V DC voltage source, a $20 \text{ }\Omega$ resistor, a $100 \text{ }\mu\text{F}$ capacitor, and a $20 \text{ }\Omega$ resistor. The voltage across the capacitor is again 0 V at the moment $t_0=0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1 \text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U = 6 \text{ V}$$

$$R_i = R_1 \parallel R_B = 10 \text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .



The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$

$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$
So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_c(t)$ which has to be $u_c(t_1) = 0.5 \cdot U$: $u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$
It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E9 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage $\underline{u}(t) = 50 \cos(300t - 10^\circ)$ V and the phasor current $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$ A are both through the components (R and X_L) shall be given.

After analysis, the full bandwidth of the circuit impedance Z can be extracted and the phase angle φ in phase with the voltage $\underline{u}(t) = 50 \cos(300t + 90^\circ)$ V shall be given.

Solution
.. Calculation of the physical values of the two components.
Solution
$$R = \frac{U}{I} = \frac{50}{0.24} = 208.33 \Omega$$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{0.24 \angle 90^\circ} = 208.33 \angle -100^\circ$$

The voltage $\underline{u}(t) = 50 \cos(300t - 10^\circ)$ V and the current $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$ A are both through the components (R and X_L) shall be given.

After analysis, the full bandwidth of the circuit impedance Z can be extracted and the phase angle φ in phase with the voltage $\underline{u}(t) = 50 \cos(300t + 90^\circ)$ V shall be given.

Solution
.. Calculation of the physical values of the two components.
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The voltage $\underline{u}(t) = 50 \cos(300t - 10^\circ)$ V and the current $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$ A are both through the components (R and X_L) shall be given.

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After analysis, the full bandwidth of the circuit impedance Z can be extracted and the phase angle φ in phase with the voltage $\underline{u}(t) = 50 \cos(300t + 90^\circ)$ V shall be given.

Solution
.. Calculation of the physical values of the two components.
Solution
$$R = \frac{U}{I} = \frac{50}{0.24} = 208.33 \Omega$$

Solution

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{0.24 \angle 90^\circ} = 208.33 \angle -100^\circ$$

The voltage $\underline{u}(t) = 50 \cos(300t - 10^\circ)$ V and the current $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$ A are both through the components (R and X_L) shall be given.

After analysis, the full bandwidth of the circuit impedance Z can be extracted and the phase angle φ in phase with the voltage $\underline{u}(t) = 50 \cos(300t + 90^\circ)$ V shall be given.

Solution
.. Calculation of the physical values of the two components.
Solution
$$R = \frac{U}{I} = \frac{50}{0.24} = 208.33 \Omega$$

The absolute value of the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ with $R = 5 \Omega$, $X_L = \omega L = 2\pi \cdot 4 \text{ MHz} \cdot 100 \text{ nH} = 2.51 \text{ m}\Omega$ and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 10 \text{ nF}} = 3.98 \text{ m}\Omega$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{2.51 \text{ m}\Omega - 3.98 \text{ m}\Omega}{5 \Omega}\right) = -0.24 \text{ rad}$.
 With the complex part comes the physical value: $I = \frac{U}{Z} = \frac{50 \text{ V}}{\sqrt{5^2 + (2.51 - 3.98)^2}} = 10 \text{ A}$.
 The phase ϕ is $\phi = -0.24 \text{ rad} = -13.7^\circ$.

Exercise E11 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with $R_1 = 1 \text{ k}\Omega$, $R_2 = 4.7 \text{ k}\Omega$, $L = 100 \text{ nH}$ and $C = 10 \text{ nF}$ at $f = 4 \text{ MHz}$, the voltage across the resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 The equivalent impedance for R_1 and R_2 combined is given by $Z_{R1R2} = R_1 + R_2 = 5.7 \text{ k}\Omega$.
 The equivalent impedance for L and C combined is given by $Z_{LC} = \frac{j\omega L}{1 - \omega^2 LC}$.
 The total impedance is $Z = Z_{R1R2} + Z_{LC}$.
 The absolute value of the impedance is $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$.
 The voltage across R_1 is $U_{R1} = I \cdot R_1$.
 The voltage across C_1 is $U_{C1} = I \cdot X_{C1}$.
 The condition is $U_{R1} = U_{C1}$, which leads to $R_1 = X_{C1}$.
 This can be simplified to $R_1 = \frac{1}{\omega C_1}$.
 Therefore, the resulting current of the parallel circuit is given as:
 $I = \frac{U}{Z} = \frac{50 \text{ V}}{\sqrt{5.7^2 + (2.51 - 3.98)^2}} = 10 \text{ A}$.
 Back to the first formula: $R_1 \cdot \frac{1}{\omega C_1} = \frac{1}{\omega C_1} \cdot \frac{1}{\sqrt{R_1^2 + (X_L - X_C)^2}}$.

Exercise E12 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution

$$R_1 = 1.00 \text{ } \Omega$$

$$R_2 = 10.0 \text{ } \Omega$$

A series circuit means that the current is constant on every component.

The equivalent impedance for R and L combined is given by

$$Z_{RL} = R + j\omega L$$

Parallel circuit means that the voltage is the same on R_2 and C_1

$$Z_{RC} = \frac{R_2 \cdot (-j/\omega C_1)}{R_2 - j/\omega C_1}$$

Since Z_{RC} is perpendicular to Z_{RL} , the resulting current of the parallel circuit is given as:

$$I_{3R} = I_{2R} + I_{3C}$$

$$I_{3R} = \frac{U}{R_2} + \frac{U}{-j\omega C_1}$$

$$I_{3R} = \frac{U}{R_2} \left(1 - j\omega R_2 C_1 \right)$$

$$I_{3R} = \frac{3.0 \text{ V}}{10 \text{ } \Omega} \left(1 - j \cdot 2\pi \cdot 4 \cdot 10^6 \cdot 40 \cdot 10^{-9} \right)$$

$$I_{3R} = 0.3 \text{ A} \cdot (1 - j0.1005)$$

$$I_{3R} = 0.3 \text{ A} - j0.03015 \text{ A}$$

$$|I_{3R}| = \sqrt{0.3^2 + 0.03015^2} = 0.3015 \text{ A}$$

Exercise E13 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the resistor R in the circuit shown below. The voltage source is $u(t) = 3.0 \text{ V} \cdot \sin(2\pi \cdot 15 \text{ kHz} \cdot t)$.

Solution

Result

$$Z = 48.2 \text{ } \Omega \quad Z_C = 19.8 \text{ } \Omega$$

Draw the circuit diagram of the given circuit

$$Z = \frac{U}{I} \quad I = \frac{U}{Z}$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}}$$

$$Z = R + j\omega L - j\omega C = 10 \text{ } \Omega + j \cdot 2\pi \cdot 15 \text{ kHz} \cdot 330 \text{ } \mu\text{H} - j \cdot 2\pi \cdot 15 \text{ kHz} \cdot 0.22 \text{ } \mu\text{F}$$

$$Z = 10 \text{ } \Omega + j19.8 \text{ } \Omega - j19.8 \text{ } \Omega = 10 \text{ } \Omega$$

$$I = \frac{3.0 \text{ V}}{10 \text{ } \Omega} = 0.3 \text{ A}$$

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Exercise E14 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ in the circuit shown in Fig. 1. The voltage source $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V is connected in series with an inductor of $330 \mu\text{H}$ and a capacitor of $0.22 \mu\text{F}$.

Result: $Z = 19.8 - j48.2 \Omega$

Draw the circuit diagram of the given circuit and label all components, voltages, and currents.

$$Z = \frac{\hat{U}}{\hat{I}} \quad \hat{I} = \frac{\hat{U}}{Z} \quad Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}}$$

$$\hat{I} = \frac{3.0}{19.8 - j48.2} = 0.61 \cdot e^{j\phi} \text{ A}$$

$$i(t) = 0.61 \cdot \sin(2\pi \cdot 15 \cdot t + \phi) \text{ A}$$

$$\underline{Z} = R + j\omega L - j\omega C = 330 \cdot 10^{-6} + j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} - j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}$$

$$\underline{Z} = 330 \cdot 10^{-6} + j \cdot 3.16 \cdot 10^{-3} - j \cdot 1.98 \cdot 10^{-3} = 330 \cdot 10^{-6} + j \cdot 1.18 \cdot 10^{-3} \Omega$$

$$\underline{Z} = R + j(X_L - X_C) = 330 \cdot 10^{-6} + j(3.16 - 1.98) \cdot 10^{-3} \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(330 \cdot 10^{-6})^2 + (1.18 \cdot 10^{-3})^2} = 1.18 \cdot 10^{-3} \Omega$$

$$\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{1.18 \cdot 10^{-3}}{330 \cdot 10^{-6}}\right) = 1.1 \text{ rad}$$

$$\hat{I} = \frac{\hat{U}}{|\underline{Z}|} = \frac{3.0}{1.18 \cdot 10^{-3}} = 2.54 \cdot 10^3 \text{ A}$$

$$i(t) = 2.54 \cdot \sin(2\pi \cdot 15 \cdot t + 1.1) \text{ A}$$

$$\underline{Z} = R + j\omega L - j\omega C = 330 \cdot 10^{-6} + j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} - j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}$$

$$\underline{Z} = 330 \cdot 10^{-6} + j \cdot 3.16 \cdot 10^{-3} - j \cdot 1.98 \cdot 10^{-3} = 330 \cdot 10^{-6} + j \cdot 1.18 \cdot 10^{-3} \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(330 \cdot 10^{-6})^2 + (1.18 \cdot 10^{-3})^2} = 1.18 \cdot 10^{-3} \Omega$$

$$\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{1.18 \cdot 10^{-3}}{330 \cdot 10^{-6}}\right) = 1.1 \text{ rad}$$

$$\hat{I} = \frac{\hat{U}}{|\underline{Z}|} = \frac{3.0}{1.18 \cdot 10^{-3}} = 2.54 \cdot 10^3 \text{ A}$$

$$i(t) = 2.54 \cdot \sin(2\pi \cdot 15 \cdot t + 1.1) \text{ A}$$

$$\underline{Z} = R + j\omega L - j\omega C = 330 \cdot 10^{-6} + j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6} - j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}$$

$$\underline{Z} = 330 \cdot 10^{-6} + j \cdot 3.16 \cdot 10^{-3} - j \cdot 1.98 \cdot 10^{-3} = 330 \cdot 10^{-6} + j \cdot 1.18 \cdot 10^{-3} \Omega$$

$$|\underline{Z}| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(330 \cdot 10^{-6})^2 + (1.18 \cdot 10^{-3})^2} = 1.18 \cdot 10^{-3} \Omega$$

$$\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{1.18 \cdot 10^{-3}}{330 \cdot 10^{-6}}\right) = 1.1 \text{ rad}$$

$$\hat{I} = \frac{\hat{U}}{|\underline{Z}|} = \frac{3.0}{1.18 \cdot 10^{-3}} = 2.54 \cdot 10^3 \text{ A}$$

$$i(t) = 2.54 \cdot \sin(2\pi \cdot 15 \cdot t + 1.1) \text{ A}$$



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