

# Exam Winter Semester 2022

## Student Group

First Name	Surname	Matrikel Nr.

## Table of Contents

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) .....	3
Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022) .....	3
Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) .....	3
Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022) .....	4
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) .....	5
Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022) .....	6
Exercise E5 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) .....	8
Exercise E6 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022) .....	12
Exercise E7 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) .....	16
Exercise E8 Charging Capacitors (written test, approx. 16 % of a 60-minute written test, WS2022) .....	17
Exercise E9 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) .....	19
Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022) .....	19
Exercise E11 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) .....	20
Exercise E12 Impedances at different Frequencies (written test, approx. 18 % of a 60-minute written test, WS2022) .....	20
Exercise E13 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written	

---

test, WS2022) .....	21
Exercise E14 Complex Impedance Circuit (written test, approx. 15 % of a 60-minute written test, WS2022) .....	24

### Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of  $d = 0.357 \text{ mm}$  and an electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary.

Determine the current  $I$  linked to the operating voltage  $U = 230 \text{ V}$  for heating elements.

The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$ .

The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 0.357 \text{ mm}$ .

∴ Calculate the resistance  $R$  of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{l}{\frac{1}{4} d^2 \cdot \pi} \quad \text{with } R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

### Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a diameter of  $d = 0.357 \text{ mm}$  and an electric power dissipation (= heat flow) of  $P = 40 \text{ W}$  is necessary.

Determine the current  $I$  linked to the operating voltage  $U = 230 \text{ V}$  for heating elements.

The Nichrome wire has a resistivity of  $\rho = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m}$ .

The heating element is  $l = 3 \text{ m}$  long and has a diameter of  $d = 0.357 \text{ mm}$ .

∴ Calculate the resistance  $R$  of the heating element.

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} \\ &= \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{l}{\frac{1}{4} d^2 \cdot \pi} \quad \text{with } R = 1.10 \cdot 10^{-6} \text{ } \Omega \cdot \text{m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

### Exercise E2 Temperature-dependent Resistance

**(written test, approx. 6 % of a 60-minute written test, WS2022)**

2. The diagram shows a thermistor with a temperature coefficient of resistance  $\alpha = 0.01 \text{ K}^{-1}$  and a reference resistance  $R_0 = 10 \text{ k}\Omega$  at  $T_0 = 25^\circ\text{C}$ . The thermistor is used in a Wheatstone bridge circuit. The bridge is powered by a 10 V DC source. The bridge output is connected to an operational amplifier configured as a voltage follower. The op-amp is powered by a 5 V supply. The bridge resistance is  $R = 6.5 \text{ k}\Omega$ . Calculate the resistance of the thermistor at  $T = -40^\circ\text{C}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermistor at  $T = -40^\circ\text{C}$ .

The power transfer resistor  $R$  is a part of the circuit and generates heat. Therefore, a solution is to use a heat sink to cool the resistor.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

```
\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{with } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ \end{align*}
```

**Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)**

2. The diagram shows a thermistor with a temperature coefficient of resistance  $\alpha = 0.01 \text{ K}^{-1}$  and a reference resistance  $R_0 = 10 \text{ k}\Omega$  at  $T_0 = 25^\circ\text{C}$ . The thermistor is used in a Wheatstone bridge circuit. The bridge is powered by a 10 V DC source. The bridge output is connected to an operational amplifier configured as a voltage follower. The op-amp is powered by a 5 V supply. The bridge resistance is  $R = 6.5 \text{ k}\Omega$ . Calculate the resistance of the thermistor at  $T = -40^\circ\text{C}$ .

Result: The temperature inside the refrigeration system can reach down to  $-40^\circ\text{C}$ .

Calculate the resistance of the thermistor at  $T = -40^\circ\text{C}$ .

The power transfer resistor  $R$  is a part of the circuit and generates heat. Therefore, a solution is to use a heat sink to cool the resistor.

Therefore, with constant  $U$  and increasing  $R$  the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

```
\begin{align*} R &= R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) && | \\ \text{with } \Delta T &= T_{\text{end}} - T_{\text{start}} \\ R &= 10 \text{ k}\Omega \cdot \left(1 + 0.01 \frac{1}{\text{K}} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \frac{1}{\text{K}^2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2\right) \\ \end{align*}
```

**Exercise E3 Pure Resistor Network Simplification**  
**(written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved at once,  $R_2 = R_3 = 100 \Omega$  and the switch is given.  $R_1 = 400 \Omega$ . Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution

$$R_{eq} = 133.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2 = R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between  $A$  and  $B$ .

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

**Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)**

The following shall be solved with  $R_1 = 200 \Omega$ ,  $R_2 = R_3 = 100 \Omega$  and the source  $B = 15 \text{ V}$ .  
 Result given:  $R_{\text{eq}} = 132.8 \Omega$ .

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.



Since  $R_2=R_3$  and based on the equations for the transformation, the transformed  $R_Y$  is given as: 
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series: 
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance  $R_{eq}$  between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel$$

**Exercise E5 Equivalent linear Source  
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.  
Result

$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



Calculate the internal resistance  $R_{in}$  and the source voltage  $U_{s}$  of an equivalent linear voltage source on the connectors  $A$  and  $B$ .  $R_1=5.0 \Omega$ ,  $U_2=6.0 \text{ V}$ ,  $R_3=10 \Omega$ ,  $I_4=4.2 \text{ A}$ ,  $R_5=10 \Omega$ ,  $R_6=7.5 \Omega$ ,  $R_7=15 \Omega$ . Use equivalent sources in order to simplify the circuit!

### Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ : 
$$R_{135} = R_1 || R_3 || R_5$$
 
$$I_{24} = I_2 - I_4$$
 The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following: 
$$U_{24} = I_{24} \cdot (R_5 || R_6)$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0\Omega$ , so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with  $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$ :

$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega} \\ R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

### Exercise E6 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.  
Result

$$U_s = U_{AB} = 4.5\text{V} \\ R_i = R_{AB} = 6\Omega$$



Calculated the internal resistance  $R_{\text{int}}$  and the source voltage  $U_{\text{oc}}$  of an equivalent linear voltage source on the connectors  $\text{A}$  and  $\text{B}$ . 
$$R_1=5.0 \text{ } \Omega, \quad U_2=6.0 \text{ V}, \quad R_3= 10 \text{ } \Omega, \quad I_4=4.2 \text{ A}, \quad R_5=10 \text{ } \Omega, \quad R_6=7.5 \text{ } \Omega, \quad R_7=15 \text{ } \Omega$$
 Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of  $U_2$  and  $R_1$  can be transformed into a current source  $I_2 = \frac{U_2}{R_1}$  and  $R_1$ :



Now a lot of them can be combined. The resistors  $R_1$ ,  $R_3$ ,  $R_5$  are in parallel, like also  $I_2$  and  $I_4$ :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4 = \frac{U_2}{R_1} - I_4$$

The resulting circuit can again be transformed:



Here, the  $U_{24}$  is calculated by  $I_{24}$  as the following:

$$U_{24} = I_{24} \cdot R_{135}$$

$$U_{AB} = R_{135} \cdot I_{24} = \left( \frac{U_2}{R_1} - I_4 \right) \cdot R_1 \parallel R_3 \parallel R_5$$

On the right side of the last circuit, there is a voltage divider given by  $R_{135}$ ,  $R_6$ , and  $R_7$ .

Therefore the voltage between  $A$  and  $B$  is given as:

$$U_{AB} = U_{24} \cdot \left\{ \frac{R_7}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \right\} = \left( \frac{U_2}{R_1} - I_4 \right) \cdot \left\{ \frac{R_7 \cdot R_1 \parallel R_3 \parallel R_5}{R_6 + R_7 + R_1 \parallel R_3 \parallel R_5} \right\}$$

For the internal resistance  $R_i$  the ideal voltage source is substituted by its resistance ( $=0 \Omega$ , so a short-circuit):

$$R_{AB} = R_7 \parallel (R_6 + R_1 \parallel R_3 \parallel R_5)$$

with  $R_1 \parallel R_3 \parallel R_5 = 5 \Omega \parallel 10 \Omega \parallel 10 \Omega = 5 \Omega \parallel 5 \Omega = 2.5 \Omega$ :

$$U_{AB} = \left( \frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left\{ \frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right\}$$

$$R_{AB} = 15 \Omega \parallel (7.5 \Omega + 2.5 \Omega)$$

**Exercise E7 Charging Capacitors**  
 (written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit below is a circuit with a DC voltage source  $U$ , a resistor  $R_1$ , a resistor  $R_2$ , a capacitor  $C$ , and a switch  $S_1$ . The switch  $S_1$  is open. The voltage across the capacitor is again  $0 \text{ V}$  at the moment  $t_0 = 0 \text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2 = 1 \text{ ms}$  after closing the switch.

**Solution:** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_2$ .

The ideal voltage source  $U_{eq}$  is given by:

$$U_{eq} = \frac{U \cdot R_2}{R_1 + R_2}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .



The circuit contains a voltage source  $U=12\text{ V}$ , a switch  $S_1$ , a resistor of  $R_1=20\text{ }\Omega$  and a capacitor of  $C=100\text{ }\mu\text{F}$ . The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first task. At the moment  $t_0=0\text{ s}$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_c(t_0)=0\text{ V}$ .



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time  $t_1$  when  $u_c(t_1)=0.5 \cdot U$ .

Solution



So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_c(t_1)=0.5 \cdot U$ : 
$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies -t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = 1/2 \cdot U$  The internal resistance is given by substituting the ideal voltage source with its resistance ( $=0\text{ }\Omega$ , short-circuit). 
$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

**Exercise E8 Charging Capacitors**  
(written test, approx. 16 % of a 60-minute written test, WS2022)

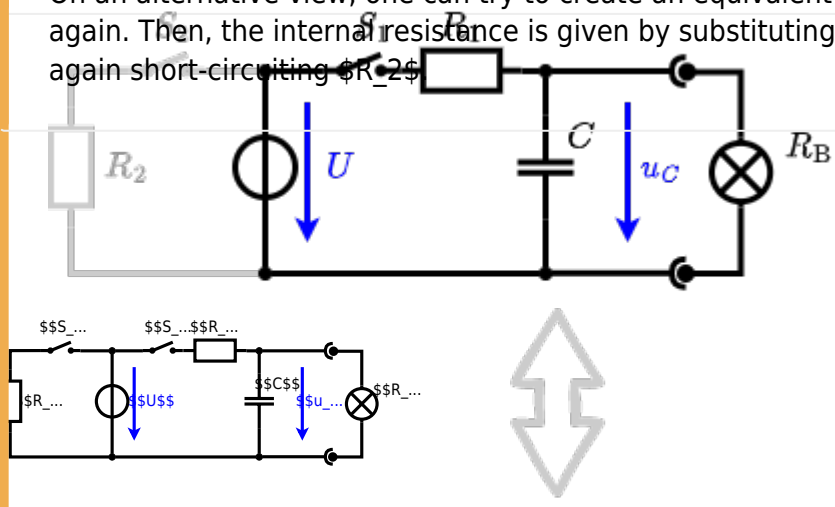
The circuit (as shown in the figure) consists of a DC voltage source  $U=6\text{ V}$ , a resistor  $R_1=20\text{ }\Omega$ , a capacitor  $C=20\text{ }\mu\text{F}$  and a light bulb  $R_B=20\text{ }\Omega$ . The voltage across the capacitor is again  $0\text{ V}$  at the moment  $t_0=0\text{ s}$  when the switch  $S_1$  is closed. Calculate the voltage  $u_c(t_2)$  across the capacitor at  $t_2=1\text{ ms}$  after closing the switch.

**Solution** To solve this, first create an equivalent linear voltage source from  $U$ ,  $R_1$ , and  $R_B$ . 
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = 2\text{ V}$$
 
$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is  $U = 12 \text{ V}$ . The internal resistance is  $R_1 = 20 \text{ }\Omega$ . The voltage across the capacitor is  $u_C$ . The voltage across the light bulb is  $u_B$ . The voltage across the resistor  $R_2$  is  $u_{R_2}$ . The voltage across the capacitor is  $u_C$ . The voltage across the light bulb is  $u_B$ . The voltage across the resistor  $R_2$  is  $u_{R_2}$ .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting  $R_2$ .

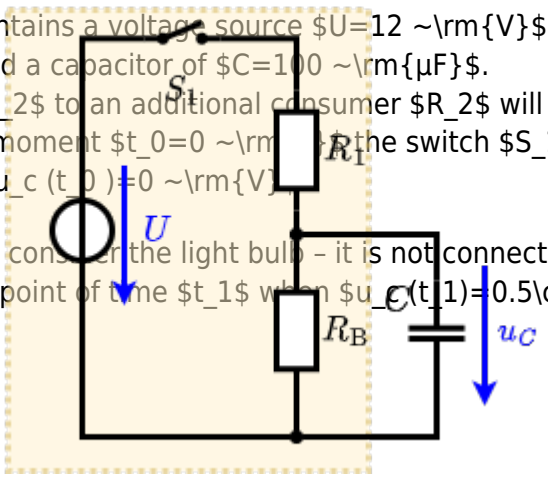


The circuit contains a voltage source  $U = 12 \text{ V}$ , a switch  $S_1$ , a resistor of  $R_1 = 20 \text{ }\Omega$  and a capacitor of  $C = 100 \text{ }\mu\text{F}$ .

The switch  $S_2$  to an additional consumer  $R_2$  will be considered to be open for the first asks. At the moment  $t_0 = 0$  the switch  $S_1$  is closed, the voltage across the capacitor is  $u_C(t_0) = 0$ .

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time  $t_1$  when  $u_C(t_1) = 0.5 \cdot U$ .



Solution

An equivalent linear voltage source can be given with  $U$ ,  $R_1$ , and  $R_B$  as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is:  $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$ . The internal resistance is given by substituting the ideal voltage source with its resistance ( $R = 0 \text{ }\Omega$ , short-circuit).  $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$ .

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2 / (R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2 / (10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only  $R_1$  and  $C$  gives the time constant:  $\tau = R_1 \cdot C$

The following formula describes the time course of  $u_C(t)$  which has to be  $u_C(t_1) = 0.5 \cdot U$ :  $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$ . It has to be rearranged to  $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

### Exercise E9 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage  $\underline{u}(t) = 50 \cos(300t - 10^\circ)$  V and the phasor current  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A are both through the components ( $R$  and  $X_L$ ) shall be given.

After analysis, the full bandwidth of the circuit impedance can be extracted and the phase shift  $\varphi$  in phase (in  $Z$ ) late  $\underline{u}(t) = 50 \cos(300t + 90^\circ)$  V and  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A.

Solution  
.. Calculation of physical values of the two components.  
Solution  $\underline{R} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{0.24 \angle 90^\circ} = 208.33 \angle -100^\circ \Omega$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{0.24 \angle 90^\circ} = 208.33 \angle -100^\circ \Omega$   
The voltage  $\underline{u}(t) = 50 \cos(300t - 10^\circ)$  V and the current  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A are both through the components ( $R$  and  $X_L$ ) shall be given.

resulting impedance  $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{0.24 \angle 90^\circ} = 208.33 \angle -100^\circ \Omega$   
Therefore, the component  $R$  is  $208.33 \cos(-100^\circ) = -68.8 \Omega$  and the component  $X_L$  is  $208.33 \sin(-100^\circ) = -208.33 \sin(100^\circ) = -208.33 \cos(10^\circ) = -208.33 \cdot 0.9848 = -205.1 \Omega$

impedance  $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{0.24 \angle 90^\circ} = 208.33 \angle -100^\circ \Omega$   
 $\underline{Z} = R + jX_L = -68.8 - j205.1 \Omega$   
The phase shift  $\varphi$  is  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{-205.1}{-68.8}\right) = \arctan(2.98) = 70.9^\circ$

With the complex part  $\underline{Z} = R + jX_L = -68.8 - j205.1 \Omega$   
 $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{-205.1}{-68.8}\right) = \arctan(2.98) = 70.9^\circ$

The phase shift  $\varphi$  can be calculated as  $\varphi = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{-205.1}{-68.8}\right) = \arctan(2.98) = 70.9^\circ$

### Exercise E10 Analyzing complex Impedances (written test, approx. 14 % of a 60-minute written test, WS2022)

2. Given that the phasor voltage  $\underline{u}(t) = 50 \cos(300t - 10^\circ)$  V and the phasor current  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A are both through the components ( $R$  and  $X_L$ ) shall be given.

After analysis, the full bandwidth of the circuit impedance can be extracted and the phase shift  $\varphi$  in phase (in  $Z$ ) late  $\underline{u}(t) = 50 \cos(300t + 90^\circ)$  V and  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A.

Solution  
.. Calculation of physical values of the two components.  
Solution  $\underline{R} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{0.24 \angle 90^\circ} = 208.33 \angle -100^\circ \Omega$

Solution

$\underline{I} = \frac{\underline{U}}{\underline{Z}} \Leftrightarrow \underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{0.24 \angle 90^\circ} = 208.33 \angle -100^\circ \Omega$   
The voltage  $\underline{u}(t) = 50 \cos(300t - 10^\circ)$  V and the current  $\underline{i}(t) = 0.24 \cos(300t + 90^\circ)$  A are both through the components ( $R$  and  $X_L$ ) shall be given.

resulting impedance  $\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{50 \angle -10^\circ}{0.24 \angle 90^\circ} = 208.33 \angle -100^\circ \Omega$   
Therefore, the component  $R$  is  $208.33 \cos(-100^\circ) = -68.8 \Omega$  and the component  $X_L$  is  $208.33 \sin(-100^\circ) = -208.33 \sin(100^\circ) = -208.33 \cos(10^\circ) = -208.33 \cdot 0.9848 = -205.1 \Omega$

The absolute value of the impedance is  $Z = \sqrt{R^2 + (X_L - X_C)^2}$  with  $R = 5 \Omega$ ,  $X_L = \omega L = 2\pi \cdot 4 \text{ MHz} \cdot 100 \text{ nH} = 2.51 \text{ m}\Omega$  and  $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 10 \text{ nF}} = 3.98 \text{ m}\Omega$ .  
 The phase  $\phi$  is given by  $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{2.51 \text{ m}\Omega - 3.98 \text{ m}\Omega}{5 \Omega}\right) = -0.24 \text{ rad}$ .  
 With the complex part comes the physical value:  $I = \frac{U}{Z} = \frac{50 \text{ V}}{\sqrt{5^2 + (2.51 - 3.98)^2}} = 9.99 \text{ A}$ .  
 The phase  $\phi$  is  $\phi = -0.24 \text{ rad} = -13.7^\circ$ .

**Exercise E11 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with  $R_1 = 1 \text{ k}\Omega$ ,  $R_2 = 4.7 \text{ k}\Omega$ ,  $L = 100 \text{ nH}$  and  $C = 10 \text{ nF}$  at  $f = 4 \text{ MHz}$ , the voltage across the capacitor  $U_C = 50 \text{ V}$  is measured. What is the voltage across the resistor  $R_2$ ?  
 Result:  $U_{R_2} = 4.7 \text{ V}$

Solution:  

$$R_1 = 1.00 \text{ k}\Omega$$

$$R_2 = 4.7 \text{ k}\Omega$$

$$L = 100 \text{ nH}$$

$$C = 10 \text{ nF}$$

$$f = 4 \text{ MHz}$$

$$U_C = 50 \text{ V}$$

A series circuit means that the current is constant on every component.  
 The equivalent impedance for  $R_1$  and  $R_2$  combined is given by  $R_{1+2} = R_1 + R_2$ .  
 Parallel circuit means that the voltage is the same on  $R_2$  and  $C$ .  

$$U_C = U_{R_2} = U_C$$
  

$$I_C = \frac{U_C}{X_C} = \frac{U_C}{\frac{1}{\omega C}} = U_C \cdot \omega C$$
  

$$I_{R_2} = \frac{U_{R_2}}{R_2} = \frac{U_C}{R_2}$$
  
 Since  $I_C = I_{R_2}$  (series circuit), we have  $U_C \cdot \omega C = \frac{U_C}{R_2}$ .  

$$\omega C = \frac{1}{R_2}$$
  

$$2\pi \cdot 4 \text{ MHz} \cdot 10 \text{ nF} = \frac{1}{4.7 \text{ k}\Omega}$$
  
 This is not true, so the circuit is not a parallel circuit.  
 The resulting current of the parallel circuit is given as:  

$$I_{R_2} = \frac{U_C}{R_2} = \frac{50 \text{ V}}{4.7 \text{ k}\Omega} = 10.6 \text{ mA}$$
  
 Back to the first formula: 
$$U_C = I_{R_2} \cdot X_C = I_{R_2} \cdot \frac{1}{\omega C}$$
  

$$50 \text{ V} = 10.6 \text{ mA} \cdot \frac{1}{2\pi \cdot 4 \text{ MHz} \cdot 10 \text{ nF}}$$
  
 This is also not true, so the circuit is not a parallel circuit.

**Exercise E12 Impedances at different Frequencies**  
 (written test, approx. 18 % of a 60-minute written test, WS2022)



□□□□□□□□□□ 10510...









From:  
<https://first.mexle.te.hs-heilbronn.de/> - **MEXLE Wiki**

Permanent link:  
[https://first.mexle.te.hs-heilbronn.de/electrical\\_engineering\\_1/ws2022\\_exam?rev=1680389116](https://first.mexle.te.hs-heilbronn.de/electrical_engineering_1/ws2022_exam?rev=1680389116)

Last update: **2023/04/02 00:45**

