

Block 07 — Power-relevant Figures

Student Group

First Name	Surname	Matrikel Nr.

Table of Contents

Block 07 — Power-relevant figures	2
<i>Learning objectives</i>	2
<i>90-minute plan</i>	2
<i>Conceptual overview</i>	2
Sign convention (recap)	3
Core content	3
Power Measurement	3
Power and Characteristics in Diagrams	4
The Characteristics: Efficiency and Utilization Rate	5
Power-flow chains (series stages)	6
Utilization rate ϵ	7
Exercise	7
Exercises	8
Exercise 7.1 Efficiency vs. maximum power (match or not?)	8
Exercise 7.2 Power-flow chain (product of efficiencies)	8
Exercise 7.3 — Simplify by Thevenin/Norton (preparation for Block 08)	8
Exercise 7.4 — Battery monitor shunt: loss vs. measurement accuracy	9
Applications & remarks	9
Summary	9
Common pitfalls checklist	10

Block 07 — Power-relevant figures

Learning objectives

- Define and compute **input/output power, losses, efficiency** η and **utilization rate** ϵ for DC sources and loads.
- Use the **real source model** with internal resistance R_{i} to compute operating point $(U_{\text{L}}, I_{\text{L}})$, P_{L} and P_{loss} .
- Understand the different design goals:
 1. **High efficiency** (power engineering): $R_{\text{L}} \gg R_{\text{i}}$.
 2. **Maximum power transfer** (communications): $R_{\text{L}} = R_{\text{i}}$.
- Combine efficiencies along a **power-flow chain**.
- Relate these figures to **Thevenin/Norton** equivalents and the **loaded voltage divider**.

90-minute plan

1. Warm-up (8 min): recall passive/active sign convention; quick unit check for $P = U \cdot I$.
2. Core concepts (35 min): real source model; definitions of η and ϵ ; design goals; chain efficiency.
3. Worked example (10 min): battery + internal resistance + load.
4. Two-port view & loaded divider (12 min): quick Thevenin/Norton recap; loaded divider formulas.
5. Practice (20 min): 3 short exercises (see panels below).
6. Wrap-up (5 min): summary + pitfalls.

Conceptual overview

1. Real sources are modeled by an **ideal source** plus **internal resistance** R_{i} ; the terminal voltage **drops under load**.
2. **Efficiency** η compares *delivered* to *drawn* power. In the simple DC source-load case, $\eta = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{i}}}$ (dimensionless). High-efficiency design wants $R_{\text{L}} \gg R_{\text{i}}$.
3. **Utilization rate** ϵ compares delivered power to the **maximum** available from the ideal source: $\epsilon = \frac{R_{\text{L}} R_{\text{i}}}{(R_{\text{L}} + R_{\text{i}})^2}$. It peaks at $R_{\text{L}} = R_{\text{i}}$ with $\epsilon_{\text{max}} = 25\%$. This is the **maximum power transfer** condition.
4. Different goals → different R_{L} :
 - **Power engineering**: maximize $\eta \rightarrow R_{\text{L}} \gg R_{\text{i}}$.
 - **Communications** (matching, antennas, RF): maximize $P_{\text{L}} \rightarrow R_{\text{L}} = R_{\text{i}}$, $\eta = 50\%$.

Sign convention (recap)

We use **conventional current** and the **passive/active sign convention** as introduced earlier. For a **consumer** (passive convention), $P = U \cdot I > 0$ means absorption; for a **source** (active convention), $P = U \cdot I > 0$ means delivery. See [sign_and_arrow-systems](#).

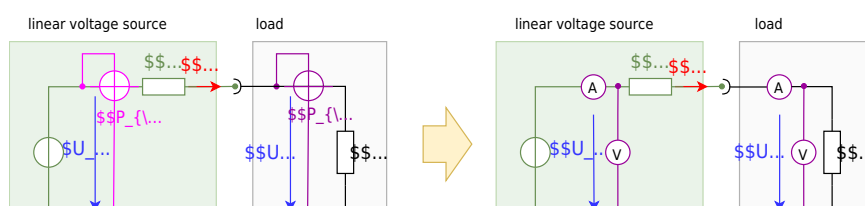
Core content

Power Measurement

First, it is necessary to consider how to determine the power. The power meter (or wattmeter) consists of a combined ammeter and voltmeter.

In [figure 1](#) the wattmeter with the circuit symbol can be seen as a round network with crossed measuring inputs. The circuit also shows one wattmeter each for the (not externally measurable) output power of the ideal source P_{S} and the input power of the load P_{L} .

Fig. 1: Power measurement on linear voltage source



Power and Characteristics in Diagrams

The simulation in [figure 2](#) shows the following:

- The circuit with linear voltage source (U_0 and R_{i}), and a resistive load R_{L} .
- A simulated wattmeter, where the ammeter is implemented by a measuring resistor R_{S} (English: shunt) and a voltage measurement for U_{S} . The power is then: $P_{\text{L}} = \frac{1}{R_{\text{S}}} \cdot U_{\text{S}} \cdot U_{\text{L}}$.
- in the oscilloscope section (below).
 - On the left is the power P_{L} plotted against time in a graph.
 - On the right is the already-known current-voltage diagram of the current values.
- The slider load resistance R_{L} , with which the value of the load resistance R_{L} can be changed.

Now try to vary the value of the load resistance R_{L} (slider) in the simulation so that the maximum power is achieved. Which resistance value is set?

Fig. 2: power adjustment

[figure 3](#) shows three diagrams:

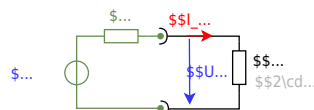
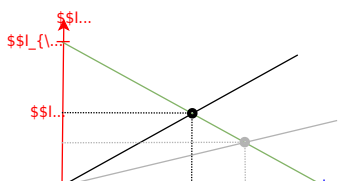
- Diagram top: current-voltage diagram of a linear voltage source.
- Diagram in the middle: source power P_{S} and consumer power P_{L} versus delivered voltage U_{L} .
- Diagram below: Reference quantities over delivered voltage U_{L} .

The two powers are defined as follows:

- source power: $P_{\text{S}} = U_0 \cdot I_{\text{L}}$
- consumer power: $P_{\text{L}} = U_{\text{L}} \cdot I_{\text{L}}$

1. Both power P_{S} and P_{L} are equal to 0 without current flow. The source power becomes maximum, at maximum current flow, that is when the load resistance $R_{\text{L}}=0$. In this case, all the power flows out through the internal resistor. The efficiency drops to 0%. This is the case, for example, with a battery shorted by a wire.
2. If the load resistance becomes just as large as the internal resistance $R_{\text{L}}=R_{\text{i}}$, the result is a voltage divider where the load voltage becomes just half the open circuit voltage: $U_{\text{L}} = \frac{1}{2} \cdot U_{\text{OC}}$. On the other hand, the current is also half the short-circuit current $I_{\text{L}}=I_{\text{SC}}$, since the resistance at the ideal voltage source is twice that in the short-circuit case.
3. If the load resistance becomes high impedance $R_{\text{L}} \rightarrow \infty$, less and less current flows, but more and more voltage drops across the load. Thus, the efficiency increases and approaches 100% for $R_{\text{L}} \rightarrow \infty$.

Fig. 3: current-voltage diagram, power-voltage diagram and efficiency-voltage diagram



The whole context can be investigated in this [Simulation with a resistor](#) or [this one with a variable load](#).

The Characteristics: Efficiency and Utilization Rate

To understand the lower diagram in [figure 3](#), the definition equations of the two reference quantities shall be described here again:

The **efficiency** η describes the delivered power (consumer power) concerning the supplied power (power of the ideal source):
$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{R_{\text{L}} \cdot I_{\text{L}}^2}{(R_{\text{L}} + R_{\text{i}}) \cdot I_{\text{L}}^2} \quad \rightarrow \quad \boxed{\eta = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{i}}}}$$

Once we want to get the **relative maximum power** out of a system (so maximum power related to the input power) the efficiency should go towards $\eta \rightarrow 100\%$. This situation close to (1.) in [figure 3](#).

Application:

1. In power engineering $\eta \rightarrow 100\%$ is often desired: We want the maximum power output with the lowest losses at the internal resistance of the source. Thus, the internal resistance of the source should be low compared to the load $R_{\text{L}} \gg R_{\text{i}}$.

The **utilization rate** ϵ describes the delivered power P_{out} concerning the maximum possible power $P_{\text{in, max}}$ of the ideal source. Here, the currently supplied power is not assumed (as in the case of efficiency), but the best possible power of the ideal source, i.e. in the short-circuit case:

$$\begin{aligned} \epsilon &= \frac{P_{\text{out}}}{P_{\text{in, max}}} = \frac{R_L \cdot I_L^2}{U_0^2 \over R_i} = \frac{R_L \cdot R_i \cdot I_L^2}{U_0^2} = \frac{R_L \cdot R_i \cdot \left(\frac{U_0}{R_L + R_i}\right)^2}{U_0^2} \\ &\quad \rightarrow \boxed{\epsilon = \frac{R_L \cdot R_i}{(R_L + R_i)^2} = \frac{R_L}{(R_L + R_i)}} \end{aligned}$$

In other applications, the **absolute maximum power** has to be taken from the source, without consideration of the losses via the internal resistance. This corresponds to the situation (2.) in [figure 3](#). For this purpose, the internal resistance of the source and the load are matched. This case is called **impedance matching** (the impedance is up to for DC circuits equal to the resistance). The utilization rate here becomes maximum: $\epsilon = 25\%$.

Application:

1. In communications engineering the impedance matching of the source (the antenna) and the load (the signal-acquiring microcontroller) uses resistors, capacitors, and inductors. There, we want to get the maximum power out of an antenna. For this purpose, the internal resistance of the source (e.g., a receiver) and the load (e.g., the downstream evaluation) are matched. An example can be seen in this [application note for near field communication](#).
2. Furthermore, also for photovoltaic cells one wants to get the maximum power out. In this case, the concept is often called **Maximum Power Point Tracking (MPPT)**

The impedance matching/power matching is also [here](#) explained in a German video.

Power-flow chains (series stages)

The usable (= outgoing) P_{O} power of a real system is always smaller than the supplied (incoming) power P_{I} . This is due to the fact, that there are additional losses in reality. The difference is called power loss P_{loss} . It is thus valid:

$$P_{\text{I}} = P_{\text{O}} + P_{\text{loss}}$$

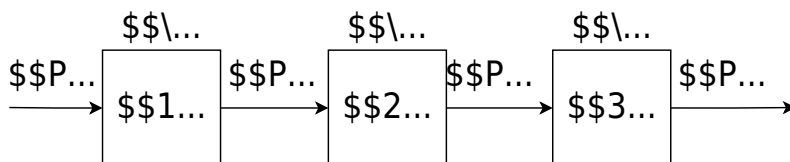
Instead of the power loss P_{loss} , the efficiency η is often given:

$$\boxed{\eta = \frac{P_{\text{O}}}{P_{\text{I}}} \overset{!}{<} 1}$$

For cascaded conversions (cf. [figure 4](#)), the **overall efficiency is the product** of stage efficiencies:

$$\boxed{\eta = \frac{P_{\text{O}}}{P_{\text{I}}} = \frac{P_1}{P_{\text{I}}} \cdot \frac{P_2}{P_1} \cdot \frac{P_{\text{O}}}{P_2} = \eta_1 \cdot \eta_2 \cdot \eta_3}$$

Fig. 4: Power flow diagram



Utilization rate ϵ

Define the **best-case input power** of the ideal source as $P_{in,max} = \frac{U_0^2}{R_i}$. Then
$$\epsilon = \frac{P_L}{P_{in,max}} = \frac{R_i R_L}{(R_L + R_i)^2}$$
 Maximization gives $R_L = R_i$ and $\epsilon_{max} = 25\%$. (At this point $\eta = 50\%$.)

Exercise

Given: $U_0 = 12.0 \text{ V}$, $R_i = 0.50 \text{ }\Omega$, $R_L = 5.0 \text{ }\Omega$. **Find:** U_L , I_L , P_L , η , ϵ .

$$\begin{aligned} I_L &= \frac{12.0 \text{ V}}{0.50 \text{ }\Omega + 5.0 \text{ }\Omega} = 2.182 \text{ A} \\ U_L &= I_L R_L = 2.182 \text{ A} \cdot 5.0 \text{ }\Omega = 10.91 \text{ V} \\ P_L &= U_L I_L = 10.91 \text{ V} \cdot 2.182 \text{ A} = 23.8 \text{ W} \\ P_{in,max} &= \frac{U_0^2}{R_i} = \frac{(12.0 \text{ V})^2}{0.50 \text{ }\Omega} = 288 \text{ W} \\ \eta &= \frac{P_L}{P_{in,max}} = \frac{23.8 \text{ W}}{288 \text{ W}} = 0.0826 = 8.26\% \\ \epsilon &= \frac{R_i R_L}{(R_L + R_i)^2} = \frac{0.50 \cdot 5.0}{(5.50)^2} = 0.0826 = 8.26\% \end{aligned}$$

Interpretation: very **efficient** (small R_i) but using only **8.26 %** of the source's ideal

maximum capability U_0^2/R_{i} —which is fine for power engineering aims.

Exercises

Exercise 7.1 Efficiency vs. maximum power (match or not?)

A source has $U_0=9.0\text{ V}$, $R_{\text{i}}=1.0\text{ }\Omega$.

1. (a) Choose $R_{\text{L}}=9.0\text{ }\Omega$. Compute I_{L} , U_{L} , P_{L} , η , ϵ .
2. (b) Choose $R_{\text{L}}=1.0\text{ }\Omega$. Repeat. Which choice maximizes P_{L} ? Which yields higher η ?

Strategy: use the boxed formulas in this block; for (b) note $R_{\text{L}}=R_{\text{i}}$
 $\rightarrow \eta=50\%$.

Exercise 7.2 Power-flow chain (product of efficiencies)

A battery (stage 1) feeds a DC/DC converter (stage 2) which feeds a sensor (stage 3). Their efficiencies are $\eta_1=0.93$, $\eta_2=0.90$, $\eta_3=0.80$.

1. Compute η_{total} .
2. If the battery provides 5.0 W , what power reaches the sensor?

Exercise 7.3 — Simplify by Thevenin/Norton (preparation for Block 08)

Simplify the following circuits (*NT* for Norton, *TT* for Thevenin) to a single source plus R_{i} , then compute U_{L} and η for a given R_{L} . Fig. ##: Simplification by Norton / Thevenin theorem

a) NT

Tip: Short ideal voltage sources and open ideal current sources to determine the internal resistance. :contentReference[oaicite:13]{index=13}

Exercise 7.4 — Battery monitor shunt: loss vs. measurement accuracy

Design the shunt R_S for the battery monitor (target range $\pm 0.20\text{ V}$ at the ADC). Compute measurable current range, minimum current step, shunt loss at maximum current, and the **efficiency penalty** due to the shunt for a given R_L .

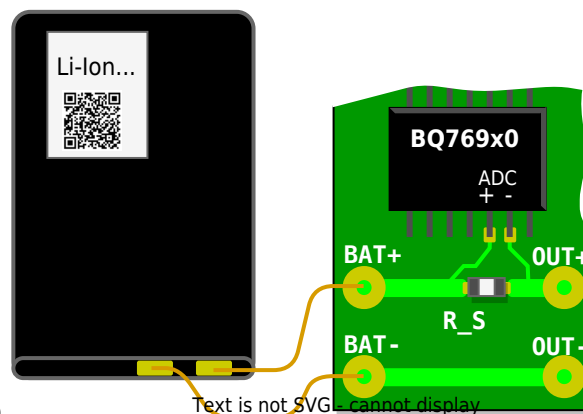


Fig. ##: Sketch of the setup

(Use the prompts inside the panel text on the source page for guidance.)

:contentReference[oaicite:14]{index=14}

Applications & remarks

- **Communications / RF:** power matching networks (R , L , C) to get maximum signal power; see [here](#) and [application note for near field communication](#). :contentReference[oaicite:15]{index=15}
- **Photovoltaics:** operation near **maximum power point** (MPPT) optimizes P_L rather than η of the source alone. :contentReference[oaicite:16]{index=16}

Summary

1. Real sources: $U_L = U_0 \frac{R_L}{R_i + R_L}$, $I_L = \frac{U_0}{R_i + R_L}$; $P_L = \frac{U_0^2 R_L}{(R_i + R_L)^2}$.
2. **Efficiency:** $\eta = \frac{R_L}{R_i + R_L}$; maximize by $R_L \gg R_i$ (power engineering). :contentReference[oaicite:17]{index=17}
3. **Utilization rate:** $\varepsilon = \frac{R_L R_i}{(R_L + R_i)^2}$; peak $\varepsilon_{\max} = 25\%$ at $R_L = R_i$ (maximum power transfer; $\eta = 50\%$). :contentReference[oaicite:18]{index=18}
4. **Chain efficiencies** multiply: $\eta_{\text{total}} = \prod \eta_i$.

:contentReference[oaicite:19]{index=19}

5. Thevenin/Norton help to **separate** source figures (U_0 , R_{i}) from the load and to reuse the same formulas. :contentReference[oaicite:20]{index=20}
6. **Max efficiency η** : $R_{\text{L}} \rightarrow \infty$ (relative to R_{i}) $\rightarrow$ small current, small loss.
7. **Max delivered power P_{L}** : $R_{\text{L}} = R_{\text{i}}$ (impedance matching).
See also [impedance matching](#) and [Maximum Power Point Tracking \(MPPT\)](#) for PV systems.

</callout>

Common pitfalls checklist

1. Forgetting **units** in intermediate results (always write $x = \text{number} \times \text{unit}$).
2. Mixing up **goals**: high η vs. high P_{L} lead to **different** R_{L} .
3. Using **ideal source** formulas for a **real** source (always include R_{i}).
4. Ignoring the **sign convention** when interpreting $P = U \cdot I$ (source vs. load).

From:

<https://first.mexle.te.hs-heilbronn.de/> - MEXLE Wiki

Permanent link:

https://first.mexle.te.hs-heilbronn.de/electrical_engineering_and_electronics_1/block07?rev=1759101218

Last update: 2025/09/29 01:13

