

Block 07 — Power-relevant Figures

Student Group

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Block 07 — Power-relevant figures

Learning objectives

- Define and compute **input/output power**, **losses**, **efficiency** η and **utilization rate** ϵ for DC sources and loads.
- Use the **real source model** with internal resistance R_{i} to compute operating point $(U_{\text{L}}, I_{\text{L}})$, P_{L} and P_{loss} .
- Understand the different design goals:
 1. **High efficiency** (power engineering): $R_{\text{L}} \gg R_{\text{i}}$.
 2. **Maximum power transfer** (communications): $R_{\text{L}} = R_{\text{i}}$.
- Combine efficiencies along a **power-flow chain**.
- Relate these figures to **Thevenin/Norton** equivalents and the **loaded voltage divider**.

Preparation at Home

And again:

- Please read through the following chapter.
- Also here, there are some clips for more clarification under 'Embedded resources'.

For checking your understanding please do the following exercises:

- 7.1
- E3.3.3

90-minute plan

1. Warm-up (8 min): recall passive/active sign convention; quick unit check for $P = U \cdot I$.
2. Core concepts (35 min): real source model; definitions of η and ϵ ; design goals; chain efficiency.
3. Worked example (10 min): battery + internal resistance + load.
4. Two-port view & loaded divider (12 min): quick Thevenin/Norton recap; loaded divider formulas.
5. Practice (20 min): 3 short exercises (see panels below).
6. Wrap-up (5 min): summary + pitfalls.

Conceptual overview

1. Real sources are modeled by an **ideal source** plus **internal resistance** R_{i} ; the terminal voltage **drops under load**.
2. **Efficiency** η compares **delivered** to **drawn** power. In the simple DC source-load case,
$$\eta = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{i}}}$$
 (dimensionless).

High-efficiency design wants $R_{\text{L}} \gg R_{\text{i}}$.

3. **Utilization rate** ϵ compares delivered power to the **maximum** available from the ideal source: $\epsilon = \frac{R_{\text{L}} R_{\text{i}}}{(R_{\text{L}} + R_{\text{i}})^2}$. It peaks at $R_{\text{L}} = R_{\text{i}}$ with $\epsilon_{\text{max}} = 25\%$. This is the **maximum power transfer** condition.
4. Different goals → different R_{L} :
 - **Power engineering**: maximize $\eta \rightarrow R_{\text{L}} \gg R_{\text{i}}$.
 - **Communications** (matching, antennas, RF): maximize $P_{\text{L}} \rightarrow R_{\text{L}} = R_{\text{i}}$, $\eta = 50\%$.

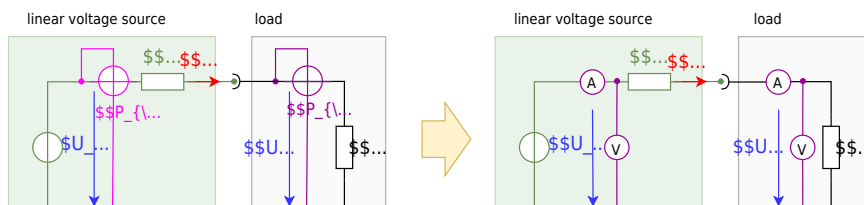
Core content

Power Measurement

First, it is necessary to consider how to determine the power. The power meter (or wattmeter) consists of a combined ammeter and voltmeter.

In [figure 1](#) the wattmeter with the circuit symbol can be seen as a round network with crossed measuring inputs. The circuit also shows one wattmeter each for the (not externally measurable) output power of the ideal source P_{S} and the input power of the load P_{L} .

Fig. 1: Power measurement on linear voltage source



Power and Characteristics in Diagrams

The simulation in [figure 2](#) shows the following:

- The circuit with linear voltage source (U_0 and R_{L} i), and a resistive load R_{L} .
- A simulated wattmeter, where the ammeter is implemented by a measuring resistor R_{S} (English: shunt) and a voltage measurement for U_{S} . The power is then: $P_{\text{L}} = \frac{1}{R_{\text{S}}} \cdot U_{\text{S}} \cdot U_{\text{L}}$.
- in the oscilloscope section (below).
 - On the left is the power P_{L} plotted against time in a graph.
 - On the right is the already-known current-voltage diagram of the current values.
- The slider load resistance R_{L} , with which the value of the load resistance R_{L} can be changed.

Now try to vary the value of the load resistance R_{L} (slider) in the simulation so that the maximum power is achieved. Which resistance value is set?

Fig. 2: power adjustment

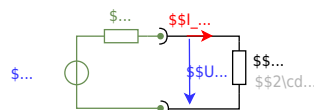
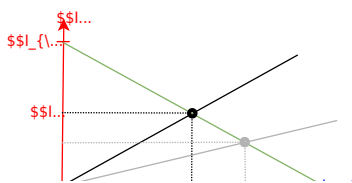
[figure 3](#) shows three diagrams:

- Diagram top: current-voltage diagram of a linear voltage source.
- Diagram in the middle: source power P_{S} and consumer power P_{L} versus delivered voltage U_{L} .
- Diagram below: Reference quantities over delivered voltage U_{L} .

The two powers are defined as follows:

- source power: $P_{\text{S}} = U_0 \cdot I_{\text{L}}$
 - consumer power: $P_{\text{L}} = U_{\text{L}} \cdot I_{\text{L}}$
1. Both power P_{S} and P_{L} are equal to 0 without current flow. The source power becomes maximum, at maximum current flow, that is when the load resistance $R_{\text{L}}=0$. In this case, all the power flows out through the internal resistor. The efficiency drops to 0%. This is the case, for example, with a battery shorted by a wire.
 2. If the load resistance becomes just as large as the internal resistance $R_{\text{L}}=R_{\text{i}}$, the result is a voltage divider where the load voltage becomes just half the open circuit voltage: $U_{\text{L}} = \frac{1}{2} \cdot U_{\text{OC}}$. On the other hand, the current is also half the short-circuit current $I_{\text{L}}=I_{\text{SC}}$, since the resistance at the ideal voltage source is twice that in the short-circuit case.
 3. If the load resistance becomes high impedance $R_{\text{L}} \rightarrow \infty$, less and less current flows, but more and more voltage drops across the load. Thus, the efficiency increases and approaches 100% for $R_{\text{L}} \rightarrow \infty$.

Fig. 3: current-voltage diagram, power-voltage diagram and efficiency-voltage diagram



The whole context can be investigated in this [Simulation with a resistor](#) or [this one with a variable load](#).

The Efficiency

To understand the lower diagram in [figure 3](#), the definition equations of the two reference quantities shall be described here again:

The **efficiency** η describes the delivered power (consumer power) concerning the supplied power (power of the ideal source):
$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{R_{\text{L}} \cdot I_{\text{L}}^2}{(R_{\text{L}} + R_{\text{i}}) \cdot I_{\text{L}}^2} \quad \rightarrow \quad \boxed{\eta = \frac{R_{\text{L}}}{R_{\text{L}} + R_{\text{i}}}}$$

Once we want to get the **relative maximum power** out of a system (so maximum power related to the input power) the efficiency should go towards $\eta \rightarrow 100\%$. This situation close to (1.) in [figure 3](#).

Application:

1. In power engineering $\eta \rightarrow 100\%$ is often desired: We want the maximum power output with the lowest losses at the internal resistance of the source. Thus, the internal resistance of the source should be low compared to the load $R_{\text{L}} \gg R_{\text{i}}$.

The Utilization Rate

The **utilization rate** ϵ describes the delivered power P_{out} concerning the maximum possible power $P_{\text{in, max}}$ of the ideal source. Here, the currently supplied power is not assumed (as in the case of efficiency), but the best possible power of the ideal source, i.e. in the short-circuit case:

$$\begin{aligned} \epsilon &= \frac{P_{\text{out}}}{P_{\text{in, max}}} = \frac{R_{\text{L}} \cdot I_{\text{L}}^2}{U_0^2 \over R_{\text{i}}} = \frac{R_{\text{L}} \cdot R_{\text{i}} \cdot I_{\text{L}}^2}{U_0^2} = \frac{R_{\text{L}} \cdot R_{\text{i}} \cdot \left(\frac{U_0}{R_{\text{L}} + R_{\text{i}}} \right)^2}{U_0^2} \quad \rightarrow \quad \boxed{\epsilon = \frac{R_{\text{L}} \cdot R_{\text{i}}}{(R_{\text{L}} + R_{\text{i}})^2}} \end{aligned}$$

In other applications, the **absolute maximum power** has to be taken from the source, without consideration of the losses via the internal resistance. This corresponds to the situation (2.) in [figure 3](#). For this purpose, the internal resistance of the source and the load are matched. This case is called **impedance matching** (the impedance is up to for DC circuits equal to the resistance). The utilization rate here becomes maximum: $\epsilon = 25\%$.

Application:

1. In communications engineering the impedance matching of the source (the antenna) and the load (the signal-acquiring microcontroller) uses resistors, capacitors, and inductors. There, we want to get the maximum power out of an antenna. For this purpose, the internal resistance of the source (e.g., a receiver) and the load (e.g., the downstream evaluation) are matched. An example can be seen in this [application note for near field communication](#).
2. Furthermore, also for photovoltaic cells one wants to get the maximum power out. In this case, the concept is often called **Maximum Power Point Tracking (MPPT)**

The impedance matching/power matching is also [here](#) explained in a German video.

Exercise

Given: $U_0 = 12.0 \text{ V}$, $R_{\text{i}} = 0.50 \text{ }\Omega$, $R_{\text{L}} = 5.0 \text{ }\Omega$. **Find:** U_{L} , I_{L} , P_{L} , η , ϵ .

$$\begin{aligned} I_{\text{L}} &= \frac{12.0 \text{ V}}{0.50 \text{ }\Omega + 5.0 \text{ }\Omega} = 2.182 \text{ A} \\ U_{\text{L}} &= I_{\text{L}} \cdot R_{\text{L}} = 2.182 \text{ A} \cdot 5.0 \text{ }\Omega = 10.91 \text{ V} \\ P_{\text{L}} &= U_{\text{L}} \cdot I_{\text{L}} = 10.91 \text{ V} \cdot 2.182 \text{ A} = 23.8 \text{ W} \\ P_{\text{in, max}} &= \frac{U_0^2}{R_{\text{i}}} = \frac{(12.0 \text{ V})^2}{0.50 \text{ }\Omega} = 288 \text{ W} \\ \eta &= \frac{P_{\text{L}}}{P_{\text{in, max}}} = \frac{23.8 \text{ W}}{288 \text{ W}} = 0.0826 = 8.26\% \\ \epsilon &= \frac{R_{\text{L}} \cdot R_{\text{i}}}{(R_{\text{L}} + R_{\text{i}})^2} = \frac{5.0 \text{ }\Omega \cdot 0.50 \text{ }\Omega}{(5.0 \text{ }\Omega + 0.50 \text{ }\Omega)^2} = 0.0826 = 8.26\% \end{aligned}$$

Interpretation: very **efficient** (small R_{i}) but using only **8.26 %** of the source's ideal maximum capability U_0^2/R_{i} —which is fine for power engineering aims.

Power-flow chains (series stages)

The usable (= outgoing) P_{O} power of a real system is always smaller than the supplied (incoming) power P_{I} . This is due to the fact, that there are additional losses in reality. The difference is called power loss P_{loss} . It is thus valid:

$$P_{\text{I}} = P_{\text{O}} + P_{\text{loss}}$$

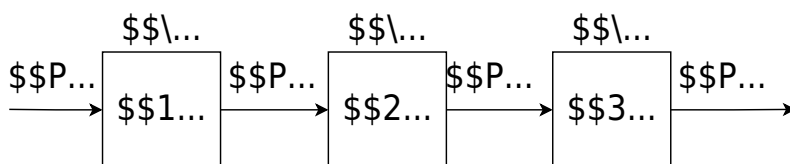
Instead of the power loss P_{loss} , the efficiency η is often given:

$$\boxed{\eta = \frac{P_{\text{O}}}{P_{\text{I}}} \overset{!}{<} 1}$$

For cascaded conversions (cf. [figure 4](#)), the **overall efficiency is the product** of stage efficiencies:

$$\boxed{\eta = \frac{P_{\text{O}}}{P_{\text{I}}} = \frac{P_{\text{O1}}}{P_{\text{I}}} \cdot \frac{P_{\text{O2}}}{P_{\text{O1}}} \cdot \frac{P_{\text{O3}}}{P_{\text{O2}}} = \eta_1 \cdot \eta_2 \cdot \eta_3}$$

Fig. 4: Power flow diagram



Exercises

Exercise 7.1 Efficiency vs. maximum power (match or not?)

A source has $U_0 = 9.0 \text{ V}$, $R_{\text{i}} = 1.0 \text{ }\Omega$.

1. (a) Choose $R_{\text{L}} = 9.0 \text{ }\Omega$. Compute I_{L} , U_{L} , P_{L} , η , ϵ .
2. (b) Choose $R_{\text{L}} = 1.0 \text{ }\Omega$. Repeat.

Which choice maximizes P_{L} ? Which yields higher η ?

Strategy: use the boxed formulas in this block; for (b) note $R_{\text{L}} = R_{\text{i}}$
 $\rightarrow \eta = 50\%$.

Exercise 7.2 Power-flow chain (product of efficiencies)

A battery (stage 1) feeds a DC/DC converter (stage 2) which feeds a sensor (stage 3). Their efficiencies are $\eta_1 = 0.93$, $\eta_2 = 0.90$, $\eta_3 = 0.80$.

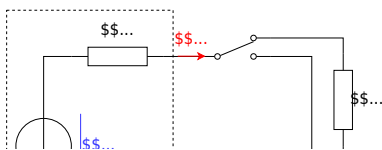
1. Compute η_{total} .
2. If the battery provides 5.0 W , what power reaches the sensor?

Exercise 3.3.2 Internal resistances and Efficiency

For the company „HHN Mechatronics & Robotics“ you shall analyze a competitor product: a simple drilling machine. This contains a battery pack, some electronics, and a motor. For this consideration, the battery pack can be treated as a linear voltage source with $U_{\text{s}} = 11 \text{ V}$ and internal resistance of $R_{\text{i}} = 0.1 \text{ }\Omega$. The used motor shall be considered as an ohmic resistance $R_{\text{m}} = 1 \text{ }\Omega$.

The drill has two speed-modes:

1. max power: here, the motor is directly connected to the battery.
2. reduced power: in this case, a shunt resistor $R_{\text{s}} = 1 \text{ }\Omega$ is connected in series to the motor.



Tasks:

1. Calculate the input and output power for both modes.
2. What are the efficiencies for both modes?
3. Which value should the shunt resistor R_s have, when the reduced power should be exactly half of the maximum power?
4. Your company uses the reduced power mode instead of the shunt resistor R_s multiple diodes in series D , which generates a constant voltage drop of $U_D = 2.8 \sim V$.

What are the input and output power, such as the efficiency in this case?

You can check your results for the currents, voltages, and powers with the following simulation:

Exercise E3.3.3 Power of two pole components

What is the maximum possible efficiency η of a power supply P_{sup} at the loads two lithium-batteries (both with $U_S = 3.3 \sim V$, $R_i = 0.1 \sim \Omega$)?

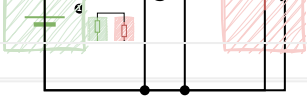
Result:

Solution: What are the possible ways to connect these components?

The following circuit is shown in the figure. The output power is $P_{\text{out}} = U_L I_L$ and the input power is $P_{\text{in}} = U_S I_S$. The efficiency is $\eta = P_{\text{out}} / P_{\text{in}}$.

The highest efficiency is achieved when the output power is maximum. At the maximum output power, the input power is $P_{\text{in}} = U_S I_S = U_S (I_L + I_S) = U_S I_L (1 + \frac{R_i}{R_L})$. The efficiency is $\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_L I_L}{U_S I_L (1 + \frac{R_i}{R_L})} = \frac{U_L}{U_S (1 + \frac{R_i}{R_L})}$. The utilization rate is given by $\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_L I_L}{U_S I_S} = \frac{U_L I_L}{U_S (I_L + I_S)} = \frac{U_L}{U_S (1 + \frac{R_i}{R_L})}$. The utilization rate is given by $\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_L I_L}{U_S I_S} = \frac{U_L I_L}{U_S (I_L + I_S)} = \frac{U_L}{U_S (1 + \frac{R_i}{R_L})}$.

Details of the maximum power transfer theorem: $P_{\text{out}} = \epsilon \cdot P_{\text{in, max}}$ where $\epsilon = \frac{R_{\text{L}}}{R_{\text{i}} + R_{\text{L}}}$. For the resulting equivalent integral resistance, the efficiency approaches the resulting equivalent load resistance, as higher the utilization rate ϵ will be. Therefore, a series configuration of the batteries ($R_{\text{i}} = 0.2 \sim \Omega$) and a parallel configuration of the load ($\frac{1}{2} R_{\text{L}} = 0.25 \sim \Omega$) will have the highest output.



Exercise E18 Efficiency

(written test, approx. 14 % of a 60-minute written test, SS2023)

1. (34 Points) Other than efficiency, the most important figure of merit for the battery is its energy density. The battery shall provide energy for a device with a load resistance of $R_{\text{L}} = 2 \sim \Omega$. The following values are from the battery data sheet.

Open-circuit voltage $U_{\text{OC}} = 3.6 \sim \text{V}$, nominal voltage $U_{\text{N}} = 3.5 \sim \text{V}$, short-circuit current $I_{\text{SC}} = 3.5 \sim \text{A}$

Capacity $Q = 2.6 \sim \text{Ah}$, internal resistance $R_{\text{i}} = 0.05 \sim \Omega$, maximum power $P_{\text{max}} = 2 \sim \text{W}$

• Type: Li-Ion

Lowest efficiency for highest output (for $R_{\text{L}} = R_{\text{i}}$ in this case), the

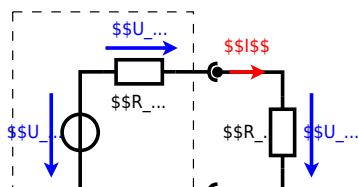
.. efficiency of the equivalent circuit diagram with the internal resistance and an external load.

Label voltage and current: $Q = 2.6 \sim \text{Ah}$, $U_{\text{OC}} = 3.6 \sim \text{V}$, $U_{\text{N}} = 3.5 \sim \text{V}$, $I_{\text{SC}} = 3.5 \sim \text{A}$

Formula: $\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_{\text{L}} \cdot I_{\text{L}}}{U_{\text{OC}} \cdot I_{\text{L}}} = \frac{U_{\text{L}}}{U_{\text{OC}}}$

$\eta = 1 - \frac{R_{\text{i}}}{R_{\text{L}} + R_{\text{i}}} = 1 - \frac{0.05}{2 + 0.05} = 1 - 0.025 = 0.975$

Result: $\eta = 97.5 \sim \%$



Summary

1. Real sources: $U_L = U_0 \frac{R_L}{R_i + R_L}$, $I = \frac{U_0}{R_i + R_L}$; $P_L = \frac{U_0^2 R_L}{(R_i + R_L)^2}$.
2. **Efficiency**: $\eta = \frac{R_L}{R_i + R_L}$; maximize by $R_L \gg R_i$ (power engineering).
3. **Utilization rate**: $\epsilon = \frac{R_L R_i}{(R_L + R_i)^2}$; peak $\epsilon_{\max} = 25\%$ at $R_L = R_i$ (maximum power transfer; $\eta = 50\%$).
4. **Chain efficiencies** multiply: $\eta_{\text{total}} = \prod \eta_i$.
5. Thevenin/Norton help to **separate** source figures (U_0 , R_i) from the load and to reuse the same formulas.
6. **Max efficiency η** : $R_L \rightarrow \infty$ (relative to R_i) $\rightarrow$ small current, small loss.
7. **Max delivered power P_L** : $R_L = R_i$ (impedance matching).
See also [impedance matching](#) and [Maximum Power Point Tracking \(MPPT\)](#) for PV

systems.

Common pitfalls checklist

1. Forgetting **units** in intermediate results (always write $x = \text{number} \times \text{unit}$).
2. Mixing up **goals**: high η vs. high P_{L} lead to **different** R_{L} .
3. Using **ideal source** formulas for a **real** source (always include R_{i}).
4. Ignoring the **sign convention** when interpreting $P = U \cdot I$ (source vs. load).

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