

Block 13 - Capacitor Circuits and Energy

Student Group

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Block 13 - Capacitor Circuits and Energy

Learning objectives

After this 90-minute block, you can

- identify series vs. parallel connections of capacitors from a circuit diagram,
- compute equivalent capacitance C_{eq} for series and parallel networks,
- use the key sharing rules: in **series** $Q_k = \text{const.}$ and voltages divide; in **parallel** $U_k = \text{const.}$ and charges divide,
- apply the capacitor divider relation (two series capacitors),
- determine stored energy, including a dimensional check to J .

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- 5.9.5

90-minute plan

1. Warm-up (10 min):
 1. Quick quiz (2–3 items): series or parallel? which rule applies (constant U or constant Q)?
 2. Recall $Q = C \cdot U$ and energy $W = \frac{1}{2} C U^2$ (units).
2. Core concepts & derivations (35 min):
 1. Derive C_{eq} for **series** from Kirchhoff's voltage law and $Q = \text{const.}$; derive voltage division $U_k = \frac{Q}{C_k}$.
 2. Derive C_{eq} for **parallel** from Kirchhoff's current/charge balance and $U = \text{const.}$; obtain $Q_k = C_k U$.
 3. Energy in the electric field: integrate $dW = U \cdot dq \rightarrow W = \frac{1}{2} C U^2$; short dimensional check.
3. Practice (35 min):
 1. Two short worked examples: mixed series/parallel network; two-capacitor divider with given U (find U_1 , U_2 , W on each).
 2. Short simulation tasks (use the two embedded Falstad circuits in this page): observe U_k , Q_k when toggling the switch or changing values.
 3. Mini-problems: "double a plate area / halve distance" reasoning on C and W .
4. Wrap-up (10 min):
 1. Common-pitfalls checklist and one exit-ticket calculation.

Conceptual overview

1. **What stays the same?** In **series** all capacitors carry the **same charge** Q ; in **parallel** all capacitors see the **same voltage** U .
2. **How do totals form?** Capacitances **add inversely** in series and **add directly** in parallel. This mirrors resistors but with the roles swapped.
3. **Voltage/charge sharing:** In series, the **smaller** C_k takes the **larger** U_k ($U_k = Q/C_k$). In parallel, the **larger** C_k takes the **larger** Q_k ($Q_k = C_k U$).
4. **Energy viewpoint:** Charging needs work against the field; $W = \frac{1}{2} C U^2 = \frac{1}{2} Q U = \frac{Q^2}{2C}$. Dimensional check: $[C] = \text{rm F} = \frac{A \cdot s}{V}$, so $[C U^2] = \frac{A \cdot s}{V} \cdot V^2 = A \cdot s \cdot V = J$.
5. **Design intuition:** Increasing plate area A or dielectric ϵ_r raises C and thus stored W at the same U ; increasing gap d lowers C .

Core content

Series Circuit of Capacitor

If capacitors are connected in series, the charging current I into the individual capacitors $C_1 \dots C_n$ is equal. Thus, the charges absorbed ΔQ are also equal:
$$\Delta Q = \Delta Q_1 = \Delta Q_2 = \dots = \Delta Q_n$$

Furthermore, after charging, a voltage is formed across the series circuit, which corresponds to the source voltage U_q . This results from the addition of partial voltages across the individual capacitors.
$$U_q = U_1 + U_2 + \dots + U_n = \sum_{k=1}^n U_k$$

It holds for the voltage $U_k = \frac{Q_k}{C_k}$.

If all capacitors are initially discharged, then $U_k = \frac{\Delta Q}{C_k}$ holds. Thus
$$U_q = U_1 + U_2 + \dots + U_n = \sum_{k=1}^n U_k \quad U_q = \frac{\Delta Q}{C_1} + \frac{\Delta Q}{C_2} + \dots + \frac{\Delta Q}{C_3} = \sum_{k=1}^n \frac{1}{C_k} \cdot \Delta Q \quad \frac{1}{C_{\text{eq}}} \cdot \Delta Q = \sum_{k=1}^n \frac{1}{C_k} \cdot \Delta Q$$

Thus, for the series connection of capacitors $C_1 \dots C_n$:

$$\frac{1}{C_{\text{eq}}} = \sum_{k=1}^n \frac{1}{C_k} \quad \Delta Q_k = \text{rm const.}$$

For initially uncharged capacitors, (voltage divider for capacitors) holds:
$$Q = Q_k \quad U_{\text{eq}} \cdot C_{\text{eq}} = U_k \cdot C_k$$

In the simulation below, besides the parallel connected capacitors C_1 , C_2 , C_3 , an ideal voltage source U_q , a resistor R , a switch S , and a lamp are installed.

- The switch S allows the voltage source to charge the capacitors.
- The resistor R is necessary because the simulation cannot represent instantaneous charging. The resistor limits the charging current to a maximum value.

- The capacitors can be discharged again via the lamp.

Parallel Circuit of Capacitors

If capacitors are connected in parallel, the voltage U across the individual capacitors $C_1 \dots C_n$ is equal. It is therefore valid:

$$U_q = U_1 = U_2 = \dots = U_n$$

Furthermore, during charging, the total charge ΔQ from the source is distributed to the individual capacitors. This gives the following for the individual charges absorbed:

$$\Delta Q = \Delta Q_1 + \Delta Q_2 + \dots + \Delta Q_n = \sum_{k=1}^n \Delta Q_k$$

If all capacitors are initially discharged, then $Q_k = \Delta Q_k = C_k \cdot U$
 Thus
$$\Delta Q = Q_1 + Q_2 + \dots + Q_n = \sum_{k=1}^n Q_k$$

$$Q = C_1 \cdot U + C_2 \cdot U + \dots + C_n \cdot U = \sum_{k=1}^n C_k \cdot U$$

$$C_{\text{eq}} \cdot U = \sum_{k=1}^n C_k \cdot U$$

Thus, for the parallel connection of capacitors $C_1 \dots C_n$:

$$C_{\text{eq}} = \sum_{k=1}^n C_k$$

$$U_k = \{\text{const.}\}$$

For initially uncharged capacitors, (charge divider for capacitors) holds:

$$\Delta Q = \sum_{k=1}^n Q_k$$

$$\frac{Q_k}{C_k} = \frac{\Delta Q}{C_{\text{eq}}}$$

In the simulation below, again, besides the parallel connected capacitors C_1 , C_2 , C_3 , an ideal voltage source U_q , a resistor R , a switch S , and a lamp are installed.

Energy in the electric Field

Now we want to see how much energy is stored in a capacitor during charging. When we want to charge a capacitor charges have to be separated (see [figure 1](#)). This gets more and more difficult as more charges were moved, since these already moved charges create an electric field.

Fig. 1: summary of electrostatics

We already had a first look onto the energy in the electric field in [block09](#).

There, we got:

$$\begin{aligned} \Delta W &= \int \vec{F} \cdot d\vec{r} \quad \&= q \int \vec{E} \cdot d\vec{r} \quad \&= q \cdot U \quad dW \\ &= dq \cdot U \end{aligned}$$

Now, For a capacitor we include the formula for the capacitor $C = \frac{q}{U}$, or better its rearranged version $U = \frac{q}{C}$:

$$\begin{aligned} dW &= dq \cdot \frac{q}{C} \quad \int dW = \int \frac{q}{C} dq \end{aligned}$$

Here we again see, that the needed energy portion dW to move a portion dq is also related to the already moved charges q .

To get the energy ΔW needed to move all of the charges $Q = \int dq$ we have to integrate from 0 to Q :

$$\begin{aligned} \Delta W &= \int_0^Q dW \quad \&= \int_0^Q \frac{q}{C} dq \quad \&= \\ \frac{1}{2} \frac{Q^2}{C} \end{aligned} \quad \boxed{\Delta W = \frac{1}{2} \frac{Q^2}{C}} = \frac{1}{2} QU = \frac{1}{2} CQ^2$$

Common pitfalls

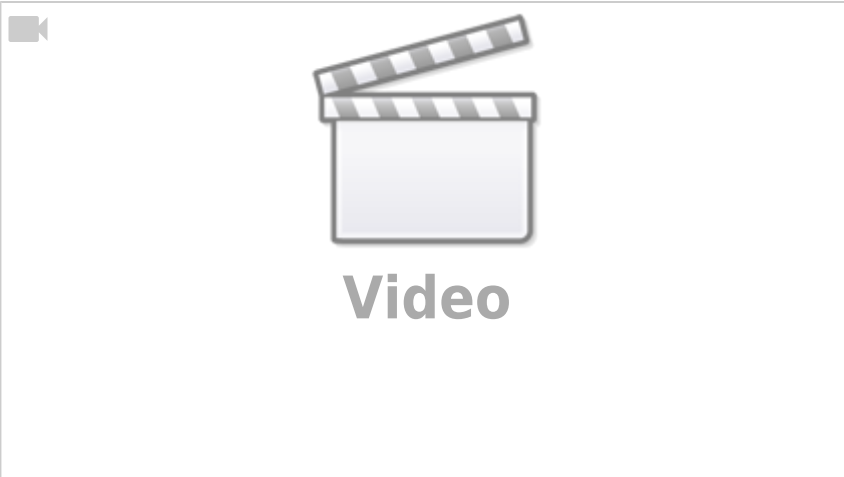
- Mixing up the rules: writing $C_{\text{eq}} = C_1 + C_2$ for **series** (wrong) or $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$ for **parallel** (wrong).
- Forgetting which quantity is equal: **series** $\rightarrow Q_k = \text{const.}$, **parallel** $\rightarrow U_k = \text{const.}$.
- Applying the **resistive** voltage divider $U_1 = \frac{R_1}{R_1 + R_2} U$ to capacitors. For capacitors in series it inverts: $U_1 = \frac{C_2}{C_1 + C_2} U$.
- Ignoring **initial charge states**: pre-charged capacitors reconnected will redistribute charge; use charge conservation on isolated nodes before using $Q = C \cdot U$.
- Dropping units or mixing forms of energy: always keep $W = \frac{1}{2} C U^2 = \frac{1}{2} Q U = \frac{Q^2}{2C}$ and check J .

Exercises

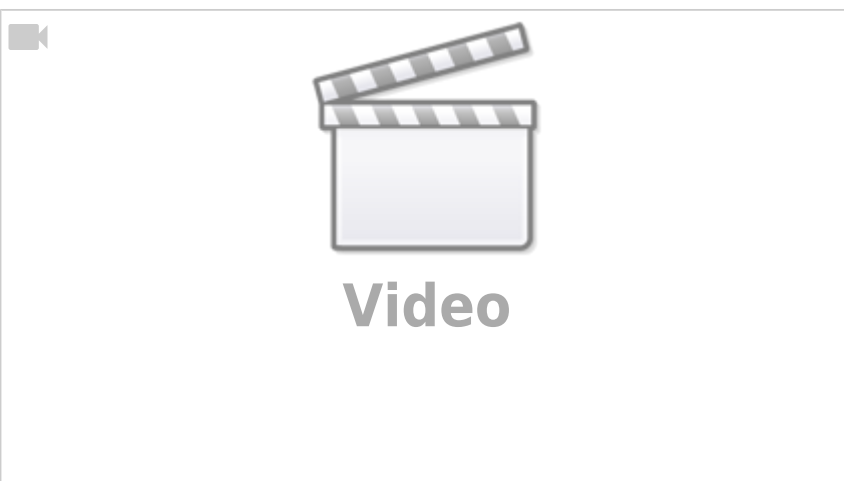
Task 5.8.1 Calculating a circuit of different capacitors

See <https://www.youtube.com/watch?v=vSeSHAmPd4Y>

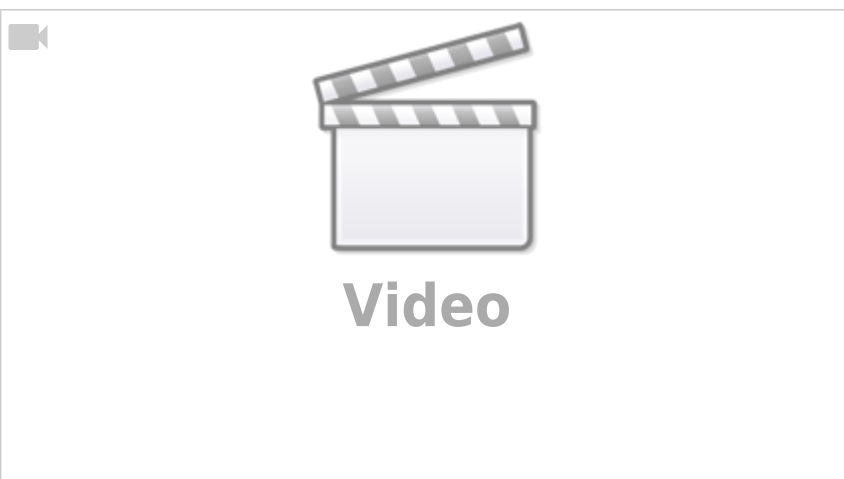
Task 5.9.1 Layered Capacitor Task



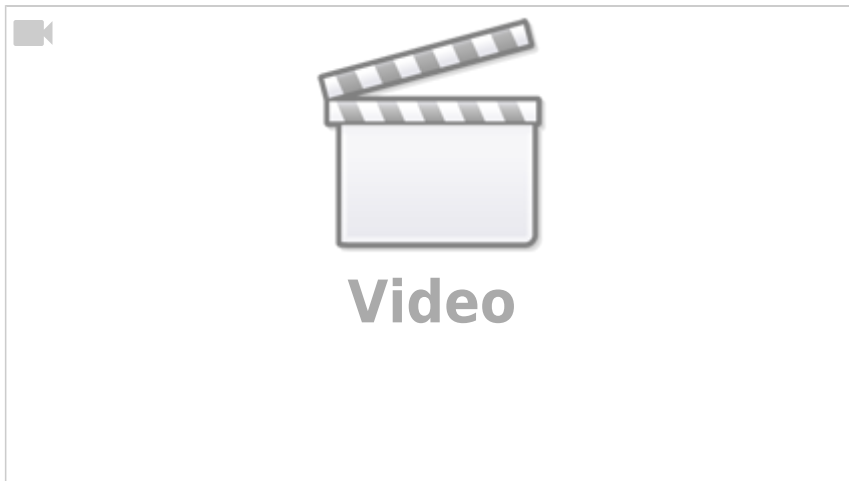
Exercise 5.9.2 Further capacitor charging/discharging practice Exercise



Exercise 5.9.3 Further practice charging the capacitor

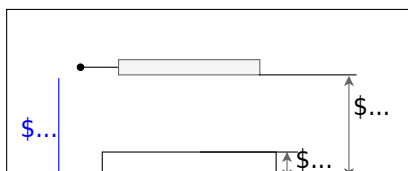


Exercise 5.9.4 Charge balance of two capacitors



Exercise 5.9.5 Capacitor with glass plate

Fig. 2: Structure of a capacitor with glass plate



Two parallel capacitor plates face each other with a distance $d_K = 10 \text{ mm}$. A voltage of $U = 3000 \text{ V}$ is applied to the capacitor. Parallel to the capacitor plates, there is a glass plate ($\epsilon_{r,G} = 8$) with a thickness $d_G = 3 \text{ mm}$ in the capacitor.

1. Calculate the partial voltages U_G in the glass and U_A in the air gap.
2. What is the maximum thickness of the glass pane if the electric field $E_{0,G}$ must not exceed 12 kV/cm ?

Exercise E1 Capacitor

(written test, approx. 12 % of a 120-minute written test, SS2024)

Result: Desimaged.

The contaminant has $\epsilon_{\text{r,c}} > \epsilon_{\text{r,air}}$, while the distance between the plates remains the same. Give a generalized formula

$$z_2 = f(A, d, x, \epsilon_{\{r, c\}}, x, \epsilon_{\{r, air\}}) \$$$

$$Q = \frac{146 \cdot \epsilon_{\{0\}} \cdot \epsilon_{\{0\}} \cdot \epsilon_{\{0\}} \cdot \epsilon_{\{0\}}}{(d-x) \cdot \epsilon_{\{r, c\}} + x \cdot \epsilon_{\{r, air\}}} \$$$

There are two ways now. Either: $Q \approx C \cdot U = 1.1 \cdot 10^{-12} \cdot 3.3 \cdot 10^{-12} = 3.63 \cdot 10^{-24} \text{ pC}$ Or: $Q \approx D \cdot A = 146 \cdot \epsilon_{\{0\}} \cdot \epsilon_{\{0\}} \cdot \epsilon_{\{0\}} \cdot \epsilon_{\{0\}} = 8.854 \cdot 10^{-12} \cdot 10^{-12} \cdot 10^{-12} \cdot 10^{-12} = 8.854 \cdot 10^{-48} \text{ pC}$

The displacement field is given by $D = \epsilon_{\{0\}} \cdot \epsilon_{\{0\}} \cdot \epsilon_{\{0\}} \cdot \epsilon_{\{0\}} = 8.854 \cdot 10^{-12} \cdot 10^{-12} \cdot 10^{-12} \cdot 10^{-12} = 8.854 \cdot 10^{-48} \text{ pC}$

The resulting capacity C is now a series circuit of $C_{\{0\}} = 1.1 \cdot 10^{-12} \text{ pF}$ and $C_{\{r\}} = 3.3 \cdot 10^{-12} \text{ pF}$.

Therefore: $C = \frac{1}{\frac{1}{C_{\{0\}}} + \frac{1}{C_{\{r\}}}} = \frac{1}{\frac{1}{1.1 \cdot 10^{-12}} + \frac{1}{3.3 \cdot 10^{-12}}} = 0.825 \cdot 10^{-12} \text{ pF}$

$$\begin{aligned} \text{With } C_{\text{air}} &= \epsilon_0 \epsilon_{\text{air}} \frac{A}{d-x} \quad \&= \epsilon_0 A \frac{\epsilon_{\text{air}}}{d-x} \quad \& \\ C_{\text{c}} &= \epsilon_0 \epsilon_{\text{c}} \frac{A}{x} \quad \&= \epsilon_0 A \frac{\epsilon_{\text{c}}}{x} \end{aligned}$$

In the following such a sensor is given, with:

the following such a sensor is given with:

This leads to:
$$C = \frac{1}{\epsilon_0 A} \left(\frac{d}{\epsilon_r} + \frac{x}{\epsilon_r} + \frac{x}{\epsilon_c} \right)$$

• Plate area: $A = 25 \text{ cm}^2$

- Distance between both plates: $d=200 \text{ }\mu\text{m}$
- Air between the plates: $\epsilon_{r,\text{air}}=1$
- Supply voltage: 3.3 V
- Boundary effects on the end of the layers shall be ignored in the following calculations.

$$\epsilon_0 = 8.854 \cdot 10^{-12} \sim \text{F/m}$$

1. Calculate the capacity \$C\$.

Path

$$C \approx \frac{\epsilon_0 \epsilon_r \{A\over{d}\}}{8.854 \cdot 10^{-12} \cdot \frac{25 \cdot 10^{-6} \cdot 1 \cdot 10^{-6}}{200 \cdot 10^{-6}}}$$

Exercise E8 Capacitor

(written test, approx. 7 % of a 120-minute written test, SS2022)

Given the dielectric permittivity of the left-side layer with the following dimensions: $\epsilon_r = 0.1$ ~\rm

Result: ϵ_r of air ($\epsilon_r = 1$), while the thickness of the dielectric material remains the same.

Length of layer overlap: $l = 1.5$ ~\rm mm

Distance between single layers: $d = 1.0$ ~\rm \mu m

Depth of component: $w = 0.7$ ~\rm mm

Number of layers (from the picture): 3 left-side and 3 right-side layers.

Path

The capacity can be derived from the geometry by:
$$C = \epsilon_0 \epsilon_r \frac{A}{d}$$

The air is another capacitor in series to the dielectric material. Therefore, the capacity can be calculated as
$$\frac{1}{C_{\text{Air}}} = \frac{1}{C_{\text{dielectric}}} + \frac{1}{C_{\text{air}}}$$

The capacity of air is
$$C_{\text{Air}} = \epsilon_0 \epsilon_r \frac{A}{d} = 8.854 \cdot 10^{-12} \cdot \frac{1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}}{0.1 \cdot 10^{-6}} = 0.465 \dots \text{ nF}$$

The material shall have a dielectric permittivity of $\epsilon_r = 3$.

By this the overall capacity is
$$C_{\text{total}} = \frac{C_{\text{Air}} \cdot C_{\text{dielectric}}}{C_{\text{Air}} + C_{\text{dielectric}}} = \frac{0.465 \dots \text{ nF} \cdot 1.39 \dots \text{ nF}}{0.465 \dots \text{ nF} + 1.39 \dots \text{ nF}} = 0.35 \dots \text{ nF}$$

For this, we have to count facing areas A_0 . There are $N = 5$.

What is the field strength in the dielectric material between the layer, when a voltage of $U = 6.3$ ~\rm V is applied?

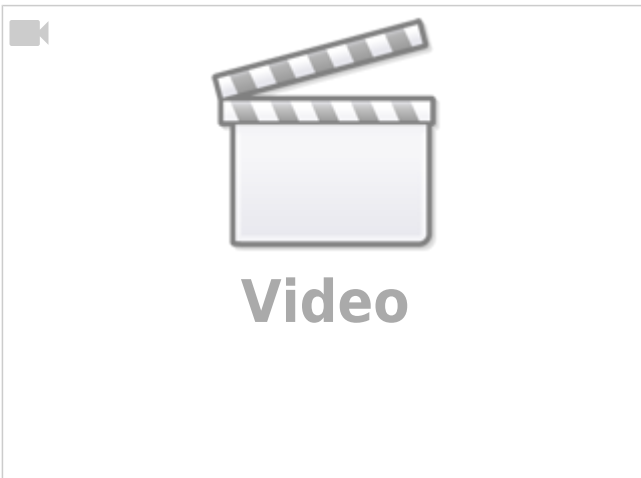
Path

The electric field strength E is given by:
$$E = \frac{U}{d} = \frac{6.3 \text{ V}}{1 \cdot 10^{-6} \text{ m}} = 6.3 \cdot 10^6 \text{ V/m}$$

Therefore, the formula is
$$C = \frac{\epsilon_0 \epsilon_r \{N \cdot l \cdot w\}}{d} \approx 8.854 \cdot 10^{-12} \frac{\text{As/Vm} \cdot 3 \cdot \{5 \cdot 1.5 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3} \cdot 0.7 \cdot 10^{-3}\}}{1 \cdot 10^{-6}}$$

Embedded resources

The equivalent capacitor for series of parallel configuration is well explained here



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