

Block 20 — Inductance and Energy

Student Group

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Block 20 — Electromagnetic Induction and Energy

Learning objectives

After this 90-minute block, you can

- ...

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- ...

90-minute plan

1. Warm-up (x min):
 1.
2. Core concepts & derivations (x min):
 1. ...
3. Practice (x min): ...
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

1. ...

Core content

Self-Induction

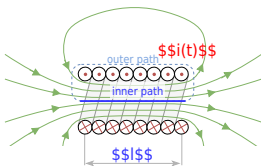
Up to now, we investigated the induction of electric voltages and currents based on the change of an external flux $\frac{d\Phi_{\text{ext}}}{dt}$. For the induced current i_{ind} , we found that it counteracts the change of the external flux (Lenz law).

But what happens, when there is no external field - only a coil which creates the flux change itself (see [figure 1](#))?

Fig. 1: Induction Phenomenons

To understand this, we will investigate the situation for a long coil ([figure 2](#)).

Fig. 2: Self-Induction of a Coil



The created field density of the coil can be derived from Ampere's Circuital Law

$$\oint \vec{H}(t) \cdot d\vec{s} = \int \vec{H}_{\text{inner}}(t) \cdot d\vec{s} + \int \vec{H}_{\text{outer}}(t) \cdot d\vec{s} = \int \vec{H}(t) \cdot d\vec{s} + 0 = H(t) \cdot l$$

With magnetic voltage $\oint \vec{H}(t) \cdot d\vec{s} = N \cdot I$ this leads to the magnetic flux density $B(t)$

$$N \cdot I = H(t) \cdot l \quad H(t) = \frac{N \cdot I}{l} \quad B(t) = \mu_0 \mu_r \cdot \frac{N \cdot I}{l}$$

Based on the magnetic flux density $B(t)$ it is possible to calculate the flux $\Phi(t)$:

$$\Phi(t) = \int_A \vec{B}(t) \cdot d\vec{A} = \int_A \mu_0 \mu_r \cdot \frac{N \cdot I}{l} \cdot dA = \mu_0 \mu_r \cdot \frac{N \cdot I}{l} \cdot A$$

The changing flux Φ is now creating an induced electric voltage and current, which counteracts the initial change of the current. This effect is called **Self Induction**. The induced electric voltage u_{ind} is given by:

$$u_{\text{ind}} = -N \cdot \frac{d\Phi(t)}{dt} = -N \cdot \frac{d(\mu_0 \mu_r \cdot \frac{N \cdot I}{l} \cdot A)}{dt} = -N \cdot \mu_0 \mu_r \cdot \frac{N \cdot A}{l} \cdot \frac{dI}{dt}$$

$$\mu_{\text{r}} \cdot \left\{ \frac{N \cdot A}{l} \right\} \cdot \left\{ \frac{d}{dt} \right\} \quad \text{\textbackslash end{align*}}$$

$$\begin{aligned} \boxed{u_{\text{ind}}} &= -\mu_0 \mu_{\text{r}} \cdot N^2 \cdot \left\{ \frac{A}{l} \right\} \cdot \left\{ \frac{d}{dt} \right\} \quad \text{\textbackslash text{for a long coil}} \quad \text{\textbackslash end{align*}} \end{aligned}$$

The result means that the induced electric voltage u_{ind} is proportional to the change of the current $\left\{ \frac{d}{dt} \right\} i$. The proportionality factor is also called **Self-inductance** L (or often simply called inductance).

4.5 Inductance

The inductance is another passive basic component of the electric circuit. Besides the ohmic resistor R and the capacitor C , the inductor L is the lump component entailing the inductance.

Generally, the inductance is defined by:
$$L = \left| \frac{u_{\text{ind}}}{\frac{d}{dt} i} \right| \quad \text{\textbackslash end{align*}}$$

The inductance L can also be described differently based on Lenz law $u_{\text{ind}} = - \left\{ \frac{d}{dt} \right\} \Psi(t)$:

$$\begin{aligned} L &= \left| \frac{u_{\text{ind}}}{\frac{d}{dt} i} \right| \quad \&= \quad \left\{ \frac{d}{dt} \right\} \frac{\Psi(t)}{\frac{d}{dt} i} \quad \text{\textbackslash end{align*}} \end{aligned}$$

$$\boxed{L = \left\{ \frac{\Psi(t)}{i} \right\}} \quad \text{\textbackslash end{align*}}$$

One can also consider an inductor a “conservative person”: it does not like to see abrupt changes in the passing current. It reacts to any change in the current with a counteracting voltage since the current change leads to a changing flux and - therefore - an induced voltage. The [figure 3](#) shows an inductor in series with a resistor and a switch (any real switch also behaves as a capacitor, when open). Once the simulation is started, the inductor directly counteracts the current, which is why the current only slowly increases.

The unit of the inductance is $1 \sim \text{Henry} = 1 \sim \text{H} = 1 \left\{ \frac{\text{Vs}}{\text{A}} \right\} = 1 \left\{ \frac{\text{Wb}}{\text{A}} \right\}$

Fig. 3: Example of a Circuit with an Inductor

Mathematically the voltages can be described in the following way:

$$\begin{aligned} u_0 &= u_R + u_L \quad \&= \quad i \cdot R + \left\{ \frac{d}{dt} \right\} \Psi \quad \&= \quad i \cdot R + L \cdot \left\{ \frac{d}{dt} \right\} i \quad \text{\textbackslash end{align*}} \end{aligned}$$

Inductance of different Components

Long Coil

In the last sub-chapter, the formula of a long coil was already investigated. By these, the inductance of a long coil is

$$\begin{aligned} \boxed{L_{\text{long ; coil}}} &= \mu_0 \mu_{\text{r}} \cdot N^2 \cdot \left\{ \frac{A}{l} \right\} \quad \text{\textbackslash end{align*}} \end{aligned}$$

Toroidal Coil

The toroidal coil was analyzed in the last chapter(see [magnetic Field Strength Part 1: Toroidal Coil](#)). Here, a rectangular intersection is assumed (see [figure 4](#)).

Fig. 4: Self-Induction of a toroidal Coil

This leads to

$$\begin{aligned} H(t) = \frac{N \cdot i}{l} \end{aligned}$$

with the mean magnetic path length (= length of the average field line) $l = \pi(r_{\text{o}} + r_{\text{i}})$:

$$\begin{aligned} H(t) = \frac{N \cdot i}{\pi(r_{\text{o}} + r_{\text{i}})} \end{aligned}$$

The inductance L can be calculated by

$$\begin{aligned} L_{\text{toroidal \; coil}} &= \frac{\Psi(t)}{i} \quad \&= \quad \frac{N \cdot \Phi(t)}{i} \end{aligned}$$

With the magnetic flux density $B(t) = \mu_0 \mu_{\text{r}} H(t) = \mu_0 \mu_{\text{r}} \frac{i \cdot N}{l}$ and the cross section $A = h(r_{\text{o}} - r_{\text{i}})$, we get:

$$\begin{aligned} \quad \quad L_{\text{toroidal \; coil}} &= \frac{N \cdot \mu_0 \mu_{\text{r}} \left(\frac{i \cdot N}{\pi(r_{\text{o}} + r_{\text{i}})} \right) \cdot h(r_{\text{o}} - r_{\text{i}})}{i} \quad \&= \quad \frac{N^2 \cdot \mu_0 \mu_{\text{r}} \cdot h(r_{\text{o}} - r_{\text{i}})}{\pi(r_{\text{o}} + r_{\text{i}})} \end{aligned}$$

$$\boxed{L_{\text{toroidal \; coil}} = \mu_0 \mu_{\text{r}} \cdot N^2 \cdot \frac{h(r_{\text{o}} - r_{\text{i}})}{\pi(r_{\text{o}} + r_{\text{i}})}} \quad \end{aligned}$$

6 Inductances in Circuits

Focus here: uncoupled inductors!

Series Circuits

Based on $L = \frac{\Psi(t)}{i}$ and Kirchhoff's mesh law ($i = \text{const}$) the series circuit of inductions can be interpreted as a single current i which generates multiple linked fluxes Ψ_i . Since the current must stay constant in the series circuit, the following applies for the equivalent inductor of a series connection of single ones:

$$L_{\text{eq}} = \frac{\sum_i \Psi_i}{i} = \sum_i L_i$$

A similar result can be derived from the induced voltage $u_{\text{ind}} = L \frac{di}{dt}$, when taking the situation of a series circuit (i.e. $i_1 = i_2 = i_3 = \dots = i_{\text{eq}}$ and $u_{\text{eq}} = u_1 + u_2 + \dots$):

$$\begin{aligned} u_{\text{eq}} &= u_1 + u_2 + \dots \quad \& L_{\text{eq}} \frac{di_{\text{eq}}}{dt} = L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + \dots \\ &= L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + \dots \quad \& L_{\text{eq}} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \dots \end{aligned}$$

Parallel Circuits

For parallel circuits, one can also start with the principles based on Kirchhoff's mesh law:

$$u_{\text{eq}} = u_1 = u_2 = \dots$$

and Kirchhoff's nodal law:

$$i_{\text{eq}} = i_1 + i_2 + \dots$$

Here, the formula for the induced voltage has to be rearranged:

$$\begin{aligned} u_{\text{ind}} &= L \frac{di}{dt} \quad \& \quad \quad \quad \int u_{\text{ind}} dt = L \cdot i \quad \& \quad \quad \quad \frac{1}{L} \int u_{\text{ind}} dt = i \end{aligned}$$

By this, we get:

$$i_{\text{eq}} = i_1 + i_2 + \dots = \frac{1}{L_{\text{eq}}} \int u_{\text{eq}} dt$$

$$\begin{aligned} \frac{d}{dt} &= \frac{1}{L_1} \cdot \int u_1 \, dt + \frac{1}{L_2} \cdot \int u_2 \, dt + \dots \parallel \frac{1}{L_{\text{eq}}} \cdot \int u \, dt = \frac{1}{L_1} \cdot \int u \, dt + \frac{1}{L_2} \cdot \int u \, dt + \dots \parallel \frac{1}{L_{\text{eq}}} \end{aligned}$$

Notice:

The inductor behaves in the parallel and series circuit similar to the resistor.

Common pitfalls

- ...

Exercises

Exercise 4.1.4 Effects of induction I

A change of magnetic flux is passing a coil with a single winding. The following pictures [figure ##](#) show different flux-time-diagrams as examples.

- Create for each $\Phi(t)$ -diagram the corresponding $u_{\text{ind}}(t)$ -diagram!
- Write down each maximum value of $u_{\text{ind}}(t)$

Fig. ##: Flux-Time-Diagrams

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Solution for (a)

For partwise linear u_{ind} one can derive:
$$u_{\text{ind}} = - \frac{d\Phi}{dt} \quad \Leftrightarrow \quad \Delta \Phi = - \frac{\Delta \Phi}{\Delta t} \Delta t$$

For diagram (a):

- $t = 0.0 \dots 0.6 \text{ s}$: $u_{\text{ind}} = - \frac{0 \text{ Vs}}{0.6 \text{ s}} = 0 \text{ V}$
- $t = 0.6 \dots 1.5 \text{ s}$: $u_{\text{ind}} = - \frac{-3.75 \cdot 10^{-3} \text{ Vs}}{0.9 \text{ s}} = +4.17 \text{ mV}$
- $t = 1.5 \dots 2.1 \text{ s}$: $u_{\text{ind}} = - \frac{0 \text{ Vs}}{0.6 \text{ s}} = 0 \text{ V}$

Final result for (a)

Fig. ##: Flux-Time-Diagrams Solution

u_{ind} ...

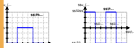
Exercise 4.1.5 Effects of induction II

A changing of magnetic flux is passing a coil with a single winding and induces the voltage $u_{\text{ind}}(t)$. The following pictures [figure ##](#) show different voltage-time diagrams as examples.

- Create for each $u_{\text{ind}}(t)$ -diagram the corresponding $\Phi(t)$ -diagram!
- Write down each maximum value of $\Phi(t)$

Note the given start value Φ_0 for each flux.

Fig. ##: Voltage-Time-Diagrams



Solution for (a)

For partwise linear u_{ind} one can derive:
$$u_{\text{ind}} = - \frac{d\Phi}{dt} \implies \Phi = - \int_0^t u_{\text{ind}} \, dt + \Phi_0 = \Phi_0 - \sum_k u_{\text{ind},k} \cdot \Delta t$$

For diagram (a):

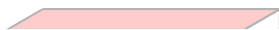
- $t = 0.00 \dots 0.04 \text{ s}$: $\Phi = \Phi_0 - 0 \cdot \Delta t = 0 \text{ Wb}$
- $t = 0.04 \dots 0.10 \text{ s}$: $\Phi = 0 \text{ Wb} - \{30 \text{ mV} \cdot (t - 0.04 \text{ s})\} = \{1.2 \text{ mWb}\} - t \cdot 30 \text{ mV}$
- $t = 0.10 \dots 0.14 \text{ s}$: $\Phi = \{1.2 \text{ mWb}\} - \{0.10 \text{ s} \cdot 30 \text{ mV}\} = - \{1.8 \text{ mWb}\}$

Exercise 4.1.6 Coil in magnetic Field I

A single winding is located in a homogenous magnetic field ($B = 0.5 \text{ T}$) between the pole pieces. The winding has a length of 150 mm and a distance between the conductors of 50 mm (see [figure ##](#)).

- Determine the function $u_{\text{ind}}(t)$, when the coil is rotating with 3000 min^{-1} .
- Given a current of 20 A through the winding: What is the torque $M(\varphi)$ depending on the angle between the surface vector of the winding and the magnetic field?

Fig. ##: Winding between Pole Pieces



Solution

$$\begin{aligned} u_{\text{ind}} &= - \frac{d\Phi}{dt} = - \frac{d}{dt} (B \cdot A) = - B \cdot \frac{dA}{dt} \\ &= - B \cdot \frac{d}{dt} (l \cdot d \cdot \cos(\omega t)) = + B \cdot l \cdot d \cdot \omega \cdot \sin(\omega t) \end{aligned}$$

Exercise 4.1.7 Coil in magnetic Field II

A rectangular coil is given by the sizes $a=10 \text{ cm}$, $b=4 \text{ cm}$, and the number of windings $N=200$. This coil moves with a constant speed of $v=2 \text{ m/s}$ perpendicular to a homogeneous magnetic field ($B=1.3 \text{ T}$ on a length of $l=5 \text{ cm}$). The area of the coil is tilted with regard to the field in $\alpha=60^\circ$ and enters the field from the left

side (see [figure ##](#)).

- Determine the function $u_{\text{ind}}(t)$ on the coil along the given path. Sketch of the $u_{\text{ind}}(t)$ diagram.
- What is the maximum induced voltage $u_{\text{ind,Max}}$?

Fig. ##: Winding between Pole Pieces

Solution

Let assume, that I is in the x -axis, $\vec{B}$ in the y -axis and a .

Step 1: Calculate the effective area, perpendicular to the $\vec{B}$ -field (independent from whether the area is in the $\vec{B}$ -field or not).

For this b has to be projected onto the plane perpendicular to the $\vec{B}$ -field: $b_{\text{eff}} = b \cdot \cos \alpha$

$$A_{\text{eff}} = a \cdot b \cdot \cos \alpha$$

Step 2: Focus on entering and exiting the $\vec{B}$ -field.

Induction only occurs for $\frac{d}{dt}(A \cdot B) \neq 0$, so here: when the area A_{eff} enters and leaves the constant $\vec{B}$ -field.

When entering the $\vec{B}$ -field the area A with $0 < A < A_{\text{eff}}$ is in the field. The area moves with v . Therefore, after $\Delta t = \frac{b_{\text{eff}}}{v}$ the full $\vec{B}$ -field is provided onto the area A_{eff} :

$$\frac{d\Phi}{dt} = -N \cdot \frac{d}{dt}(B \cdot A) = -N \cdot B \cdot \frac{dA}{dt} = -N \cdot B \cdot \frac{d}{dt}(a \cdot b \cdot \cos \alpha) = -N \cdot B \cdot \frac{da}{dt}$$

The following diagram shows ...

- ... how one can derive the effective width b_{eff} , which is projected onto the plane perpendicular to the $\vec{B}$ -field: $b_{\text{eff}} = b \cdot \cos \alpha$
- ... what happens on the effective area A_{eff} : there is a change of the field lines in the area only for entering and leaving the $\vec{B}$ -field.
- ... how the $u_{\text{ind}}(t)$ looks as a graph: the part of A_{eff} , where the $\vec{B}$ -field passes through increase linearly due to the constant speed v

Be aware, that the task did not provide a clue for the direction of windings and therefore it provides no clue for the polarization of the induced voltage.

So, the course of the voltage when entering or exiting is not uniquely given.

Fig. ##: Solution

◀ ◻ ▶

Exercise E1 Self-Induction

(written test, approx. 8 % of a 120-minute written test, SS2024)

2. Determine the induced voltage in the coil using the change in current.

Result: The current through the coil changes linearly from 0 A to 3 A in 0.02 ms.

The arrangement is located in air ($\mu_{\text{r}} = 1$).

Path

$$\mu_0 = 4\pi \cdot 10^{-7} \text{ Vs/Am}$$

$$U_{\text{ind}} = 1.32 \text{ mV}$$

3. Calculate the (self-)inductance of the coil.

For the linear change of the current the formula of the induced voltage can also be linearized:

$$u_{\text{ind}} = -L \cdot \frac{di}{dt} \quad \text{or} \quad U_{\text{ind}} = -L \cdot \frac{\Delta i}{\Delta t}$$

$$U_{\text{ind}} = -L \cdot \frac{\Delta i}{\Delta t} \quad \text{or} \quad U_{\text{ind}} = -L \cdot \frac{3 \text{ A}}{0.02 \cdot 10^{-3} \text{ s}}$$

The formula for the induction of a long coil is:
$$L = \mu_0 \mu_r \cdot N^2 \cdot \frac{A}{l} \quad \&= 4\pi \cdot 10^{-7} \cdot \frac{\text{Vs/A} \cdot (500)^2}{2 \cdot 10^{-2} \cdot \pi \cdot (2 \cdot 10^{-2} \cdot \text{m})^2}$$

Exercise E12 Self Induction

(written test, approx. 8 % of a 120-minute written test, SS2022)

2. A motor is in a room with a voltage of $U = 50 \text{ V}$, which the electrician has to DC voltage source, which is fused with a circuit breaker.

Sketch the diagram of the circuit with $i(t) = 0 \text{ A}$ with a percentage of 63 % of the induced voltage is reduced linearly down to 0 A within 1 ms .

(The inner resistance of the motor shall be neglected.)

$$u_{\text{ind}}(t) = 3150 \text{ V}$$

Path

.. Draw the circuit (the circuit breaker can be drawn as a switch), with all voltage and current arrows.

For the maximum voltage on the circuit breaker one has to consider the following:

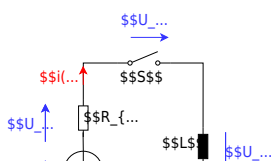
Result

- external voltage of the voltage source U
- voltage $u_{\text{ind}}(t)$ induced by the change of the current

The first one is not given in the exercise, and therefore not considered here.

The induced voltage can be calculated by linearizing the following:
$$u_{\text{ind}}(t) = -L \frac{di}{dt} \rightarrow u_{\text{ind}}(t) = -L \frac{\Delta i}{\Delta t}$$

With the given details:
$$u_{\text{ind}}(t) = -L \frac{0 - I}{t_1 - t_0} = 50 \cdot 10^{-6} \cdot \frac{63 \text{ A}}{1 \cdot 10^{-6} \text{ s}} = 3150 \frac{\text{Vs}}{\text{A}} \cdot \frac{\text{A}}{\text{s}}$$



Exercise 4.5.1 Self Induction I

2. Calculate the self inductance of a long solenoid with a square cross-section of side $a = 10 \text{ cm}$ and a length $l = 1 \text{ m}$. The number of windings is $N = 1000$.

1. Cylindrical long air coil with $N = 390$, winding diameter $d = 3.0 \text{ cm}$ and length $l = 0.18 \text{ m}$.

Solution

Solution
$$L_1 = \mu_0 \mu_r \frac{N^2}{l} \cdot \frac{\pi d^2}{4}$$

$$\begin{aligned} L_1 &= \mu_0 \mu_r \frac{N^2}{l} \cdot \frac{\pi d^2}{4} \\ L_1 &= 4\pi \cdot 10^{-7} \text{ Vs/Am} \cdot 1 \cdot \frac{\pi (0.03 \text{ m})^2}{4} \cdot (390)^2 \\ L_1 &= 4\pi \cdot 10^{-7} \cdot \frac{\pi}{4} \cdot 0.0009 \cdot 152100 \\ L_1 &= 4\pi \cdot 10^{-7} \cdot 0.0009 \cdot 152100 \end{aligned}$$

Exercise 4.5.2 Self Induction II

A cylindrical air coil (length $l = 40 \text{ cm}$, diameter $d = 5.0 \text{ cm}$, and a number of windings $N = 300$) passes a current of 30 A . The current shall be reduced linearly in 2.0 ms down to 0.0 A .

What is the amount of the induced voltage u_{ind} ?

$$|u_{\text{ind}}| = 33 \text{ V}$$

Solution

The requested induced voltage can be derived by:

$$\begin{aligned} L &= \frac{u_{\text{ind}}}{\frac{di}{dt}} \\ \Rightarrow |u_{\text{ind}}| &= L \cdot \left| \frac{di}{dt} \right| \\ &= L \cdot \left| \frac{\Delta i}{\Delta t} \right| \end{aligned}$$

Therefore, we just need the inductance L , since $\frac{\Delta i}{\Delta t}$ is defined as 30 A per 2 ms :

$$L = \mu_0 \mu_r \frac{N^2}{l} \cdot \frac{\pi d^2}{4}$$

$$\begin{aligned} |u_{\text{ind}}| &= \mu_0 \mu_r \frac{N^2}{l} \cdot \frac{\pi d^2}{4} \cdot \left| \frac{\Delta i}{\Delta t} \right| \\ &= 4\pi \cdot 10^{-7} \cdot \frac{\pi}{4} \cdot 1 \cdot (300)^2 \cdot \frac{30}{0.002} \end{aligned}$$

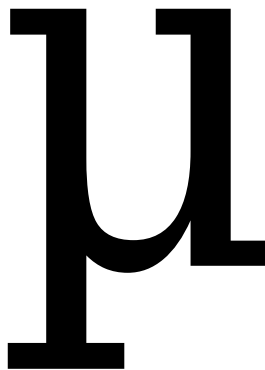
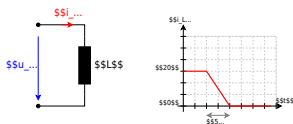
$$\left\{ \frac{\pi \cdot (0.05 \text{ m})^2}{0.40 \text{ m}} \right\} \cdot \left\{ \frac{30 \text{ A}}{2 \text{ ms}} \right\} \quad \text{\texttt{\textbackslash end\{align*\}}}$$

Exercise 4.5.3 Self Induction III

A coil with the inductance $L = 20 \text{ } \mu\text{H}$ passes a current of 40 A . The current shall be reduced linearly in $5 \text{ } \mu\text{s}$ down to 0 A (see [figure 5](#)).

- What is the amount of the induced voltage u_{ind} ?
- Sketch the course of $u_{\text{ind}}(t)$!

Fig. 5: Circuit and timing Diagram



Embedded resources

Explanation (video): ...

From:
<https://first.mexle.te.hs-heilbronn.de/> - MEXLE Wiki

Permanent link:
https://first.mexle.te.hs-heilbronn.de/electrical_engineering_and_electronics_1/block20?rev=1764697668

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