

Exam Winter Semester 2022

Student Group

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Exam Winter Semester 2022

Additional permitted Aids

- non-programmable calculator,
- formulary (2 DIN A4 pages)

Hits

- The duration of the exam is 60 min.
- Attempts to cheat will lead to exclusion and failure of the exam.
- Withdrawal is no longer possible after these exam has been handed out.
- Please write down intermediate calculations and results on the assignment sheet. (when more space is needed also on the reverse side. In this case: Mark it clearly).
- Always use units in the calculation.
- Use a document-proof, non-red pen.

Tasks

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-4} \text{ K}^{-1}$ is selected. The power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary. Calculate the current I needed to operate it.

The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.
The heating element is 3 m long and has a diameter of 3.57 mm .
Solution: $R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$

Solution

$$\begin{aligned} P &= U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}} \end{aligned}$$

$$\begin{aligned} R &= \rho \cdot \frac{l}{A} \quad \text{with } A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \\ R &= \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad \text{and } R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi} \end{aligned}$$

Exercise E1 Resistance of a Wire by Resistivity (written test, approx. 6 % of a 60-minute written test, WS2022)

A heating element made of solid nichrome wire with a temperature coefficient of $1.80 \cdot 10^{-4} \text{ K}^{-1}$ is selected.

Result power dissipation (= heat flow) of $P=40 \text{ W}$ is necessary.
 Calculate the current I needed to operate it for heating elements.
 The Nichrome wire has a resistivity of $1.10 \cdot 10^{-6} \text{ } \Omega \text{ m}$.
 The heating element is 3 m long and has a diameter of 3.57 mm .
 Solution: $R = 10^{-3} \text{ } \Omega$ (Wrong calculation)

Solution

$$P = U \cdot I = R \cdot I^2 \quad \rightarrow \quad I = \sqrt{\frac{P}{R}} = \sqrt{\frac{40 \text{ W}}{0.33 \text{ } \Omega}}$$

$$R = \rho \cdot \frac{l}{A} \quad | \quad A = r^2 \cdot \pi = \frac{1}{4} d^2 \cdot \pi \quad || \quad R = \rho \cdot \frac{4 \cdot l}{d^2 \cdot \pi} \quad || \quad R = 1.10 \cdot 10^{-6} \text{ } \Omega \text{ m} \cdot \frac{4 \cdot 3 \text{ m}}{(3.57 \cdot 10^{-3} \text{ m})^2 \cdot \pi}$$

Exercise E2 Temperature-dependent Resistance (written test, approx. 6 % of a 60-minute written test, WS2022)

2. Regulation explains a temperature sensitive component for a refrigeration system. The thermistor has a resistance of $65 \text{ k} \Omega$ at 25°C . Its temperature coefficients are: $\alpha = 0.01 \text{ } \frac{1}{\text{K}}$ and $\beta = 71 \cdot 10^{-6} \text{ } \frac{1}{\text{K}^2}$

Result The temperature inside the refrigeration system can reach down to -40°C .
 Calculate the resistance of the thermistor at -40°C .
 Resistance of the resistor R depends on the current I and generated heat. Therefore, a solution is to use the heat flow up the refrigeration system.
 Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 \cdot (1 + \alpha \cdot \Delta T + \beta \cdot \Delta T^2) \quad | \quad \Delta T = T_{\text{end}} - T_{\text{start}} \quad || \quad R = 10 \text{ k} \Omega \cdot \left(1 + 0.01 \cdot (-40^\circ \text{C} - 25^\circ \text{C}) + 71 \cdot 10^{-6} \cdot (-40^\circ \text{C} - 25^\circ \text{C})^2\right)$$

Exercise E2 Temperature-dependent Resistance

(written test, approx. 6 % of a 60-minute written test, WS2022)

2. A resistor exhibits a temperature coefficient of resistance of $\alpha = 0.01 \text{ K}^{-1}$ and a nominal resistance of $R_0 = 10 \text{ k}\Omega$ at $T_0 = 25^\circ\text{C}$. Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.
 Result: The temperature inside the refrigeration system can reach down to -40°C .

Its temperature coefficients are: $\alpha = 0.01 \text{ K}^{-1}$ and $\beta = 71 \cdot 10^{-6} \text{ K}^{-2}$.
 Result: The temperature inside the refrigeration system can reach down to -40°C .

Result: The temperature inside the refrigeration system can reach down to -40°C .

$$R = R_0 (1 + \alpha \Delta T + \beta \Delta T^2)$$

Resistance transfer resistor R is part of the circuit and generates heat. Therefore, a solution is to use a heat sink up the refrigeration system.

Therefore, with constant U and increasing R the power decreases. Ten times more resistance decreases the heat flow to one-tenth.

$$R = R_0 (1 + \alpha \Delta T + \beta \Delta T^2)$$

with $\Delta T = T_{\text{end}} - T_{\text{start}}$

$$R = 10 \text{ k}\Omega \cdot \left(1 + 0.01 \text{ K}^{-1} \cdot (-40^\circ\text{C} - 25^\circ\text{C}) + 71 \cdot 10^{-6} \text{ K}^{-2} \cdot (-40^\circ\text{C} - 25^\circ\text{C})^2 \right)$$

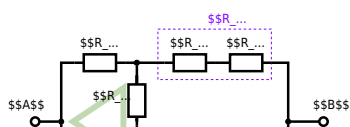
Exercise E3 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall hold: $R_1 = 200 \text{ }\Omega$, $R_2 = R_3 = 100 \text{ }\Omega$, $R_4 = 100 \text{ }\Omega$ and the voltage $U = 10 \text{ V}$.
 Result: $R_B = 132.8 \text{ }\Omega$.

Solution

$$R_{\text{eq}} = 132.8 \text{ }\Omega$$

Now a wye-delta transformation is necessary.

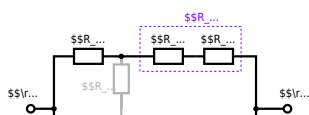


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_1) \parallel (R_2 + R_2) \parallel (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel (500 \Omega) \parallel (200 \Omega) \parallel (500 \Omega \cdot 200 \Omega) / (500 \Omega + 200 \Omega)$$

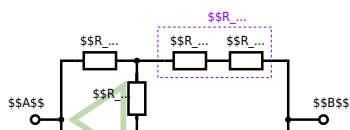
Exercise E4 Pure Resistor Network Simplification (written test, approx. 13 % of a 60-minute written test, WS2022)

The following shall be solved with $R_1 = 200 \Omega$, $R_2 = R_3 = 100 \Omega$ and the source $B = 15 \text{ V}$.
 Result given: $R_{\text{eq}} = 132.8 \Omega$.

Solution

$$R_{\text{eq}} = 132.8 \Omega$$

Now a wye-delta transformation is necessary.

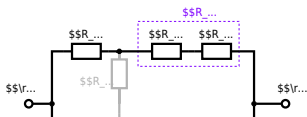


Since $R_2=R_3$ and based on the equations for the transformation, the transformed R_Y is given as:
$$R_Y = \frac{R_2 \cdot R_2}{R_2 + R_2 + R_2} = \frac{(100 \Omega)^2}{3 \cdot 100 \Omega} = \frac{1}{3} \cdot 100 \Omega = 33.33 \Omega$$

The equivalent resistor is given by a parallel configuration of resistors in series:
$$R_{eq} = R_Y + (R_Y + R_1 + R_1) \parallel (R_Y + R_2) \parallel R_{eq} = 33.33 \Omega + (33.33 \Omega + 400 \Omega) \parallel (33.33 \Omega + 100 \Omega)$$

1. The switch shall now be open. Calculate the equivalent resistance R_{eq} between A and B.

Solution



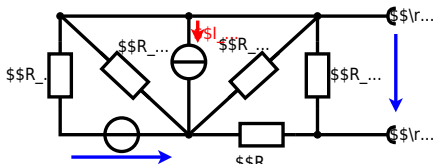
The equivalent resistor is given by a parallel configuration of resistors in series:

$$R_{\text{eq}} = (R_2 + R_1 + R_{-1}) \parallel (R_2 + R_2) \parallel R_{\text{eq}} = (100 \Omega + 200 \Omega + 200 \Omega) \parallel (100 \Omega + 100 \Omega) \parallel R_{\text{eq}} = (500 \Omega) \parallel (200 \Omega) \parallel R_{\text{eq}} = \frac{500 \Omega \cdot 200 \Omega}{500 \Omega + 200 \Omega} \parallel R_{\text{eq}}$$

**Exercise E5 Equivalent linear Source
(written test, approx. 14 % of a 60-minute written test, WS2022)**

The circuit in the following has to be simplified.
Result

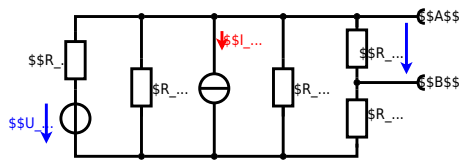
$$U_{\text{s}} = U_{\text{AB}} = 4.5 \text{ V} \quad R_{\text{i}} = R_{\text{AB}} = 6 \Omega$$



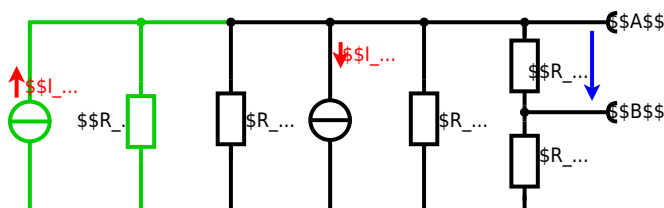
Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $\begin{aligned} R_1 &= 5.0 \text{ } \Omega, & U_2 &= 6.0 \text{ V}, & R_3 &= 10 \text{ } \Omega, & I_4 &= 4.2 \text{ A}, & \\ R_5 &= 10 \text{ } \Omega, & R_6 &= 7.5 \text{ } \Omega, & R_7 &= 15 \text{ } \Omega & \end{aligned}$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_6 + R_7)$$

$$U_{24} = U_2 \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} - I_4 \cdot \frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5}$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0\Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5\Omega || 10\Omega || 10\Omega = 5\Omega || 5\Omega = 2.5\Omega$:

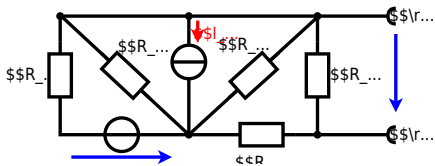
$$U_{AB} = \frac{6.0\text{V}}{5.0\Omega} - 4.2\Omega \cdot \frac{15\Omega \cdot 2.5\Omega}{7.5\Omega + 15\Omega + 2.5\Omega}$$

$$R_{AB} = 15\Omega || (7.5\Omega + 2.5\Omega)$$

Exercise E6 Equivalent linear Source (written test, approx. 14 % of a 60-minute written test, WS2022)

The circuit in the following has to be simplified.
Result

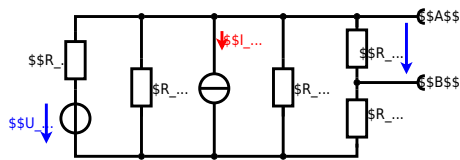
$$U_s = U_{AB} = 4.5\text{V} \quad R_i = R_{AB} = 6\Omega$$



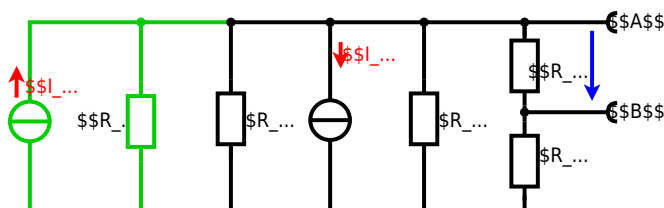
Calculated the internal resistance R_{int} and the source voltage U_{oc} of an equivalent linear voltage source on the connectors A and B . $R_1=5.0 \text{ }\Omega$, $U_2=6.0 \text{ V}$, $R_3= 10 \text{ }\Omega$, $I_4=4.2 \text{ A}$, $R_5=10 \text{ }\Omega$, $R_6=7.5 \text{ }\Omega$, $R_7=15 \text{ }\Omega$ Use equivalent sources in order to simplify the circuit!

Solution

The best thing is to re-think the wiring like rubber bands and adjust them:



The linear voltage source of U_2 and R_1 can be transformed into a current source $I_2 = \frac{U_2}{R_1}$ and R_1 :



Now a lot of them can be combined. The resistors R_1 , R_3 , R_5 are in parallel, like also I_2 and I_4 :

$$R_{135} = R_1 || R_3 || R_5$$

$$I_{24} = I_2 - I_4$$

The resulting circuit can again be transformed:



Here, the U_{24} is calculated by I_{24} as the following:

$$U_{24} = I_{24} \cdot (R_6 || R_7)$$

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - I_4 \cdot R_1 || R_3 || R_5$$

On the right side of the last circuit, there is a voltage divider given by R_{135} , R_6 , and R_7 .

Therefore the voltage between A and B is given as:

$$U_{AB} = U_{24} \cdot \left(\frac{R_7}{R_6 + R_7 + R_1 || R_3 || R_5} \right) - \left(\frac{U_2}{R_1} - I_4 \right) \cdot \left(\frac{R_7 \cdot R_1 || R_3 || R_5}{R_6 + R_7 + R_1 || R_3 || R_5} \right)$$

For the internal resistance R_i the ideal voltage source is substituted by its resistance ($=0 \Omega$, so a short-circuit):

$$R_{AB} = R_7 || (R_6 + R_1 || R_3 || R_5)$$

with $R_1 || R_3 || R_5 = 5 \Omega || 10 \Omega || 10 \Omega = 5 \Omega || 5 \Omega = 2.5 \Omega$:

$$U_{AB} = \left(\frac{6.0 \text{ V}}{5.0 \Omega} - 4.2 \text{ A} \right) \cdot \left(\frac{15 \Omega \cdot 2.5 \Omega}{7.5 \Omega + 15 \Omega + 2.5 \Omega} \right) || R_{AB} = 15 \Omega || (7.5 \Omega + 2.5 \Omega)$$

Exercise E7 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

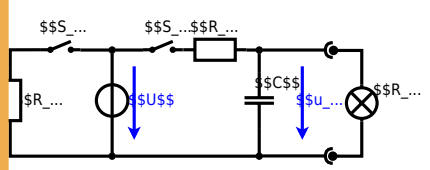
The circuit below is a fully discharging RC circuit. The capacitor is initially uncharged. At $t = 0$ s, the switch is closed. The voltage across the capacitor is again 0 V at the moment $t_0 = 0 \text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2 = 1 \text{ ms}$ after closing the switch.

Solution: To solve this, first create an equivalent linear voltage source from U , R_1 , and R_2 .

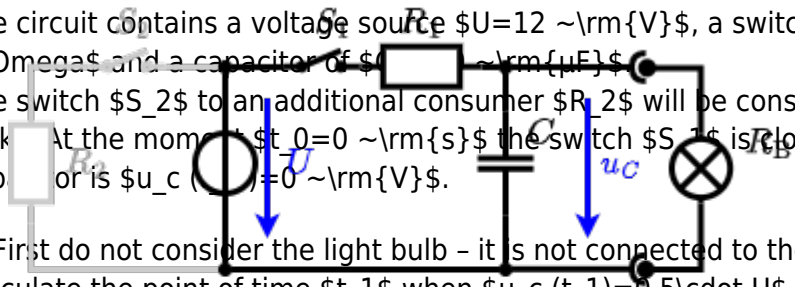
The ideal voltage source U_{eq} is given by:

$$U_{eq} = \frac{U \cdot R_2}{R_1 + R_2}$$

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

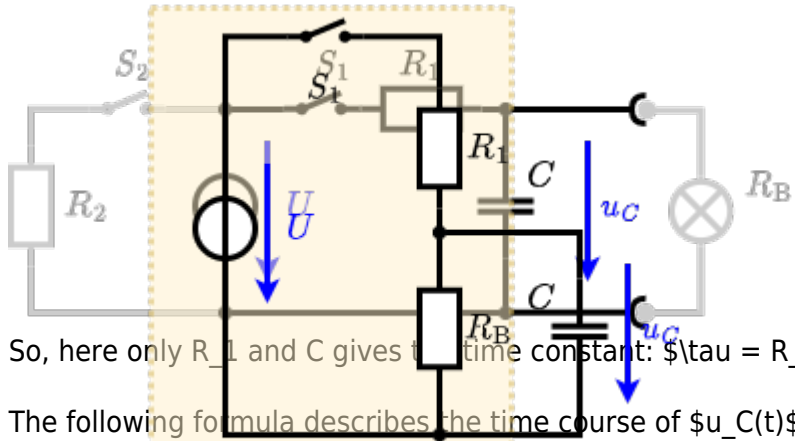


The circuit contains a voltage source $U=12\text{ V}$, a switch S_1 , a resistor of $R_1=20\text{ }\Omega$ and a capacitor of $C=100\text{ }\mu\text{F}$. The switch S_2 to an additional consumer R_2 will be considered to be open for the first task. At the moment $t_0=0\text{ s}$ the switch S_1 is closed, the voltage across the capacitor is $u_c(t_0)=0\text{ V}$.



First do not consider the light bulb - it is not connected to the RC circuit. Calculate the point of time t_1 when $u_c(t_1)=0.5 \cdot U$.

Solution



So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_c(t_1)=0.5 \cdot U$:

$$u_c(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$$
 It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5$.
 An equivalent linear voltage source can be given with U_s , R_1 and R_B as seen in yellow:

$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$$
 The internal resistance is given by substituting the ideal voltage source with its resistance ($r=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($r=0\text{ }\Omega$, short-circuit).

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

$$u_c(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-1\text{ ms}/(10\text{ }\Omega \cdot 100\text{ }\mu\text{F})})$$

Exercise E8 Charging Capacitors
(written test, approx. 16 % of a 60-minute written test, WS2022)

The circuit (see the solution) consists of a 12 V DC voltage source, a $20\text{ }\Omega$ resistor, a $100\text{ }\mu\text{F}$ capacitor, and a $20\text{ }\Omega$ resistor. The voltage across the capacitor is again 0 V at the moment $t_0=0\text{ s}$ when the switch S_1 is closed. Calculate the voltage $u_c(t_2)$ across the capacitor at $t_2=1\text{ ms}$ after closing the switch.

Solution To solve this, first create an equivalent linear voltage source from U , R_1 , and R_B .

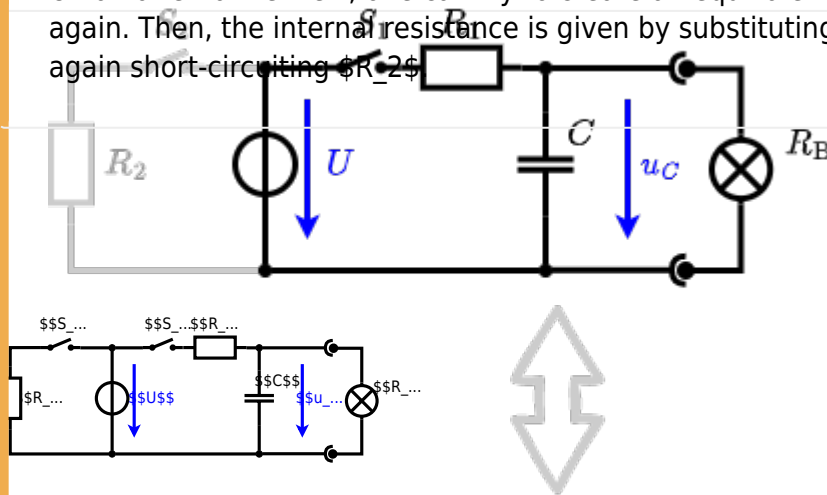
$$U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U = 6\text{ V}$$

$$R_i = R_1 \parallel R_B = 10\text{ }\Omega$$

Solution

The ideal voltage source is $U = 12 \text{ V}$. The internal resistance is $R_1 = 20 \text{ }\Omega$. The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} . The voltage across the capacitor is u_C . The voltage across the light bulb is u_B . The voltage across the resistor R_2 is u_{R_2} .

On an alternative view, one can try to create an equivalent linear voltage source again. Then, the internal resistance is given by substituting the ideal voltage source is again short-circuiting R_2 .

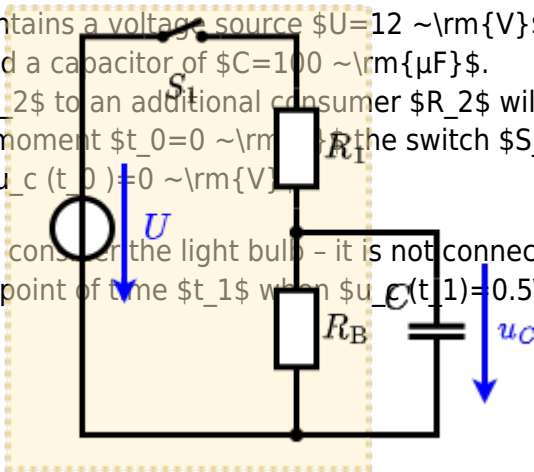


The circuit contains a voltage source $U = 12 \text{ V}$, a switch S_1 , a resistor of $R_1 = 20 \text{ }\Omega$ and a capacitor of $C = 100 \text{ }\mu\text{F}$.

The switch S_2 to an additional consumer R_2 will be considered to be open for the first asks. At the moment $t_0 = 0$ the switch S_1 is closed, the voltage across the capacitor is $u_C(t_0) = 0$.

First do not consider the light bulb - it is not connected to the RC circuit.

Calculate the point of time t_1 when $u_C(t_1) = 0.5 \cdot U$.



Solution

An equivalent linear voltage source can be given with U , R_1 , and R_B as seen in yellow.

Therefore, the voltage of the equivalent linear voltage source is: $U_s = U \cdot \frac{R_B}{R_1 + R_B} = \frac{1}{2} \cdot U$. The internal resistance is given by substituting the ideal voltage source with its resistance ($R = 0 \text{ }\Omega$, short-circuit). $R_i = R_1 \parallel R_B = 10 \text{ }\Omega$.

$$u_C(t_2) = U_s \cdot (1 - e^{-t_2/(R_i \cdot C)}) = \frac{1}{2} \cdot U \cdot (1 - e^{-t_2/(10 \text{ }\Omega \cdot 100 \text{ }\mu\text{F})})$$

So, here only R_1 and C gives the time constant: $\tau = R_1 \cdot C$

The following formula describes the time course of $u_C(t)$ which has to be $u_C(t_1) = 0.5 \cdot U$: $u_C(t) = U \cdot (1 - e^{-t/\tau}) = 0.5 \cdot U$. It has to be rearranged to $(1 - e^{-t/\tau}) = 0.5 \implies e^{-t/\tau} = 0.5 \implies t/\tau = \ln(0.5) \implies t = \tau \cdot \ln(0.5) = R_1 \cdot C \cdot \ln(0.5)$

Exercise E9 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{U} across the $30\ \Omega$ resistor in the circuit shown in the figure. The current \underline{i} is given by $i(t) = 0.24 \cos(300t - 4\pi t)$ A. The angular frequency is $\omega = 300$ rad/s. The result \underline{U} shall be given.

After analysis, the full phasor voltage \underline{U} shall be given. The phase angle φ shall be given in degrees. The result φ shall be given.

Solution
$$\underline{U} = \underline{I} \cdot \underline{Z} = 0.24 \cdot (30 + j4) = 7.2 + j0.96$$

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The voltage \underline{U} across the $30\ \Omega$ resistor is $7.2 + j0.96$ V. The phase angle φ is $\arctan(0.96/7.2) = 7.6^\circ$.

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With the complex part $\varphi = \arctan(\frac{0.96}{7.2}) = 7.6^\circ$ the physical value $u(t)$ can be calculated as $u(t) = 7.2 \cos(300t - 4\pi t + 7.6^\circ)$.

Exercise E10 Analyzing complex Impedances
(written test, approx. 14 % of a 60-minute written test, WS2022)

2. Calculate the phasor voltage \underline{U} across the $30\ \Omega$ resistor in the circuit shown in the figure. The current \underline{i} is given by $i(t) = 0.24 \cos(300t - 4\pi t)$ A. The angular frequency is $\omega = 300$ rad/s. The result \underline{U} shall be given.

After analysis, the full phasor voltage \underline{U} shall be given. The phase angle φ shall be given in degrees. The result φ shall be given.

Solution
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The absolute value of the impedance is $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$ and the phase angle is $\phi = \arctan\left(\frac{X_L - X_C}{R}\right)$.
 With the complex part comes the physical value: $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$.
 The phase ϕ is given by $\phi = \arctan\left(\frac{X_L - X_C}{R}\right) = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$.

Exercise E11 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

At a series circuit with $R = 1.00 \text{ k}\Omega$, $L = 4.7 \text{ }\mu\text{H}$, and $C = 40 \text{ nF}$ at $f = 4 \text{ MHz}$, the impedance of the capacitor Z_C shall have the same absolute value of the impedance as a resistor R .
 Result: $Z_C = 1.00 \text{ k}\Omega$, $Z_L = 17.2 \text{ }\Omega$, $Z_R = 1.00 \text{ k}\Omega$.
 A resistor R_1 shall have the same absolute value of the impedance as a capacitor $C_1 = 40 \text{ nF}$ at $f_1 = 4 \text{ MHz}$.

Solution
 Solution: $R_1 = 1.00 \text{ k}\Omega$
 Solution: $R_2 = 10.0 \text{ }\Omega$

A series circuit means that the current is constant on every component.
 The equivalent impedance for R and L combined is given by $Z_{RL} = \sqrt{R^2 + X_L^2}$.
 Parallel circuit means that the voltage is the same on R_2 and C_1 .
 The equivalent impedance for R_2 and C_1 combined is given by $Z_{R_2C_1} = \frac{R_2 \cdot X_C}{\sqrt{R_2^2 + X_C^2}}$.
 Since $Z_{RL} = Z_{R_2C_1}$, we have $\sqrt{R^2 + X_L^2} = \frac{R_2 \cdot X_C}{\sqrt{R_2^2 + X_C^2}}$.
 Squaring both sides: $R^2 + X_L^2 = \frac{R_2^2 \cdot X_C^2}{R_2^2 + X_C^2}$.
 Multiplying both sides by $R_2^2 + X_C^2$: $(R^2 + X_L^2)(R_2^2 + X_C^2) = R_2^2 \cdot X_C^2$.
 Expanding: $R^2 R_2^2 + R^2 X_C^2 + X_L^2 R_2^2 + X_L^2 X_C^2 = R_2^2 X_C^2$.
 Simplifying: $R^2 R_2^2 + R^2 X_C^2 + X_L^2 R_2^2 + X_L^2 X_C^2 - R_2^2 X_C^2 = 0$.
 $R^2 R_2^2 + R^2 X_C^2 + X_L^2 R_2^2 - R_2^2 X_C^2 + X_L^2 X_C^2 = 0$.
 $R^2 R_2^2 + R^2 X_C^2 + X_L^2 R_2^2 - R_2^2 X_C^2 = -X_L^2 X_C^2$.
 Dividing by R_2^2 : $R^2 + R^2 \frac{X_C^2}{R_2^2} + X_L^2 - X_C^2 = -\frac{X_L^2 X_C^2}{R_2^2}$.
 Rearranging: $R^2 + X_L^2 - X_C^2 = -\frac{X_L^2 X_C^2}{R_2^2} - R^2 \frac{X_C^2}{R_2^2}$.
 Multiplying by R_2^2 : $R^2 R_2^2 + X_L^2 R_2^2 - X_C^2 R_2^2 = -X_L^2 X_C^2 - R^2 X_C^2$.
 Rearranging: $R^2 R_2^2 + X_L^2 R_2^2 + X_L^2 X_C^2 + R^2 X_C^2 = X_C^2 R_2^2$.
 Dividing by X_C^2 : $R^2 \frac{R_2^2}{X_C^2} + X_L^2 \frac{R_2^2}{X_C^2} + X_L^2 + R^2 = R_2^2$.
 Rearranging: $R^2 \frac{R_2^2}{X_C^2} + X_L^2 \frac{R_2^2}{X_C^2} = R_2^2 - X_L^2 - R^2$.
 Dividing by R_2^2 : $R^2 \frac{1}{X_C^2} + X_L^2 \frac{1}{X_C^2} = 1 - \frac{X_L^2}{R_2^2} - \frac{R^2}{R_2^2}$.
 Rearranging: $R^2 \frac{1}{X_C^2} + X_L^2 \frac{1}{X_C^2} + \frac{X_L^2}{R_2^2} + \frac{R^2}{R_2^2} = 1$.
 Multiplying by R_2^2 : $R^2 \frac{R_2^2}{X_C^2} + X_L^2 \frac{R_2^2}{X_C^2} + X_L^2 + R^2 = R_2^2$.
 Rearranging: $R^2 \frac{R_2^2}{X_C^2} + X_L^2 \frac{R_2^2}{X_C^2} = R_2^2 - X_L^2 - R^2$.
 Dividing by R_2^2 : $R^2 \frac{1}{X_C^2} + X_L^2 \frac{1}{X_C^2} = 1 - \frac{X_L^2}{R_2^2} - \frac{R^2}{R_2^2}$.
 Rearranging: $R^2 \frac{1}{X_C^2} + X_L^2 \frac{1}{X_C^2} + \frac{X_L^2}{R_2^2} + \frac{R^2}{R_2^2} = 1$.
 Multiplying by R_2^2 : $R^2 \frac{R_2^2}{X_C^2} + X_L^2 \frac{R_2^2}{X_C^2} + X_L^2 + R^2 = R_2^2$.
 Rearranging: $R^2 \frac{R_2^2}{X_C^2} + X_L^2 \frac{R_2^2}{X_C^2} = R_2^2 - X_L^2 - R^2$.
 Dividing by R_2^2 : $R^2 \frac{1}{X_C^2} + X_L^2 \frac{1}{X_C^2} = 1 - \frac{X_L^2}{R_2^2} - \frac{R^2}{R_2^2}$.
 Rearranging: $R^2 \frac{1}{X_C^2} + X_L^2 \frac{1}{X_C^2} + \frac{X_L^2}{R_2^2} + \frac{R^2}{R_2^2} = 1$.

Exercise E12 Impedances at different Frequencies
 (written test, approx. 18 % of a 60-minute written test, WS2022)

Problem 13 The resistor values $R_1 = 100 \Omega$, $R_2 = 10 \Omega$, and $R_3 = 10 \Omega$ are in an AC network as shown in the following circuit. The voltage source is $u(t) = 3.0 \sin(2\pi \cdot 15 \text{ kHz} \cdot t) \text{ V}$. The current $i(t)$ is the current through the resistor R_3 . The high frequency approximation is $f \gg 1/(RC)$.

Solution

$$\begin{aligned} R_1 &= 1.00 \Omega \\ R_2 &= 10.0 \Omega \end{aligned}$$

Solution

A series circuit means that the current is constant on every component.

The equivalent impedance for R_2 and R_3 combined is given by $Z_{L2} = R_2 + jX_{L2}$. Parallel circuit means that the voltage is the same on R_2 and R_3 . $Z_{L2} = R_2 + jX_{L2} = R_2 + j\omega L_2$. Since L_2 is perpendicular to R_2 , this can be simplified to $Z_{L2} = R_2 + j\omega L_2$. It has to, since R_3 is perpendicular to Z_{L2} . Therefore, the resulting current of the parallel circuit is given as:

$$i_{L3} = i_{L2} + i_{L3} = \frac{u(t)}{Z_{L2}} + \frac{u(t)}{R_3} = \frac{3.0 \sin(2\pi \cdot 15 \text{ kHz} \cdot t)}{10 + j2\pi \cdot 15 \cdot 10^{-6}} + \frac{3.0 \sin(2\pi \cdot 15 \text{ kHz} \cdot t)}{10}$$

Back to the first formula: $R_3 \cdot i_{L3} = Z_{L3} \cdot i_{L3} = \sqrt{R^2 + X^2} \cdot i_{L3}$

Exercise E13 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ through the resistor R_3 . The voltage source is $u(t) = 3.0 \sin(2\pi \cdot 15 \text{ kHz} \cdot t) \text{ V}$. The current $i(t)$ is the current through the resistor R_3 . The high frequency approximation is $f \gg 1/(RC)$.

Solution

Result

$$Z = 48.2 \Omega \quad Z = 19.8 \Omega$$

Draw the circuit diagram of the given circuit. Label all components, voltages, and currents.

$$Z = \frac{u}{i} = \frac{3.0 \sin(2\pi \cdot 15 \text{ kHz} \cdot t)}{i}$$

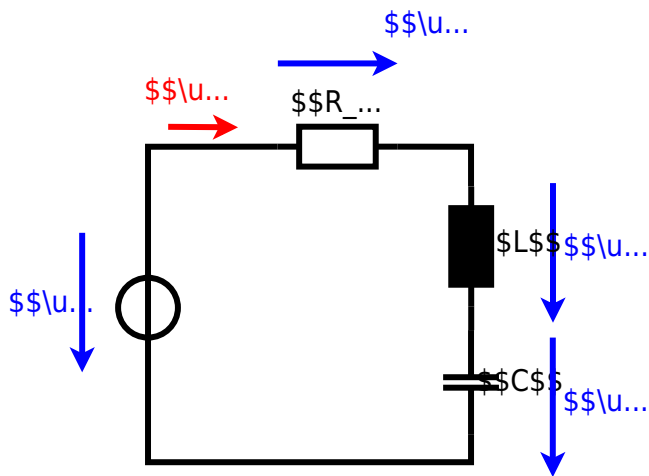
$$Z_C = \frac{1}{2\pi \cdot f \cdot C} = \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \mu\text{F}}$$

Result

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + (2\pi \cdot 15 \text{ kHz} \cdot 330 \mu\text{H} - \frac{1}{2\pi \cdot 15 \text{ kHz} \cdot 0.22 \mu\text{F}})^2}$$

$$\underline{Z} = R + jZ_L - jZ_C = R + j(\omega L - \frac{1}{\omega C})$$

$$\underline{Z} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$



Exercise E14 Complex Impedance Circuit
(written test, approx. 15 % of a 60-minute written test, WS2022)

1. Calculate the current $i(t)$ in the circuit shown in Fig. 1. The voltage source $u(t) = 3.0 \cdot \sin(2\pi \cdot 15 \cdot t)$ V is connected in series with an inductor of $330 \mu\text{H}$ and a capacitor of $0.22 \mu\text{F}$.

Result: $Z = 19.8 - j48.2 \Omega$

Draw the circuit diagram of the given circuit and label all components, voltages, and currents.

$$Z = \frac{U}{I} \quad I = \frac{U}{Z} \quad Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 2\pi \cdot 15 \cdot 0.22 \cdot 10^{-6}}$$

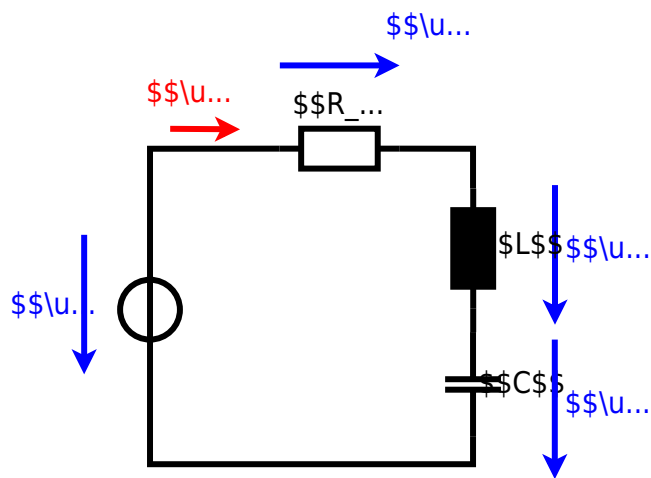
$$Z_L = j\omega L = j \cdot 2\pi \cdot 15 \cdot 330 \cdot 10^{-6}$$

$$Z = R + j\omega L - \frac{1}{j\omega C} = R + j(\omega L - \frac{1}{\omega C})$$

$$\underline{Z} = R + j(\omega L - \frac{1}{\omega C})$$

$$\underline{Z} = R + j(\omega L - \frac{1}{\omega C}) \quad |Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$





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