Block 06 - Complex Power

Student Group

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Learning objectives

> After this 90-minute block, you
>
> 1. Know the formula of the instantaneous power of the resistor, inductor, and capacitor and be able to determine its values.

Preparation at Home

Well, again

- read through the present chapter and write down anything you did not understand.
- Also here, there are some clips for more clarification under 'Embedded resources' (check the text above/below, sometimes only part of the clip is interesting).

For checking your understanding please do the following exercises:

- …

90-minute plan

1. Warm-up (x min):
    1. ….
2. Core concepts & derivations (x min):
    1. …
3. Practice (x min): …
4. Wrap-up (x min): Summary box; common pitfalls checklist.

Conceptual overview

> 1. …

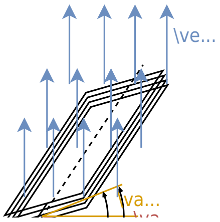
Core content

Last semester in block18 we investigated induciton effects have into coils. When the coil is rotating constantly, These can be considered as the AC voltace source on a circuit:

- the sinusoidal alternating voltage is produced by the rotation of a coil in a homogeneous magnetic field, and
- the sinusoidal alternating current is formed by a connected load (or complex impedance).

This will be briefly illustrated here. In figure 1 a coil with w windings is seen in a magnetic field with a magnetic flux density $\vec{B}$. The coil rotates - starting from φ_0 with angular velocity ω. The rotation changes the chained flux Ψ through the coil and thus a voltage $u(t)$ is induced.

Fig. 1: voltage generation in generator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For the rotation angle φ holds: \begin{align*} \varphi(t)=\omega t + \varphi_0 \quad \text{with} \quad \varphi_0=\varphi(t=0) \\ \end{align*}

Thus, the induced voltage $u(t)$ is given by: \begin{align*} u(t) &= -\frac{{\rm d} \Psi} {{\rm d}t} \\ &= -N \cdot \frac{{\rm d} \Phi} {{\rm d}t} \\ &= -NBA\cdot \frac{{\rm d} \cos \varphi(t)} {{\rm d}t} \\ &= -\hat{\Psi}\cdot\frac{{\rm {\rm d}} \cos (\omega t + \varphi_0)}{{\rm d}t} \\ &= \omega \hat{\Psi} \cdot \sin (\omega t + \varphi_0) \\ &= \hat{U} \cdot \sin (\omega t + \varphi_0) \\ \end{align*}

Such single-phase systems are therefore alternating current systems, which use one outgoing line and one return line each for the current conduction.
Out of the last formula we derived the following instantaneous voltage $u(t)$ \begin{align*} u(t) &= \hat{U} \cdot \sin (\omega t + \varphi_0) \\ &= \sqrt{2} U\cdot \sin (\omega t + \varphi_0) \\ \end{align*}

In block03, we used DC power $P = U \cdot I$ and compared it with the instantaneous power $p(t)$ of an AC circuit:

\begin{align*} p(t) &= \color{blue}{u(t)} \cdot \color{red}{i(t)} \end{align*}

Additionally we used a bit later in block03 the functions representing the instantaneous signals to derive the inductance: $x(t)= \sqrt{2}{X}\cdot \sin(\omega t + \varphi_x)$

Now, we will combine both to analyze the AC power on the resistor, capacitor and inductivity in more detail.

Ideal Ohmic resistance R

The simplest component to look at for the instantaneous power is the resistor. For this, we start with the basic definition of the instantaneous voltage $u_R(t)$ (which was given in the last semester) as

\begin{align*} \color{blue}{u_R(t)} &= \sqrt{2}U \sin(\omega t + \varphi_u) \end{align*}

With the defining formula for the resistor, we get:

\begin{align*} \color{blue}{u_R(t)} &= R \cdot \color{red}{i(t)} \\ \rightarrow \color{red}{i(t)} &= {{\color{blue}{u_R(t)}}\over{R}} \\ &= \sqrt{2}{ {U} \over{R}} \sin(\omega t + \varphi_u) \end{align*}

This leads to an instantaneous power $p_R(t)$ of \begin{align*} p_R(t) &= \color{blue}{u_R(t)} \cdot \color{red}{i_R(t)} \\ &= 2\cdot {{U^2}\over{R}} \sin^2(\omega t + \varphi_u) \\ &= {{U^2}\over{R}}\left(1- \cos\left(2\cdot (\omega t + \varphi_u)\right)\right) \\ \end{align*}

For the last step the Double-angle formula “$\cos(2x) = 1 - 2 \sin^2(x)$” was used.

This result is interesting in the following ways:

1. The part $1- \cos (2\cdot (\omega t + \varphi_u))$ is always non-negative and a shifted sinusoidal function between $0...2$. The average value of this part is 1.
2. The average value of $p_R(t)$ is then: $P_R = {{U^2}\over{R}}$
3. The use of the $\sqrt{2}$ in the definition $\color{blue}{u_R(t)} = \sqrt{2} U \sin(\omega t + \varphi_u)$ leads to the average power as $P_R = {{U^2}\over{R}}$. This formula for the power is exactly like the formula for the power in pure DC situations.

Ideal Inductivity L

A similar approach is done for the ideal inductivity. We again start with the basic definition of the instantaneous voltage

\begin{align*} \color{blue}{u_{\rm L}(t)} &= \sqrt{2}U \sin(\omega t + \varphi_u) \end{align*}

With the defining formula for inductivity, we get: \begin{align*} \color{blue}{u_{\rm L}(t)} &= L\cdot {{{\rm d}\color{red} {i_{\rm L}(t)}}\over{{\rm d}t}} \\ \rightarrow \color{red}{i_{\rm L}(t)} &= {{1}\over{L}} \int \color{blue}{u_{\rm L}(t)} {\rm d}t \\ &= - \sqrt{2} {{U}\over{\omega L}} \cos(\omega t + \varphi_u) \end{align*}

This leads to an instantaneous power $p_L(t)$ of

\begin{align*} p_L(t) &= \color{blue}{u(t)} \cdot \color{red}{i(t)} \\ &= - 2\cdot {{U^2}\over{\omega L}} \cdot \sin(\omega t + \varphi_u) \cos(\omega t + \varphi_u) \\ &= - {{U^2}\over{\omega L}} \cdot \sin(2\cdot (\omega t + \varphi_u)) \\ \end{align*}

Again a trigonometric identity (Double-angle formula "$\sin(2x) = 2 \sin(x)\cos(x)$") was used.

Also, this result is interesting:

1. The part $\sin(2\cdot (\omega t + \varphi_u))$ has an average value of 0.
2. Therefore, the average value of $p_L(t)=0$

Ideal Capacity C

Also here, we start with the basic definition of the instantaneous voltage

\begin{align*} \color{blue}{u_C(t)} &= \sqrt{2}U \sin(\omega t + \varphi_u) \end{align*}

With the defining formula for the capacity, we get: \begin{align*} \color{red}{i_C(t)} &= C {{{\rm d}\color{blue}{u_C(t)}}\over{{\rm d}t}} \\ &= \sqrt{2} U \omega C \cos(\omega t + \varphi_u) \end{align*}

This leads to an instantaneous power $p_C(t)$ of

\begin{align*} p_C(t) &= \color{blue}{u_C(t)} \cdot \color{red}{i_C(t)} \\ &= 2\cdot U^2 \omega C \cdot \sin(\omega t + \varphi_u) \cos(\omega t + \varphi_u) \\ &= + U^2 \omega C \cdot \sin(2\cdot (\omega t + \varphi_u)) \\ \end{align*}

Again this result leads to:

1. The part $\sin(2\cdot (\omega t + \varphi_u))$ has an average value of 0.
2. Therefore, also the average value of $p_C(t)=0$

- Instantaneous values of power at R, L, C
- Active, reactive, apparent, and complex power

This effect can also be seen in the following simulation: The simulation shows three loads, all with an impedance of $|Z| = 1 ~\rm k\Omega$. The diagram on top of each circuit shows the instantaneous **voltage**, **current** and **power**.

1. Ohmic load: The instantaneous voltage is in phase with the instantaneous current. The

instantaneous power is always non-negative. The average power is $P=U^2/R = {{1}\over{2}} \hat{U}^2/R= {{1}\over{2}}(6V)^2/1 ~\rm k\Omega = 18 ~\rm mW$

2. Inductive load: The voltage is ahead of the current. The phase angle is $+90°$ (which also reflects the $+{\rm j}$ in the inductive impedance $+{\rm j}\omega L$). The instantaneous is half positive, half negative; the average power is zero (in the simulation not completely visible).
3. Capacitive load: The voltage is lagging the current. The phase angle is $-90°$ (which also reflects the $-{\rm j}$ in the capacitive impedance ${{1}\over{{\rm j}\omega C}}$). The instantaneous is again half positive, half negative; the average power is zero (in the simulation not completely visible).

arbitrary two-terminal Component

For an arbitrary component, we do not have any defining formula. But, the $u(t)$ and $i(t)$ can generally be defined as:

\begin{align*} \color{blue}{u(t)} &= \sqrt{2}U \sin(\omega t + \varphi_u) \\ \color{red }{i(t)} &= \sqrt{2}I \sin(\omega t + \varphi_i) \\ \end{align*}

This leads to an instantaneous power $p(t)$ of

\begin{align*} p(t) &= \color{blue}{u(t)} \cdot \color{red}{i(t)} \\ &= 2\cdot UI \sin(\omega t + \varphi_u) \sin(\omega t + \varphi_i) \\ \end{align*}

The formula can be further simplified with the help of the following equations

- $\varphi = \varphi_u - \varphi_i \quad \rightarrow \varphi_i = \varphi_u - \varphi$
- $ \sin(\Box - \varphi) = \sin(\Box) \cos \varphi - \cos(\Box) \sin \varphi $
- $2\sin\Box \sin\Box = 1 - \cos(2\Box)$
- $2\sin\Box \cos\Box = \sin(2\Box)$

\begin{align*} p(t) = UI\big(\cos\varphi \left(1- \cos(2(\omega t + \varphi_u)\right) - \sin\varphi \cdot \sin(2(\omega t + \varphi_u) \big) \\ \end{align*}

This result is twofold:

1. The part $ \cos\varphi \; \cdot \; \left(1- \cos(2(\omega t + \varphi_u)\right)$ results into a non-zero average - explicitly this part is 1 in average. On average the first part of the formula results in $UI \cos\varphi$.
2. The part $- \sin\varphi \; \cdot \; \sin(2(\omega t + \varphi_u))$ is zero on average, so the second part of the formula results in zero. The amplitude of the second part is $UI \sin\varphi$

Notice:

A distinction is now made between:

- An **active power** (alternatively real or true power, in German: *Wirkleistung*): $P = UI \cos \varphi$
    - The active power represents a pulsed energy drain out of the electrical system (commonly by an ohmic resistor).
    - The active power transforms the electric energy permanently into thermal or

    - mechanical energy
    - Therefore, the unit of the active power is $\rm Watt$.
- A **reactive power** (in German: *Blindleistung*): $Q = UI \sin \varphi$
    - The reactive power describes the “sloshing back and forth” of the energy into the electric and/or magnetic fields.
    - The reactive power is completely regained by the electric circuit.
    - To distinguish the values, the unit of the reactive power is $\rm VAr$ (or $\rm Var$) for **V** olt**a** mpere-**r** eactive.
- An **apparent power** (in German *Scheinleistung*): $S = UI $
    - The apparent power is the simple multiplication of the RMS values from the current and the voltage.
    - The apparent power shows only what seems to be a value of power, but can deviate from usable power when inductors or capacitors are used in the circuit.
    - The unit of the apparent power is $\rm VA$ for **V**olt**a**mpere

Similarly, the currents and voltages can be separated into active, reactive, and apparent values.

Based on the given formulas the three types of power are connected with each other. Since the apparent power is given by $S=U\cdot I$, the active power $P = U\cdot I \cdot \cos \varphi = S \cdot \cos \varphi $ and the reactive power $Q = S \cdot \sin \varphi $, the relationship can be shown in a triangle (see figure 2).

Fig. 2: Power Triangle of active, reactive and apparent power

Generally, the apparent power can also be interpreted as a complex value:

$$\begin{align*} \underline{S} &= S \cdot {\rm e}^{{\rm j}\varphi} \\ &= U \cdot I \cdot {\rm e}^{{\rm j}\varphi} \end{align*}$$

Based on the definition of the phase angle $\varphi = \varphi_U - \varphi_I$, this can be divided into:

$$\begin{align*} \underline{S} &= U \cdot I \cdot {\rm e}^{ {\rm j}(\varphi_U - \varphi_I)} \\ &= \underbrace{U \cdot {\rm e}^{{\rm j}\varphi_U}}_{\underline{U}} \cdot \underbrace{I \cdot {\rm e}^{-{\rm j}\varphi_I}}_{\underline{I}^*} \end{align*}$$

where $\underline{I}^*$ is the complex conjungated value of $\underline{I}$.

Notice:

The apparent power $\underline{S} $ is given by:

- $\underline{S} = UI \cdot {\rm e}^{{\rm j}\varphi}$
- $\underline{S} = UI \cdot (\cos\varphi + {\rm j} \sin\varphi)$
- $\underline{S} = P + {\rm j}Q$
- $\underline{S} = \underline{U} \cdot \underline{I}^*$

The following simulation shows three ohmic-inductive loads, all with an impedance of $|Z| = 1 ~\rm k\Omega$, however with different phase angles φ. The diagram on top of each circuit shows the instantaneous **voltage**, **current** and **power**. Similar to the last simulation, a pure ohmic resistance would consume an average power of $P=U^2/R = {{1}\over{2}} \hat{U}^2/R= {{1}\over{2}}(6~\rm V)^2/1~\rm k\Omega = 18~\rm mW$. The three diagrams shall be discussed shortly.

1. Phase angle $\varphi = 10°$: Nearly all of the impedance is given by the resistance and therefore the real part of the impedance. The instantaneous voltage is nearly in phase with the current. The instantaneous power is almost always larger than zero. The average power with $17.47~\rm mW$ is about the same as for an ohmic impedance.
2. Phase angle $\varphi = 60°$: It is clearly visible, that instantaneous voltage and current are out of phase. The instantaneous power is often lower than zero. The ohmic resistor has $500 ~\rm \Omega = {{1}\over{2}}|Z|$, but does not show half of the voltage! This is because the addition has to respect the complex behavior of the values. The complex part is $90°$ perpendicular to the real part - so they generate a right-angled triangle. The average power with $9~\rm mW$ is exactly half of the power for an ohmic impedance since only the resistance provides a way for consuming power permanently.
3. Phase angle $\varphi = 84.28°$: The phase angle is calculated in such a way, that the resistance is only 10% of the amplitude of the impedance $|Z|$. In this case, the load is nearly pure inductive. The instantaneous power is consequently almost half of the time lower than zero. The average power here is also only $10%$ of the power for a pure ohmic impedance.

The next simulation enables us to play around with the phase angle of an impedance. The circuit on the left side is a bit harder to understand but consists of a resistive (real) impedance and a complex impedance, which are driven by an AC voltage source. All of these components are parameterizable in such a way that the phase angle can be manipulated by the slider on the right side.
In the middle part reflects the time course of:

- The instantaneous power p of the **real part (active power)**, the **imaginary part (reactive power)** and **overall power**.
- The instantaneous **voltage** and **current**.

On the right-hand side, the impedance Phasor is shown (lower diagram). The upper diagram depicts the u-i-diagram, which would be a perfect line for a pure ohmic resistance (since $u_R = R \cdot i_R$) and a circle for a pure complex impedance (since the phase angle of $\pm 90°$ between $u_{L, C}$ and $i_{L, C}$). The simulation is in this part not completely perfect: The pure line and circle are sometimes not reachable.

The following questions can be solved with this simulation:

1. How does the amplitude of the active and reactive instantaneous power change, when the phase angle is changed between $-90°...+90°$?
2. What is the phase shift between the active and reactive instantaneous power?

1)

Also, the last simulation shows the relation between the phase angle (here: α) and instantaneous values, like power, voltage, and current.

Fig. 3: Simulation of instantaneous power as a function of phase

Change the phase angle with the slider under α
press here for the animation

Applications

Power Factor Correction

Cables and components have to conduct the sum of active and reactive currents, but only the active current is used outside of the circuit. Therefore, a common goal is to minimize the reactive part. The technical way to represent this is the **power factor** pf is used.

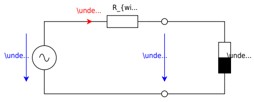
Notice:
The power factor is given by:

\begin{align*} pf &= \cos \varphi \\ &= {{P}\over{|\underline{S}|}} \end{align*}

The power factor shows how much real power one gets out of the needed apparent power.

How does the power factor show the problematic effects? For this one can investigate the situation of an ohmic-inductive load $\underline{Z}_L$ which is connected to a voltage $\underline{U}_0$ source with a wire R_{wire}. This circuit is shown in figure 4.

Fig. 4: Power Factor of a Power Line



The usable output power is $P_L = U_{\rm L} \cdot I \cdot \cos \varphi$. Based on this, the current $\underline{I}$ is:

\begin{align*} I = {{P_L}\over{U_{\rm L} \cdot \cos \varphi}} \end{align*}

The power loss of the wire P_{wire} is therefore:

\begin{align*} P_{\rm wire} &= R_{\rm wire} \cdot I^2 \\ &= R_{\rm wire} \cdot {{P_L^2}\over{U_{\rm L}^2 \cdot \cos^2 \varphi}} \end{align*}

This means: As smaller, the power factor $\cos \varphi$, as more power losses $P_{\rm wire}$ will be generated. More power losses $P_{\rm wire}$ lead to more heat up to or even beyond the maximum temperature. To compensate for this, the cross-section of the wire has to be increased, which means more copper.

Alternatively, a bad power factor can be compensated with a counteracting complex impedance. This compensating impedance has to provide enough power with the opposite sign to cancel out the unwanted reactive power. The following simulation shows an uncompensated circuit and a circuit with power factor correction. In the ladder, the voltage on the load resistor is the same, but the current provided by the power supply is smaller.

Another explanation of the power factor can be seen here:

Video

Impedance matching

not covered in this course

Common pitfalls

- …

Exercises

Worked examples

Exercise 7.1.1 Power and Power Factor I

A passive component, fed by a sinusoidal AC voltage based on the RMS value $U=230~\rm V$ and $f=50.0~\rm Hz$. The RMS current on this component is $I=5.00~\rm A$ with a phase angle of $\varphi=+60°$.

1. Calculate the active, reactive and apparent power.

Solution

Result

active power: \begin{align*} P &= U \cdot I \cdot \cos \varphi \\ &= 230{~\rm V} \cdot 5{~\rm A} \cos 60° \\ &= 575 {~\rm W} \end{align*}

reactive power: \begin{align*} Q &= U \cdot I \cdot \sin \varphi \\ &= 230{~\rm V} \cdot 5{~\rm A} \sin 60° \\ &= 996 {~\rm var} \end{align*}

apparent power: \begin{align*} S &= U \cdot I \\ &= 230{~\rm V} \cdot 5{~\rm A} \\ &= 1150{~\rm VA} \end{align*}

2. Draw the equivalent circuits based on a series and a parallel circuit of two components.

Solution

Result

For the series circuit: \begin{align*} R_s &= {23 ~\Omega} \\ L_s &= {127 ~\rm mH} \\ \end{align*}

For the parallel circuit: \begin{align*} R_p &= {92 ~\Omega} \\ L_p &= {169 ~\rm mH} \\ \end{align*}

The apparent impedance is: \begin{align*} |\underline{Z}| &= {{U}\over{I}} = {{230~\rm V}\over{5~\rm A}} = 46 ~\Omega \\ \end{align*}

By this, the following can be derived: \begin{align*} \underline{S} &= U \cdot I \cdot e^{{\rm j}\varphi} \\ &= Z \cdot I^2 \cdot e^{{\rm j}\varphi} &&= \underline{Z} \cdot I^2 \\ &= {{U^2}\over{Z}} \cdot e^{{\rm j}\varphi} &&= {{U^2}\over{\underline{Z}^{*}}} \\ \end{align*}

These formulas are handy for both types of circuits to separate the apparent power into real part (real power) and complex part (apparent power):

1. for **series circuit**: $\underline{S} = \underline{Z} \cdot I^2$ with $\underline{Z} = R + {\rm j} X_L$
2. for **parallel circuit**: $\underline{S}

For the **series circuit**, the impedances add up like: $R_s + {\rm j}\cdot X_{Ls} = \underline{Z} $, and $R_s = |\underline{Z}| \cos\varphi$ such as $X_{Ls} = |\underline{Z}| \sin\varphi$. Therefore: \begin{align*} R_s &=& {{U}\over{I}} \cdot \cos \varphi &=& {{230~\rm V}\over{5.00~\rm A}} \cdot \cos 60° &=& \boldsymbol{23.0 ~\Omega} & \\ X_{Ls} &=& {{U}\over{I}} \cdot \sin \varphi &=& {{230~\rm V}\over{5.00~\rm A}} \cdot \sin 60° &=& 39.8 ~\Omega &= \omega \cdot L_s \\ \rightarrow L_s &=& {{X_{Ls}}\over{2\pi f}} &=& {{{{U}\over{I}} \cdot \sin \varphi}\over{2\pi f}} &=& \boldsymbol{127 ~\rm mH} \end{align*}

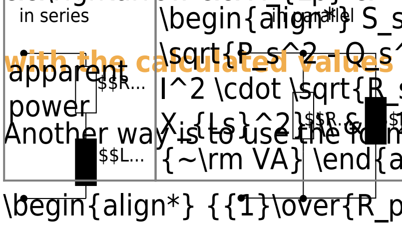
For the **parallel circuit**, the impedances add up like $ {{1}\over{\underline{Z}^{}}} = {{1}\over{R}} + {{1}\over{{\rm j}\cdot X_{L}}} $ with $\underline{Z} = {{U}\over{I}} \cdot e^{j\cdot \varphi}$.

= {{U^2}\over{\underline{Z}^{*}}} $ with ${{1}\over{\underline{Z}^{*}}} = {{1}\over{R}} - {{1}\over{{\rm j}\cdot X_L}} \rightarrow {{1}\over{\underline{Z}^{*}}} } = {{1}\over{R}} + {{\rm j}\over{ X_L}} $

There are multiple ways to solve this problem. Two ways shall be shown here:

Therefore:

	series circuit	**parallel circuit**
active power	\begin{align*} P_s &= R_s \cdot I^2 \\ &= 23 ~\Omega \cdot (5{~\rm A})^2 \\ &= 575 {~\rm W} \end{align*}	\begin{align*} P_p &= {{U_p^2}\over{R_p}} \\ &= {{(230{~\rm V})^2}\over{92~\Omega}} = 575 {~\rm W} \end{align*}
reactive power	\begin{align*} Q_s &= X_{Ls} \cdot I^2 \\ &= 39.8 ~\Omega \cdot (5{~\rm A})^2 \\ &= 996 {~\rm Var} \end{align*}	\begin{align*} Q_p &= {{U_p^2}\over{X_{Lp}}} \\ &= {{(230{~\rm V})^2}\over{53.1 ~\Omega}} = 996 {~\rm Var} \end{align*}
apparent power	\begin{align*} S_s &= \sqrt{P_s^2 - Q_s^2} \\ &= I^2 \cdot \sqrt{R_s^2 + X_{Ls}^2} = 1150 {~\rm VA} \end{align*}	\begin{align*} S_p &= \sqrt{P_s^2 - Q_s^2} \\ &= U^2 \cdot \sqrt{{{1}\over{R_p^2}} + {{1}\over{X_{Lp}^2}}} \\ &= 1150 {~\rm VA} \end{align*}



With the Euler representation

Given the formula $\underline{Z} = {{U}\over{I}} \cdot e^{j\cdot \varphi}$ the following can be derived: \begin{align*} {{1}\over{\underline{Z}^{}}} &= {{I}\over{U}} \cdot e^{-j\cdot \varphi} \\ &= {{1}\over{Z}}\cdot e^{-j\cdot \varphi} &&= {{1}\over{R_p}} + {{1}\over{{\rm j}\cdot X_{Lp}}} \\ &= {{1}\over{Z}}\cdot \left(\cos(\varphi) - {\rm j}\cdot \sin(\varphi) \right) &&= {{1}\over{R_p}} - {{\rm j}\over{X_{Lp}}} \\ \end{align*}

Therefore, the following can be concluded: \begin{align*} {{1}\over{Z}}\cdot \cos(\varphi) &= {{1}\over{R_p}} &&\rightarrow && R_p &= {{Z}\over{\cos(\varphi)}} \\ - {{1}\over{Z}}\cdot {\rm j}\cdot \sin(\varphi) &= - {{\rm j}\over{ X_{Lp}}} &&\rightarrow && X_{Lp} &= {{Z}\over{\sin(\varphi)}} \\ \end{align*}

with the calculated values of the series circuit

Another way is to use the formulas of R_s and X_{Ls} from before.

\begin{align*} {{1}\over{R_p}} + {{1}\over{{\rm j}\cdot X_{Lp}}} &=& {{1}\over{R_s + {\rm j}\cdot X_{Ls}}} \\ {{1}\over{R_p}} - {\rm j} {{1}\over{X_{Lp}}} &=& {{R_s - {\rm j}\cdot X_{Ls}}\over{R_s^2 + X_{Ls}^2}} \\ &=& {{Z \cdot \cos \varphi - {\rm j}\cdot Z \cdot \sin \varphi }\over{Z^2}} \\ &=& { {\cos \varphi - {\rm j} \cdot \sin \varphi } \over{Z}} \\ \end{align*}

Therefore

Now, the real and imaginary part is analyzed individually. First the real part:

\begin{align*} {{1}\over{R_p}} &=& {{\cos \varphi}\over{Z}} \\ \rightarrow R_p &=& {{Z}\over{\cos \varphi}} &=& {{46 ~\Omega}\over{\cos 60°}} \end{align*}

\begin{align*} {{1}\over{X_{Lp}}} &= {{\sin \varphi}\over{Z}} & \\ \rightarrow X_{Lp} &= {{Z}\over{\sin \varphi}} = {{46 ~\Omega}\over{\sin 60°}} \\ \rightarrow L_p &= {{46 ~\Omega}\over{2\pi \cdot 50~\rm Hz \cdot \sin 60°}} \end{align*}

Exercise 7.1.2 Power and Power Factor II

A magnetic coil shows at a frequency of $f=50.0 {~\rm Hz}$ the voltage of $U=115{~\rm V}$ and the current $I=2.60{~\rm A}$ with a power factor of $\cos \varphi = 0.30$

1. Calculate the real power, the reactive power, and the apparent power.
2. Draw the equivalent parallel circuit. Calculate the active and reactive part of the current.

3. Draw the equivalent series circuit. Calculate the ohmic and inductive impedance and the value of the inductivity.

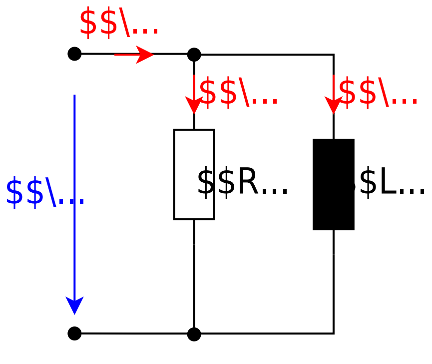
Result for 1.

The real power is \begin{align*} P &= U \cdot I \cdot \cos \varphi \\ &= 115{~\rm V} \cdot 2.6{~\rm A} \cdot 0.3 \\ &= 89.7 {~\rm W} \end{align*}

The reactive power is \begin{align*} Q &= U \cdot I \cdot \sin \varphi \\ &= 115{~\rm V} \cdot 2.6{~\rm V} \cdot \sqrt{1 - 0.3^2} \\ &= 285 {~\rm Var} \end{align*}

The apparent power is \begin{align*} S &= U \cdot I \\ &= 115{~\rm V} \cdot 2.6{~\rm A} \\ &= 299 {~\rm VA} \end{align*}

Result for 2.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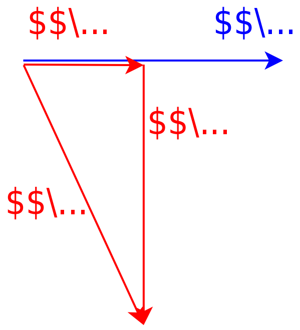
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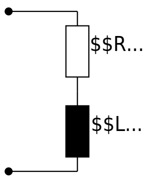
The complex current $\underline{I}$ is given as:

\begin{align*} \underline{I} &= I_R + {\rm j} \cdot I_L \\ &= I \cdot \cos\varphi - {\rm j} \cdot I \cdot \sin\varphi \end{align*}

The active and reactive part of the current is therefore: \begin{align*} I_R &= 2.60{~\rm A} \cdot 0.30 &= 0.78 {~\rm A} \\ I_L &= - 2.60{~\rm A} \cdot \sqrt{1 - 0.30^2} &= 2.48 {~\rm A} \end{align*}

Result for 3.

Important: The cosine function is ambiguous! Based on $\cos \varphi = 0.30$ it is unclear, whether φ is positive or negative.
Therefore, only based on the power factor it is unclear whether the circuit is ohmic-inductive or ohmic-capacitive! However, this is explicitly given in the problem definition.



\begin{align*} Z_s &= {{U}\over{I}} &=& {{115{~\rm V}}\over{2.60{~\rm A}}} &=& 44.2 ~\Omega \\ R_s &= {{U}\over{I}} \cdot \cos \varphi &=& {{115{~\rm V}}\over{2.60{~\rm A}}} \cdot 0.30 &=& 13.3 ~\Omega \\ X_{Ls} &= {{U}\over{I}} \cdot \sin \varphi &=& {{115{~\rm V}}\over{2.60{~\rm A}}} \cdot \sqrt{1 - 0.30^2} &=& 42.2 ~\Omega \\ \\ L_s &= 134~\rm mH \end{align*}

Exercise 7.1.3 Power and Power Factor III

A consumer is connected to a $220~\rm V$ / $50 ~\rm Hz$ network. A current of $20.0~\rm A$ and a power of $1800 ~\rm W$ is measured.

1. What is the value of the active power, the reactive power, and the power factor?
2. Assume that the consumer is a parallel circuit.
    1. Calculate the resistance and reactance.
    2. Calculate the necessary inductance/capacitance.
3. Assume that the consumer is a series circuit.
    1. Calculate the resistance and reactance.
    2. Calculate the necessary inductance/capacitance.

Result for 1.

The active power is $P = 1.80 ~\rm kW$.

The apparent power is $S = U \cdot I = 220 ~\rm V \cdot 20 ~\rm A = 4.40 ~\rm kVA$.

The reactive power is $Q = \sqrt{S^2 - P^2} = \sqrt{(4.40 ~\rm kVA)^2 - (1.80 ~\rm kW)^2} = 4.01 ~\rm kVar$

The power factor is $\cos \varphi = {{P}\over{S}} = {{1.80 ~\rm kW}\over{4.40 ~\rm kVA}} = 0.41$.

Result for 2.

Important: The cosine function is ambiguous! Based on $\cos \varphi = 0.30$ it is unclear, whether φ is positive or negative.
Therefore, only based on the power factor it is unclear whether the circuit is ohmic-inductive or ohmic-capacitive!

The consumer is a parallel circuit of the resistance $R_\rm p$ and the reactance $X_\rm p$ on the voltage U. Both values can be calculated based on the real and reactive power: \begin{align*} P &= {{U^2}\over{R_\rm p}} \rightarrow & R_p &= {{U^2}\over{P}} &= 26.9 ~\Omega \\ Q &= {{U^2}\over{X_\rm p}} \rightarrow & X_p &= {{U^2}\over{Q}} &= 12.1 ~\Omega \\ \end{align*}

The respective values for inductance/capacitance are: \begin{align*} L &= {{X_p}\over{2\pi \cdot f}} &= 38.4 {~\rm mH} \\ C &= {{1}\over{2\pi \cdot f \cdot X_p}} &= 263 ~\rm µF \\ \end{align*}

Result for 3.

The consumer is a series circuit of the resistance $R_\rm s$ and the reactance $X_\rm s$ with the current I. Both values can be calculated based on the real and reactive power: \begin{align*} P &= I^2 \cdot R_{\rm s} \rightarrow & R_{\rm s} &= {{P}\over{I^2}} &= 4.50 ~\Omega \\ Q &= I^2 \cdot X_{\rm s} \rightarrow & X_{\rm s} &= {{Q}\over{I^2}} &= 10.0 ~\Omega \\ \end{align*}

The respective values for inductance/capacitance are: \begin{align*} L &= {{X_{\rm s}}\over{2\pi \cdot f}} &= 31.9 {~\rm mH} \\ C &= {{1}\over{2\pi \cdot f \cdot X_{\rm s}}} &= 318 ~\rm µF \\ \end{align*}

Exercise 7.1.4 Power and Power Factor IV

An uncompensated ohmic-inductive series circuit shows at $U=230~\rm V$, $f=50 ~\rm Hz$ the current $I_{RL}=7 ~\rm A$, $P_{RL}=1.3 ~\rm kW$

The power factor shall be compensated to $\cos\varphi = 1$ via a parallel compensation.

1. Calculate the apparent power, the reactive power, the phase angle, and the power factor before the compensation.
2. Calculate the capacity C which has to be connected in parallel to get $\cos\varphi=1$.

Solution

\begin{align*} S &= U \cdot I_{RL} \\ Q &= \sqrt{S^2 - P_{RL}^2} \\ \varphi &= \arctan\left({{Q}\over{P}}\right) = \arccos\left({{P}\over{S}}\right) \end{align*}

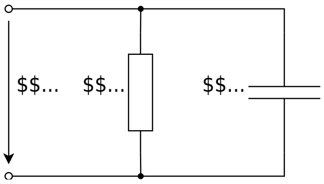
The inductor L creates the reactive power $Q = Q_L$. To compensate for a equivalent reactive power $|Q_C| = |Q_L|$ has to be given by a capacitor. The reactive power is given by: \begin{align*} Q &= \Re (U) \cdot \Im (I) \\ &= U \cdot {{U}\over{X}} \\ &= {{U^2}\over{X}} \\ \end{align*}

The capacity can therefore be calculated by \begin{align*} X_C &= {{U^2}\over{Q_L}} = {{1}\over{\omega C}} \quad \rightarrow \quad C = {{1}\over{\omega U^2}} \end{align*}

Result

\begin{align*} S &= 1.62 {~\rm kVA} \\ Q &= 0.95 {~\rm kVAr} \\ \varphi &= +36° \\ \\ C &= 57.2 ~\rm µF \end{align*}

Exercise E1 Mains Input of a Control Cabinet: Resistor-Capacitor Parallel Branch



At the input of an industrial control cabinet, an RC parallel branch is connected to the AC mains. The resistor represents a damping and discharge path, while the capacitor models an EMI suppression capacitor. For thermal design and power-quality assessment, the effective currents and powers of this branch shall be determined.

Data: \begin{align*} u_1 &= \hat U_1 \cos(\omega t) \\ \hat U_1 &= 325~{\rm V} \\ f &= 50~{\rm Hz} \\ R &= 220~{\rm \Omega} \\ C &= 4.7~{\rm \mu F} \end{align*}

1. Determine the RMS values of the voltage U_1 and of the currents I, I_R, and I_C.

SolutionResult

First convert the voltage amplitude into the RMS value: \begin{align*} U_1 &= \frac{\hat U_1}{\sqrt{2}} = \frac{325~{\rm V}}{\sqrt{2}} = 229.8~{\rm V} \end{align*}

The angular frequency is \begin{align*} \omega &= 2\pi f = 2\pi \cdot 50~{\rm s^{-1}} = 314.16~{\rm s^{-1}} \end{align*}

The resistor current is \begin{align*} I_R &= \frac{U_1}{R} = \frac{229.8~{\rm V}}{220~{\rm \Omega}} = 1.045~{\rm A}

\begin{align*} U_1 &= 229.8~{\rm V} \\ I_R &= 1.045~{\rm A} \\ I_C &= 0.339~{\rm A} \\ I &= 1.098~{\rm A} \end{align*}

\end{align*}

The capacitor current is \begin{align*} I_C &= U_1 \omega C \\ &= 229.8~{\rm V}\cdot 314.16~{\rm s^{-1}}\cdot 4.7\cdot 10^{-6}~{\rm F} \\ &= 0.339~{\rm A} \end{align*}

Because I_R and I_C are perpendicular in the phasor diagram, the total current is \begin{align*} I &= \sqrt{I_R^2+I_C^2} \\ &= \sqrt{(1.045~{\rm A})^2 + (0.339~{\rm A})^2} \\ &= 1.098~{\rm A} \end{align*}

2. What active, reactive, and apparent power does the circuit absorb?

SolutionResult

The resistor absorbs only active power: \begin{align*} P &= \frac{U_1^2}{R} = \frac{(229.8~{\rm V})^2}{220~{\rm \Omega}} \\ &= 240.1~{\rm W} \end{align*}

The capacitor absorbs only reactive power. For a capacitor, the reactive power is negative: \begin{align*} Q &= -U_1^2 \omega C \\ &= -(229.8~{\rm V})^2 \cdot 314.16~{\rm s^{-1}} \cdot 4.7\cdot 10^{-6}~{\rm F} \\ &= -78.0~{\rm var} \end{align*}

The apparent power is \begin{align*} S &= U_1 I \\ &= 229.8~{\rm V}\cdot 1.098~{\rm A} \\ &= 252.4~{\rm VA} \end{align*}

Check with the power triangle:

\begin{align*} P &= 240.1~{\rm W} \\ Q &= -78.0~{\rm var} \\ S &= 252.4~{\rm VA} \end{align*}

\begin{align*} S &= \sqrt{P^2+Q^2} = \sqrt{(240.1~{\rm W})^2+(-78.0~{\rm var})^2} \\ &= 252.4~{\rm VA} \end{align*}

3. Determine the maximum and minimum value of the instantaneous power.

SolutionResult

For a sinusoidal AC circuit, the instantaneous power can be written as \begin{align*} p(t) = P + S\cos(2\omega t + \varphi) \end{align*} where P is the active power, S is the apparent power, and φ is the phase angle.

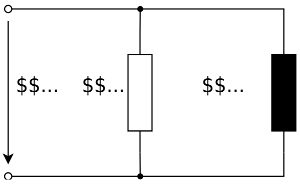
Thus the oscillating part has the amplitude S, so \begin{align*} p_{\rm max} &= P + S \\ p_{\rm min} &= P - S \end{align*}

Insert the values: \begin{align*} p_{\rm max} &= 240.1~{\rm W} + 252.4~{\rm W} = 492.5~{\rm W} \\ p_{\rm min} &= 240.1~{\rm W} - 252.4~{\rm W} = -12.3~{\rm W} \end{align*}

The negative minimum value means that for a short time interval, reactive energy is fed back from the capacitor to the source.

\begin{align*} p_{\rm max} &= 492.5~{\rm W} \\ p_{\rm min} &= -12.3~{\rm W} \end{align*}

Exercise E2 AC Solenoid Branch of a Valve Driver: Parallel Resistor-Inductor Circuit



An industrial valve driver contains an AC solenoid branch that can be modeled by a resistor in parallel with an inductor. The resistor represents electrical losses, while the inductive branch represents the magnetizing behavior of the actuator. For thermal design and grid-side power assessment, the RMS currents and powers of the branch shall be determined.

Data: \begin{align*} u_1 &= \hat U_1 \cos(\omega t) \\ \hat U_1 &= 170~{\rm V} \\ f &= 60~{\rm Hz} \\ R &= 220~{\rm \Omega} \\ L &= 325~{\rm mH} \end{align*}

1. Determine the RMS values of the voltage U_1 and of the currents I, I_R, and I_L.

SolutionResult

First convert the voltage amplitude into the RMS value: \begin{align*} U_1 &= \frac{\hat U_1}{\sqrt{2}} = \frac{170~{\rm V}}{\sqrt{2}} = 120.2~{\rm V} \end{align*}

The angular frequency is \begin{align*} \omega &= 2\pi f = 2\pi \cdot 60~{\rm s^{-1}} = 377.0~{\rm s^{-1}} \end{align*}

The inductive reactance is \begin{align*} \omega L &=

\begin{align*} U_1 &= 120.2~{\rm V} \\ I_R &= 0.546~{\rm A} \\ I_L &= 0.981~{\rm A} \\ I &= 1.123~{\rm A} \end{align*}

377.0~{\rm s^{-1}} \cdot 0.325~{\rm H} = 122.5~{\rm \Omega} \end{align*}

The resistor current is \begin{align*} I_R &= \frac{U_1}{R} = \frac{120.2~{\rm V}}{220~{\rm \Omega}} = 0.546~{\rm A} \end{align*}

The inductor current is \begin{align*} I_L &= \frac{U_1}{\omega L} = \frac{120.2~{\rm V}}{122.5~{\rm \Omega}} = 0.981~{\rm A} \end{align*}

Because I_R and I_L are perpendicular in the phasor diagram, the total current is \begin{align*} I &= \sqrt{I_R^2+I_L^2} \\ &= \sqrt{(0.546~{\rm A})^2 + (0.981~{\rm A})^2} \\ &= 1.123~{\rm A} \end{align*}

2. What active, reactive, and apparent power does the circuit absorb?

SolutionResult

The resistor absorbs only active power: \begin{align*} P &= \frac{U_1^2}{R} = \frac{(120.2~{\rm V})^2}{220~{\rm \Omega}} \\ &= 65.7~{\rm W} \end{align*}

The inductor absorbs only reactive power. For an inductor, reactive power is positive: \begin{align*} Q &= \frac{U_1^2}{\omega L} = \frac{(120.2~{\rm V})^2}{122.5~{\rm \Omega}} \\ &= 117.9~{\rm var} \end{align*}

\begin{align*} P &= 65.7~{\rm W} \\ Q &= 117.9~{\rm var} \\ S &= 135.0~{\rm VA} \end{align*}

The apparent power is \begin{align*} S &= U_1 I \\ &= 120.2~{\rm V}\cdot 1.123~{\rm A} \\ &= 135.0~{\rm VA} \end{align*}

Check with the power triangle: \begin{align*} S &= \sqrt{P^2+Q^2} = \sqrt{(65.7~{\rm W})^2+(117.9~{\rm var})^2} \\ &= 135.0~{\rm VA} \end{align*}

3. Determine the maximum and minimum value of the instantaneous power.

SolutionResult

For a sinusoidal AC circuit, the instantaneous power can be written as \begin{align*} p(t) = P + S\cos(2\omega t + \varphi_p) \end{align*}

Thus the oscillating part has the amplitude S, so \begin{align*} p_{\rm max} &= P + S \\ p_{\rm min} &= P - S \end{align*}

Insert the values: \begin{align*} p_{\rm max} &= 65.7~{\rm W} + 135.0~{\rm W} = 200.7~{\rm W} \\ p_{\rm min} &= 65.7~{\rm W} - 135.0~{\rm W} = -69.3~{\rm W} \end{align*}

The negative minimum value means that magnetic energy stored in the inductor is temporarily fed back to the source.

\begin{align*} p_{\rm max} &= 200.7~{\rm W} \\ p_{\rm min} &= -69.3~{\rm W} \end{align*}

Exercise E3 AC Filter and Sensor Front-End: Power Flow in a Composite Reactive Network

An industrial AC sensor front-end contains a compensation capacitor, a series inductor, and an output branch made of a resistor and another inductor. For thermal design and reactive-power assessment, the active, reactive, and apparent powers at the input and at each component shall be determined.

Data:

$$\begin{align*} \omega C &= 0.01~{\rm S} \\ \omega L_1 &= 50~{\rm \Omega} \\ \omega L_2 &= 200~{\rm \Omega} \\ R &= 100~{\rm \Omega} \\ \hat U_1 &= 325~{\rm V} \end{align*}$$

1. Calculate the active, reactive, and apparent power at the input and at the individual components of the circuit.

SolutionResult

For power calculations, we use RMS values. Therefore, the source voltage is \begin{align*} U_1 &= \frac{\hat U_1}{\sqrt{2}} = \frac{325~{\rm V}}{\sqrt{2}} = 229.8~{\rm V} \end{align*}

First determine the impedances of the individual elements: \begin{align*} \underline{Z}_C &= \frac{1}{j\omega C} = -j\frac{1}{\omega C} = -j100~{\rm \Omega} \\ \underline{Z}_{L_1} &= j\omega L_1 = j50~{\rm \Omega} \\ \underline{Z}_{L_2} &= j\omega L_2 = j200~{\rm \Omega} \end{align*}

Now the parallel branch: \begin{align*} \underline{Z}_{R\parallel L_2} &= \frac{R\underline{Z}_{L_2}}{R+\underline{Z}_{L_2}} \\ &= \frac{100\cdot j200}{100+j200} = 80+j40~{\rm \Omega} \end{align*}

Thus the total input impedance is \begin{align*} \underline{Z}_{\rm in} &= \underline{Z}_C + \underline{Z}_{L_1} + \underline{Z}_{R\parallel L_2} \\ &= (-j100) + (j50) + (80+j40) \\ &= 80-j10~{\rm \Omega} \end{align*}

The input current is therefore \begin{align*} \underline{I} &= \frac{\underline{U}_1}{\underline{Z}_{\rm in}} = \frac{229.8~{\rm V}}{80-j10~{\rm \Omega}} \\ &= 2.828 + j0.354~{\rm A} \end{align*}

Its magnitude is \begin{align*} I = 2.85~{\rm A} \end{align*}

The complex input power is \begin{align*} \underline{S}_{\rm in} &= \underline{U}_1 \underline{I}^{\,*} \\ &= 229.8 \cdot

\begin{align*} P_{\rm in} &= 650~{\rm W} \\ Q_{\rm in} &= -81.25~{\rm var} \\ S_{\rm in} &= 655.1~{\rm VA} \end{align*}

\begin{align*} P_C &= 0~{\rm W}, & Q_C &= -812.5~{\rm var}, & S_C &= 812.5~{\rm VA} \\ P_{L_1} &= 0~{\rm W}, & Q_{L_1} &= 406.25~{\rm var}, & S_{L_1} &= 406.25~{\rm VA} \\ P_R &= 650~{\rm W}, & Q_R &= 0~{\rm var}, & S_R &= 650~{\rm VA} \\ P_{L_2} &= 0~{\rm W}, & Q_{L_2} &= 325~{\rm var}, & S_{L_2} &= 325~{\rm VA} \end{align*}

(2.828-j0.354)~{\rm VA} \\ &= 650 - j81.25~{\rm VA} \end{align*}

Hence \begin{align*} P_{\rm in} &= 650~{\rm W} \\ Q_{\rm in} &= -81.25~{\rm var} \\ S_{\rm in} &= \sqrt{P_{\rm in}^2+Q_{\rm in}^2} = 655.1~{\rm VA} \end{align*}

Now calculate the powers of the individual components.

For the capacitor: \begin{align*} \underline{S}_C &= \underline{U}_C \underline{I}^{\,*} = -j812.5~{\rm VA} \end{align*} So \begin{align*} P_C &= 0~{\rm W} \\ Q_C &= -812.5~{\rm var} \\ S_C &= 812.5~{\rm VA} \end{align*}

For the inductor L_1: \begin{align*} \underline{S}_{L_1} &= \underline{U}_{L_1}\underline{I}^{\,*} = j406.25~{\rm VA} \end{align*} Thus \begin{align*} P_{L_1} &= 0~{\rm W} \\ Q_{L_1} &= 406.25~{\rm var} \\ S_{L_1} &= 406.25~{\rm VA} \end{align*}

The voltage across the parallel branch is \begin{align*} \underline{U}_{R\parallel L_2} &= \underline{I}\,\underline{Z}_{R\parallel L_2} \\ &= (2.828+j0.354)(80+j40) \\ &= 212.13 + j141.42~{\rm V} \end{align*}

The resistor current is \begin{align*} \underline{I}_R = \frac{\underline{U}_{R\parallel L_2}}{R} = 2.121 + j1.414~{\rm A} \end{align*}

The inductor current is \begin{align*} \underline{I}_{L_2} = \frac{\underline{U}_{R\parallel L_2}}{j200} = 0.707 - j1.061~{\rm A} \end{align*}

For the resistor: \begin{align*} \underline{S}_R &=

\underline{U}_{R\parallel L_2}\underline{I}_R^{\,*} = 650 + j0~{\rm VA} \end{align*} So \begin{align*} P_R &= 650~{\rm W} \\ Q_R &= 0~{\rm var} \\ S_R &= 650~{\rm VA} \end{align*}

For the inductor L_2: \begin{align*} \underline{S}_{L_2} &= \underline{U}_{R\parallel L_2}\underline{I}_{L_2}^{\,*} = j325~{\rm VA} \end{align*} Thus \begin{align*} P_{L_2} &= 0~{\rm W} \\ Q_{L_2} &= 325~{\rm var} \\ S_{L_2} &= 325~{\rm VA} \end{align*}

2. Verify the validity of the following reactive-power balance: \[Q = Q_C + Q_{L_1} + Q_{L_2} \]

SolutionResult

Insert the reactive powers found above: \begin{align*} Q_C &= -812.5~{\rm var} \\ Q_{L_1} &= 406.25~{\rm var} \\ Q_{L_2} &= 325~{\rm var} \end{align*}

Then \begin{align*} Q_C + Q_{L_1} + Q_{L_2} &= -812.5 + 406.25 + 325 \\ &= -81.25~{\rm var} \end{align*}

But the input reactive power is \begin{align*} Q_{\rm in} = -81.25~{\rm var} \end{align*}

Therefore, \begin{align*} Q_{\rm in} = Q_C + Q_{L_1} + Q_{L_2} \end{align*}

So the reactive-power balance is fulfilled.

\begin{align*} Q_C + Q_{L_1} + Q_{L_2} = -812.5 + 406.25 + 325 = -81.25~{\rm var} \end{align*}

\begin{align*} Q_{\rm in} = -81.25~{\rm var} \end{align*}

\begin{align*} Q_{\rm in} = Q_C + Q_{L_1} + Q_{L_2} \end{align*}

Exercise E4 Real Current-Sense Choke: Series Model of an Industrial Coil

A small current-sense choke in an industrial electronics module is used at low frequency for filtering and current shaping. In practice, the coil is not ideal: besides its inductance, it also has a winding resistance. Therefore, the real coil is modeled as a series connection of an inductance and an ohmic resistance.

Data: \begin{align*} L_{\rm sp} &= 2.5~{\rm mH} \\ R_{\rm sp} &= 100~{\rm m\Omega} \\ I_{\rm sp} &= 0.5~{\rm A} \\ f &= 50~{\rm Hz} \end{align*}

1. Draw the circuit and place all current and voltage phasors in the phasor diagram.

SolutionResult

The real coil is modeled as a series connection of \begin{align*} R_{\rm sp} \text{ and } L_{\rm sp} \end{align*}

Because this is a series circuit, the same current flows through both elements: \begin{align*} I_R = I_L = I_{\rm sp} = 0.5~{\rm A} \end{align*}

Choose the current as the reference phasor: \begin{align*} \underline{I}_{\rm sp} = 0.5~{\rm A}\angle 0^\circ \end{align*}

Then: \begin{align*} \underline{U}_R &\text{ is in phase with } \underline{I}_{\rm sp} \\ \underline{U}_L &\text{ leads } \underline{I}_{\rm sp} \text{ by } 90^\circ \\ \underline{U}_{\rm sp} &= \underline{U}_R + \underline{U}_L \end{align*}

So in the phasor diagram: \begin{align*} \underline{I}_{\rm sp}

\begin{align*} \underline{I}_{\rm sp} &= 0.5~{\rm A}\angle 0^\circ \\ \underline{U}_R &\parallel \underline{I}_{\rm sp} \\ \underline{U}_L &\text{ leads by }90^\circ \\ \underline{U}_{\rm sp} &= \underline{U}_R+\underline{U}_L \end{align*}

&: \text{horizontal to the right} \\ \underline{U}_R &: \text{same direction as } \underline{I}_{\rm sp} \\ \underline{U}_L &: \text{vertical upward} \\ \underline{U}_{\rm sp} &: \text{diagonal sum of } \underline{U}_R \text{ and } \underline{U}_L \end{align*}

2. Calculate the complex input impedance.

SolutionResult

First calculate the angular frequency: \begin{align*} \omega &= 2\pi f = 2\pi \cdot 50~{\rm s^{-1}} = 314.16~{\rm s^{-1}} \end{align*}

Then the inductive reactance is \begin{align*} X_L = \omega L_{\rm sp} = 314.16~{\rm s^{-1}} \cdot 2.5\cdot 10^{-3}~{\rm H} = 0.785~{\rm \Omega} \end{align*}

The complex input impedance of the real coil is \begin{align*} \underline{Z}_{\rm sp} &= R_{\rm sp} + jX_L \\ &= 0.100 + j0.785~{\rm \Omega} \end{align*}

Its magnitude is \begin{align*} \left|\underline{Z}_{\rm sp}\right| &= \sqrt{0.100^2+0.785^2} \\ &= 0.792~{\rm \Omega} \end{align*}

\begin{align*} \underline{Z}_{\rm sp} &= 0.100 + j0.785~{\rm \Omega} \\ \left|\underline{Z}_{\rm sp}\right| &= 0.792~{\rm \Omega} \end{align*}

3. How large is the voltage across the coil, and what is the phase-shift angle?

SolutionResult

The voltage across the winding resistance is \begin{align*} U_R &= I_{\rm sp} R_{\rm sp} = 0.5~{\rm A}\cdot 0.100~{\rm \Omega} = 0.050~{\rm V} \end{align*}

The voltage across the inductance is \begin{align*} U_L &= I_{\rm sp} X_L = 0.5~{\rm A}\cdot 0.785~{\rm \Omega} = 0.3927~{\rm V} \end{align*}

The total coil voltage is the vector sum: \begin{align*} U_{\rm sp} &= I_{\rm sp}\left|\underline{Z}_{\rm sp}\right| \\ &= 0.5~{\rm A}\cdot 0.792~{\rm \Omega} = 0.396~{\rm V} \end{align*}

The phase-shift angle between coil voltage and current is \begin{align*} \varphi &= \arctan\left(\frac{X_L}{R_{\rm sp}}\right) = \arctan\left(\frac{0.785}{0.100}\right) = 82.74^\circ \end{align*}

So the coil voltage leads the current by about 82.7°.

\begin{align*} U_R &= 0.050~{\rm V} \\ U_L &= 0.3927~{\rm V} \\ U_{\rm sp} &= 0.396~{\rm V} \\ \varphi &= 82.74^\circ \end{align*}

4. Calculate the active, reactive, and apparent power absorbed by the coil, and determine the power factor.

SolutionResult

The active power is dissipated only in the resistance: \begin{align*} P &= I_{\rm sp}^2 R_{\rm sp} \\ &=

\begin{align*} P &= 0.0250~{\rm W} \\ Q &= 0.196~{\rm var} \\ S &= 0.198~{\rm VA} \\ \lambda &= \cos\varphi = 0.126 \end{align*}

(0.5~{\rm A})^2 \cdot 0.100~{\rm \Omega} \\ &= 0.0250~{\rm W} \end{align*}

The reactive power is taken by the inductance: \begin{align*} Q &= I_{\rm sp}^2 X_L \\ &= (0.5~{\rm A})^2 \cdot 0.785~{\rm \Omega} \\ &= 0.196~{\rm var} \end{align*}

The apparent power is \begin{align*} S &= U_{\rm sp} I_{\rm sp} \\ &= 0.396~{\rm V}\cdot 0.5~{\rm A} \\ &= 0.198~{\rm VA} \end{align*}

The power factor is \begin{align*} \lambda = \cos\varphi = \frac{P}{S} = \frac{0.0250}{0.198} = 0.126 \end{align*}

5. Determine the loss factor and the loss angle.

SolutionResult

For a real inductor, the loss factor is \begin{align*} d = \frac{P}{Q} = \frac{R_{\rm sp}}{X_L} \end{align*}

Thus, \begin{align*} d &= \frac{0.100}{0.785} = 0.127 \end{align*}

The loss angle δ is related to the loss factor by \begin{align*} \tan\delta = d \end{align*}

Hence, \begin{align*} \delta &= \arctan(d) = \arctan(0.127) = 7.26^\circ \end{align*}

As a check: \begin{align*} \delta = 90^\circ - \varphi = 90^\circ - 82.74^\circ = 7.26^\circ \end{align*}

\begin{align*} d &= 0.127 \\ \delta &= 7.26^\circ \end{align*}

6. Draw the power triangle and indicate the loss angle.

SolutionResult

In the power triangle: \begin{align*} P &= 0.0250~{\rm W} \qquad \text{horizontal axis} \\ Q &= 0.196~{\rm var} \qquad \text{vertical axis upward} \\ S &= 0.198~{\rm VA} \qquad \text{hypotenuse} \end{align*}

The angle between S and the horizontal axis is \begin{align*} \varphi = 82.74^\circ \end{align*}

The loss angle is the complementary angle: \begin{align*} \delta = 7.26^\circ \end{align*}

So in the sketch, mark δ between the vertical reactive-power axis and the apparent-power vector.

\begin{align*} P &= 0.0250~{\rm W} \\ Q &= 0.196~{\rm var} \\ S &= 0.198~{\rm VA} \\ \varphi &= 82.74^\circ \\ \delta &= 7.26^\circ \end{align*}

Embedded resources

Explanation (video): …

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[1] For further development: CalcImpedances VarResistor varLC SineGenerator full simu

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