3. Linear sources and dipoles

Student Group

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Table of Contents

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It is known from everyday life that battery voltages drop under heavy load. This can be seen, for example, when turning the ignition key in winter: The load from the starter motor is sometimes so great that the low beam or radio briefly cuts out.
Another example are $1.5V$ batteries: If such a battery is short-circuited by a piece of wire, not so much current flows that the piece of wire glows, but noticeably less.

So it makes sense here to develop the concept of the ideal voltage source further. In addition, we will see that this also opens up a possibility to convert and simplify more complicated circuits.

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Fig. 1: passive two-pole

First, the concept of the two-terminal from the chapter basics and basic concepts is to be expanded (figure 1).

1. As **passive two-terminal** in the following a two-terminal is called, which acts exclusively as a consumer. Thus it is valid for the passive two-terminal that the current-voltage-characteristic always runs through the origin (see also chapter simple circuits).
2. **Active two-terminal circuits**, on the other hand, also act as generators of electrical energy. Thus, the current-voltage characteristic there does not pass through the origin. Active dipoles always contain at least one source (i.e. at least one current or voltage source).

3.1 linear sources

Goals

After this lesson, you should:

1. Be able to describe the difference between an ideal and a linear voltage or current source.
2. Know and be able to apply the relationship between output voltage, source voltage U_q and internal resistance R_i.
3. know and be able to apply the relationship between the current supplied, the source current I_q and the internal conductance G_i.
4. be able to represent the voltage curve of the linear (voltage/current) source using open-circuit voltage and short-circuit current.
5. be able to determine the open-circuit voltage and the short-circuit current using two current/voltage measuring points.
6. Be able to explain the reason for the duality of current and voltage sources.
7. Be able to convert a linear current source into a linear voltage source and vice versa.

DC Voltage & Current Source Theory

practical example of a realistic source

Fig. 2: Battery model with load resistor

For the ideal voltage source it was defined that it always supplies the same voltage independent of the load. In figure 2, in contrast, an example of a “realistic” voltage source is shown as an active two-terminal.

1. This active two-terminal generates a voltage of $1.5V$ and a current of $0A$ when the circuit is open.
2. If a resistor is added, the voltage decreases and the current increases. For example, a voltage of $1.2V$ is applied to the resistor of 2Ω and a current of $0.6A$ flows.
3. The terminals of the active two-terminal can be directly connected via the outer switch. Then a current of $3A$ flows at a voltage of $0V$.

This realization shall now be described with some technical terms:

- One speaks of **open circuit** when no current is drawn from an active two-terminal: $I_{LL}=0$.
  The voltage corresponds to the **open circuit voltage** $U=U_{LL}$ (English: OCV for Open Circuit Voltage).
  The open circuit power is $P_{LL}=U_{LL} \cdot I_{LL} = 0$.
- The term **short circuit** is used when the terminals of the two-pole are bridged without resistance. The current then flowing is called the **short-circuit current** $I=I_{KS}$.
  The short-circuit voltage is $U_{KS}=0V$.
  Also, the short-circuit power is $P_{KS}=U_{KS} \cdot I_{KS} = 0$.
- In the region between no-load and short-circuit, the active two-terminal outputs power to a connected load.

Important: As will be seen in the following, the short-circuit current inside the two-terminal can cause considerable power loss and thus a lot of waste heat. Not every real two-terminal is designed for this.

Fig. 3: Current-voltage characteristic of a linear voltage source

What is interesting now is the current-voltage characteristic of the circuit in figure 2. This can be seen in the simulation below. The result is a linear curve (see figure 3).

From a purely mathematical point of view, the course can be represented by the basic equation of linear graphs with the y-axis intercept I_{KS} and a slope of $-{{I_{KS}}\over{U_{LL}}}$:

$$\begin{align*} I = I_{KS} - {{I_{KS}}\over{U_{LL}}}\cdot U \tag{3.1.1} \end{align*}$$

On the other hand, the formula can also be resolved to U:

$$\begin{align*} U = U_{LL} - {{U_{LL}}\over{I_{KS}}}\cdot I \tag{3.1.2} \end{align*}$$

Remember:

If a two-terminal results in a linear curve between U_{LL} and I_{KS}, it is called a **linear source**. This curve describes in good approximation the behavior of many real sources. Often one finds synonymous to the term 'linear source' also the term 'real (voltage) source'. However, this is somewhat misleading as it is a simplified model for reality.

Fig. 4: equivalent circuit images of linear sources

So what does the inside of the linear source look like? In figure 4 two possible linear sources are shown, which will be considered in the following.

Linear voltage source

The linear voltage source consists of a series connection of an ideal voltage source with the source voltage U_0 (English: EMF for Electro-Magnetic-Force) and the internal resistance R_i. To determine the voltage outside the active two-terminal, the system can be considered as a voltage divider. The following applies:

$$\begin{align*} U = U_0 - R_i \cdot I \end{align*}$$

The source voltage U_0 of the ideal voltage source is to be measured at the terminals of the two-terminal, if this is unloaded. Then no current flows through the internal resistor R_i and there is no voltage drop there. Therefore: The source voltage is equal to the open circuit voltage $U_0 = U_{LL}$.

$$\begin{align*} U = U_{LL} - R_i \cdot I \end{align*}$$

When the external voltage $U=0$, it is the short circuit case. In this case, $0 = U_{LL} - R_i \cdot I_{KS}$ and transform $R_i = {{U_{LL}}\over{I_{KS}}}$. Thus, equation $(3.1.2)$ is obtained: \begin{align*} U = U_{LL} - {{U_{LL}}\over{I_{KS}}} \cdot I \end{align*}

Is this the structure of the linear source we are looking for? To verify this, we will now look at the second linear source.

Linear current source

The linear current source now consists of a parallel circuit of an ideal current source with source current I_0 and internal resistance R_i, or internal conductance $G_i = {{1}\over{R_i}}$. To determine the voltage outside the active two-terminal, the system can be considered as a current divider. Here, the following holds:

\begin{align*} I = I_0 - G_i \cdot U \end{align*}

Here, the source current can be measured at the terminals in the event of a short circuit. The following therefore applies: $I_{KS}= I_0$

\begin{align*} I = I_{KS} - G_i \cdot U \end{align*}

When the external current $I=0$, it is the no-load case. In this case, $0 = I_{KS} - G_i \cdot U_{LL}$ and transform $G_i = {{I_{KS}}\over{U_{LL}}}$.
Thus, equation $(3.1.1)$ is obtained: \begin{align*} I = I_{KS} - {{I_{KS}}\over{U_{LL}}} \cdot U \end{align*}

So it seems that the two linear sources describe the same thing.

Duality of linear sources

Fig. 5: duality of linear sources

Through the previous calculations, we came to the interesting realization that both the linear voltage source and the linear current source provide the same result. It is true: For a linear source, both a linear voltage source and a linear current source can be specified as an equivalent circuit! As already in the case of the star-delta transformation, this not only provides two explanations for a black box. Also here linear voltage sources can be transformed into linear linear current sources and vice versa.

The figure 5 compares again the two linear sources and their characteristics:

1. The linear voltage source is given by the source voltage U_0, or the open circuit voltage U_{LL} and the internal resistance R_i.
2. The linear current source is given by the source current I_0, or the short-circuit current I_{KS} and the internal conductance G_i.

The conversion is now done in such a way that the same characteristic curve is obtained:

1. From linear voltage source to linear current source:
   Given: Source voltage U_0, or open circuit voltage U_{LL}, internal resistance R_i
   \Looked for: source current I_0, or short circuit current I_{KS} , internal conductance G_i
   $\boxed{I_{KS} = {{U_{LL}}\over{R_i}}}$, $\boxed{G_i = {{1}\over{R_i}}}$
2. From linear current source to linear voltage source:
   Given: Source current I_0, resp. short-circuit current I_{KS} , internal resistance G_i
   Sought: Source voltage U_0, resp. open-circuit voltage U_{LL}, internal resistance R_i $
   $\boxed{U_{LL} = i_ks\over{G_i}}}$, $\boxed{R_i = 1_over_g_i}$

Operating point of a real voltage source

Fig. 6: Source and consumer characteristics

figure 6 shows the characteristics of the linear voltage source (left) and a resistive resistor (right). For this purpose, both are connected to a test system in the simulation: In the case of the source with a variable ohmic resistor, in the case of the load with a variable source. The characteristic curves formed in this way were described in the previous chapter.

Fig. 7: Determining the operating point

The operating point can be determined from both characteristic curves. This is assumed when both the linear voltage source is connected to the ohmic resistor (without the respective test systems). In figure 7 both characteristic curves are drawn in a current-voltage diagram. The point of intersection is just the operating point that sets in. If the load resistance is varied, the slope changes in inverse proportion and a new operating point is established (light grey in the figure).

The derivation of the working point is also here explained again in a video.

Fig. 8: Straight line arrays for source parameter variation

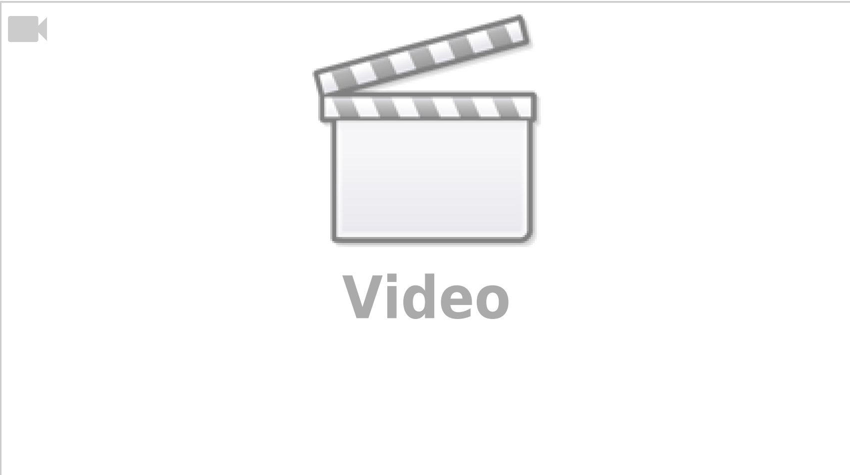
The variation of the different source parameters will be briefly discussed.

For the linear current source, the source current I_0 and the internal conductance G_i can be varied. This results in the straight line arrays in figure 8 above. The source current shifts the straight

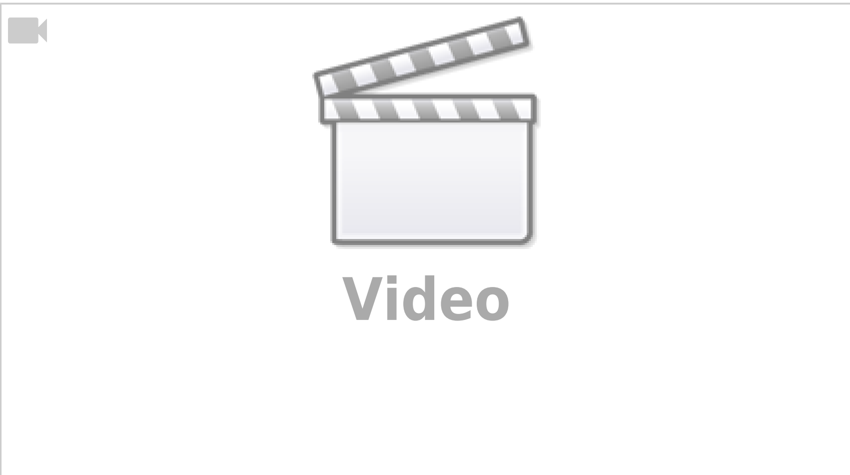
lines while keeping the slope constant. The internal conductance changes only the slope; this results in a straight line array around the intersection $I_0 = I_{KS}$.
Since an ideal current source should always supply the source current, its internal conductance $G_i=0$.

For the linear voltage source, the source voltage U_0 and the internal resistance R_i can be varied. This results in the straight line arcs in figure 8 below. The source voltage shifts the straight lines while keeping the slope constant. The internal resistance changes only the slope; this results in a straight line array around the intersection $U_0 = U_{LL}$.
Since an ideal voltage source should always supply the source voltage, its internal resistance $R_i=0$.

Task 3.1.1 Convert current source to voltage source



Task 3.1.2 Convert voltage source to current source



3.2 Conversion of any linear two-terminal

Objectives

After this lesson you should:

1. know that any linear circuit with two connections of ohmic resistors and sources can be understood as a linear current source or linear voltage source.
2. Be able to apply source conversion to more complicated circuits with multiple current sources or voltage sources.
3. know how to determine the open circuit voltage U_{LL} and the short circuit current I_{KS}).
4. be able to calculate the parameters of the equivalent voltage source (internal resistance R_i and source voltage U_q) of any linear circuit.
5. understand and be able to draw the graphical interpretation of voltage and current at the linear two-terminal in the form of a characteristic curve.

Fig. 5: Resistance of linear sources

In figure 5, it can be seen that the internal resistance of the linear current source measured by the ohmmeter (resistance meter) is exactly equal to that of the linear voltage source.

If you look at the properties of the ohmmeter in the simulation, you will see that a measuring current is used there to determine the resistance value. This concept will still be part of the electrical engineering lab experiment on resistors in the 2nd semester. However, a very large measuring current of $1A$ is used here. This could lead to high voltages or destruction of components in real setups.

Why is this nevertheless chosen so high in the simulation? Set the measuring current for both linear sources to (more realistic) $1mA$. What do you notice?

Fig. 10: circuit with two current sources

The circuit in figure 10 shows this circuit again. The ohmmeter is replaced by a current source and a voltmeter, since only the electrical properties are important in the following. In this setup, it can be seen that the current through G_i is just given by $I_i = I_0 + I_\Omega$ (node theorem). Thus, the two sources in the circuit can be reduced.

This should make the situation clear with a measuring current of $1mA$. The voltage at the resistor is now given by $U_\Omega = R \cdot (I_0 + I_\Omega)$. Only when I_Ω is very large does I_0 become negligible. The current of a conventional ohmmeter cannot guarantee this for every measurement.

Note:

If resistors are to be measured in a circuit, at least one terminal of the resistor must be disconnected from the circuit. Otherwise, other sources or resistors may falsify the measurement result.

More complex example

Fig. 8: circuit with multiple sources

But this knowledge can now be used for more complicated circuits. In figure 8 such a circuit is drawn. This is to be converted into a searched equivalent conductance G_g and a searched equivalent current source with I_g.

Important here: Only two-terminals can be converted via source duality. This means that only 2 nodes may act as output terminals for selected sections of the circuit. If there are more nodes the conversion is not possible.

1. As a first step, sources are to be converted in such a way that resistors can be combined after the conversion. In this example this is done by:
   1. converting the linear voltage source R_1 and U_1 into a linear current source with $I_1={{U_1}\over{R_1}}$ and R_1 (or $G_1={{1}\over{R_1}}$)
   2. converting the linear current source R_4 and I_4 into a linear voltage source with $U_4={{I_4}\cdot{R_4}}$ and R_4

2. In the second step, the linear voltage source U_4 formed in 1. with R_4 can be connected to the resistor R_3. From this again a linear current source can be created. This now has a resistance of $R_5 = R_3+R_4$ and an ideal current source with $I_5 = {{U_4}\over{R_3+R_4}}= {{I_4}\cdot{R_4}\over{R_3+R_4}} $.

3. The circuit diagram that now emerges is a parallel circuit of ideal current sources and resistors. This can be used to determine the values of the ideal equivalent current source and the

equivalent resistance:

1. ideal equivalent current source I_g: \begin{align*} I_g = I_1 + I_3 + I_5 = I_1 + I_3 + I_4\cdot{{R_4}\over{R_3+R_4}} \end{align*}
2. Substitute conductance G_g: \begin{align*} G_g = \Sigma G_i = {{1}\over{R_1}}+{{1}\over{R_2}}+{{1}\over{R_5}}={{1}\over{R_1}}+{{1}\over{R_2}}+{{1}\over{R_3+R_4}} \end{align*}

Note:

Fig. 12: Spare voltage and current source

Any interconnection of linear voltage sources, current sources, and ohmic resistors can be.

- as a single, linear voltage source (Thévenin theorem) or
- as a single, linear current source (Norton theorem)

In figure 12 it can be seen that the three circuits give the same result (voltage / current) with the same load. This is also true when an (AC) source is used instead of the load.

Simplified determination of the internal resistance

Note:

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Fig. 13: equivalent resistance of ideal sources

If only the equivalent resistance of a more complex circuit is sought, the following approach can be used:

1. Replace all ideal voltage sources by a short circuit (= internal resistance of the ideal voltage source).
2. Replace all ideal current sources by an open contact (= internal resistance of the ideal current source)
3. Add the remaining resistors to an equivalent resistance using the rules for parallel and series connection.

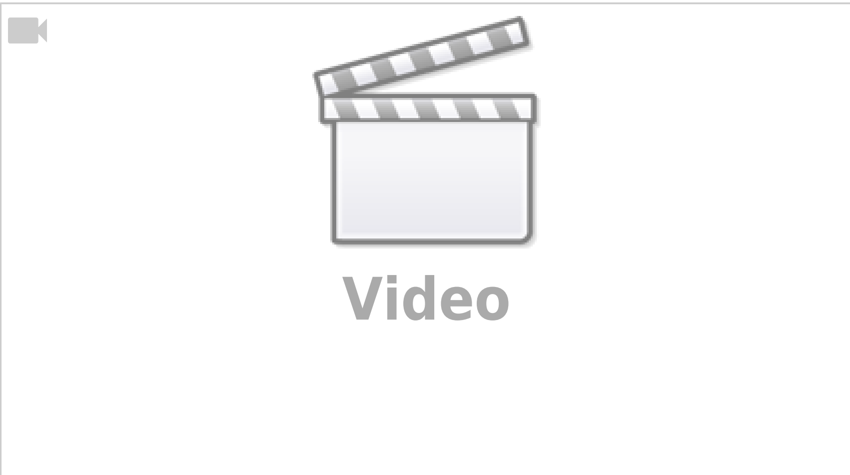
The equivalent circuits for the ideal sources can be seen via the circuit diagrams (see figure 13).

Thus also the equivalent resistance of the complex circuit above can be derived quickly.
For the source current I_0 ideal equivalent current source resp. the source voltage U_0 ideal equivalent voltage source this derivation can not be used.

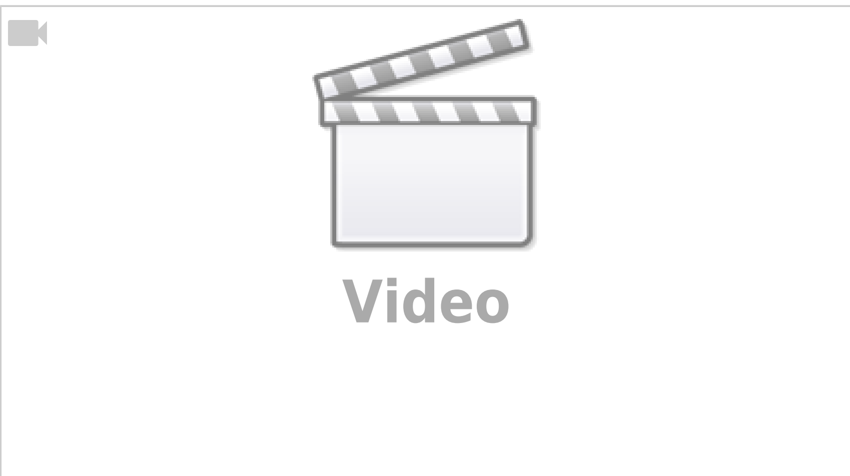
The reason that the internal resistance can be determined in this simple way will be explained in the next chapter analysis_of_dc_networks: Superposition method is explained.

Fig. 14: Simplified determination of internal resistance

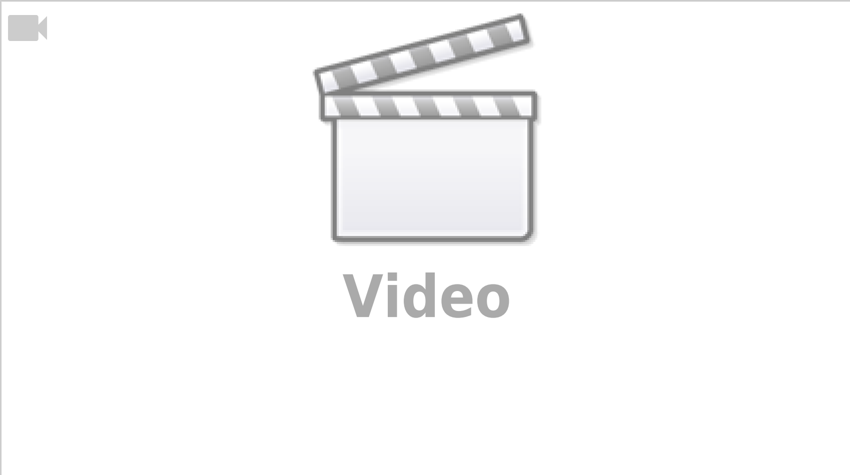
Task 3.2.1 Solving a circuit simplification I



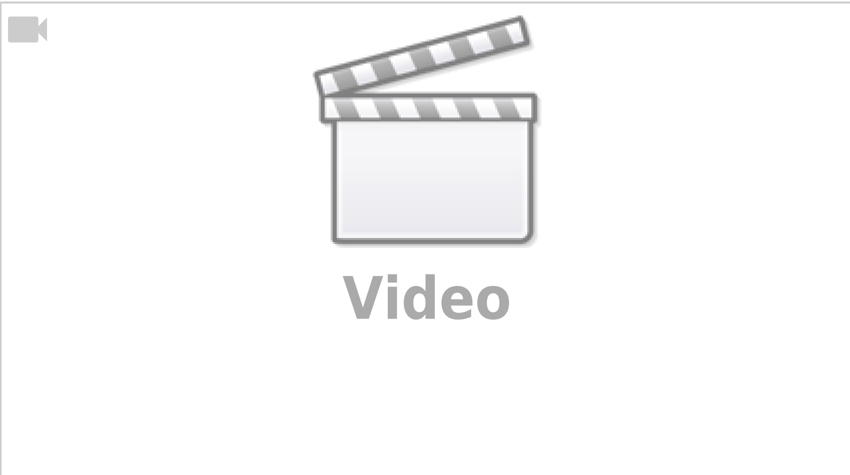
Task 3.2.2 Solving a circuit simplification II



Task 3.2.3 Solution sketch for a more difficult circuit simplification



Task 3.2.4 3 short but interesting circuit tasks



3.3 Power at two poles and reference values

> **Goals**
>
> After this lesson, you should:
>
> 1. be able to calculate the source power and consumer power.
> 2. be able to distinguish between the optimisation objectives for power engineering and communications engineering.
> 3. be able to calculate the efficiency and utilization factor.

Power and efficiency have already been considered in 1st chapter and 2nd chapter for a simple dc circuit. In the following, this will be analyzed again with the knowledge of two-terminals. This is especially important for the fields of communications and power engineering. The goals here are different:

1. In power engineering, power transmission is the goal. Power is thus to be delivered without losses as far as possible.

2. In communications engineering, the focus is on information transmission. So that, for example, the best possible signal can be extracted from an antenna, the maximum power must be extracted here.

These two goals seem similar at first, but they are quite different, as will be seen in a moment.

Power measurement

Fig. 15: Power measurement on linear voltage source

First, it is necessary to consider how to determine the power. The power meter (or wattmeter) consists of a combined ammeter and voltmeter.

In figure 15 the wattmeter with the circuit symbol can be seen as a round element with crossed measuring inputs. The circuit also shows one wattmeter each for the (not externally measurable) output power of the ideal source P_Q and the input power of the load P_R.

Power and reference variables in the diagram

Fig. 16: power adjustment

The simulation in figure 16 shows the following:

- The circuit with linear voltage source (U_0 and R_i), and a resistive load R_L.
- A simulated wattmeter, where the ammeter is implemented by a measuring resistor R_S (English: shunt) and a voltage measurement for U_S. The power is then: $P_L = {{1}\over{R_S}}\cdot U_S \cdot U_L$.
- in the oscilloscope section (below).
  - On the left is the power P_L plotted against time in a graph.
  - On the right is the already known current-voltage diagram of the current values.
- The slider `Load resistance R_L`, with which the value of the load resistance R_L can be changed.

Now try to vary the value of the load resistance R_L (slider) in the simulation so that the maximum power is achieved. Which resistance value is set?

Fig. 17: current-voltage diagram, power-voltage diagram and efficiency-voltage diagram

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figure 17 shows three diagrams:

- Diagram top: current-voltage diagram of linear voltage source.
- Diagram in the middle: source power P_Q and consumer power P_L versus delivered voltage U_L.
- Diagram below: Reference quantities over delivered voltage U_L.

The two powers are defined as follows:

- source power: $\, \, \large{ P_Q = U_0 \cdot I_L} $
- consumer power: $\large{ P_L = U_L \cdot I_L} $

1. Both power P_Q and P_L are equal to 0 without current flow. The source power becomes maximum, at maximum current flow, that is, when the load resistance $R_L=0$. In this case, all the power flows out through the internal resistor. The efficiency drops to 0%. This is the case, for example, with a battery shorted by a wire.
2. If the load resistance becomes just as large as the internal resistance $R_L=R_i$, the result is a voltage divider where the load voltage becomes just half the open circuit voltage: $U_L = {{1}\over{2}}\cdot U_{LL}$. On the other hand, the current is also half the short-circuit current $I_L=I_{KS}$, since the resistance at the ideal voltage source is twice that in the short-circuit case.
3. If the load resistance becomes high impedance $R_L\rightarrow\infty$, less and less current flows, but more and more voltage drops across the load. Thus, the efficiency increases and approaches 100% for $R_L\rightarrow\infty$.

The whole context can be seen in a extensive Simulation will be analyzed again.

The reference values efficiency and utilisation rate

In order to understand the lower diagram in figure 17, the definition equations of the two reference quantities shall be described here again:

The **efficiency** η describes the delivered power (consumer power) in relation to the supplied power (power of the ideal source): \begin{align*}\eta = {{P_{out}}\over{P_{in}}} = {{R_L\cdot I_L^2}\over{(R_L+R_i) \cdot I_L^2}} \quad \Rightarrow \quad \boxed{ \eta = {{R_L}\over{R_L+R_i}} } \end{align*}

The **efficiency** ε describes the delivered power in relation to the maximum possible power of the ideal source. Here, the currently supplied power is not assumed (as in the case of efficiency), but the best possible power of the ideal source, i.e. in the short-circuit case: \begin{align*}\varepsilon = {{P_{out}}\over{P_{in,max}}} = {{R_L\cdot I_L^2}\over{{U_0^2}\over{R_i}}} = {{R_L\cdot R_i \cdot I_L^2} \over {U_0^2}} = {{R_L\cdot R_i \cdot \left({{U_0}\over{R_L+R_i}}\right)^2} \over {U_0^2}} \quad \Rightarrow \quad \boxed{\varepsilon = {{R_L\cdot R_i } \over {(R_L+R_i)^2}} = {{R_L} \over {(R_L+R_i)}}\cdot {{R_i} \over {(R_L+R_i)}}} \end{align*}

In power engineering a situation close to (1.) in figure 17 is desired: maximum power output with lowest losses at the internal resistance of the source. Thus, the internal resistance of the source should be low compared to the load $R_L \gg R_i $. The efficiency should go towards $\eta \rightarrow 100\%$.

In communications engineering, one situation is different and corresponds to situation (2.): The maximum power is to be taken from the source, without consideration of the losses via the internal resistance. For this purpose, the internal resistance of the source (e.g., a receiver) and the load (e.g., the downstream evaluation) are matched. This case is called **{wpde>power matching| power matching or resistance matching}}**. The utilization factor here becomes maximum: $\varepsilon = 25\%$

The power adjustment is also here explained again in a video.